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Physics

for the IB Diploma
Exam Preparation Guide

Second Edition

K. A. Tsokos

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- assist students in approaching complex questions, applying critical-thinking skills and forming reasoned answers.

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

Information on this title: education.cambridge.org

First published 2016

Printed in the United Kingdom by Latimer Trend

A catalogue record for this publication is available from the British Library

ISBN 9781316629802 Exam Preparation Guide Cambridge Elevate edition
ISBN 9781107495760 Exam Preparation Guide Digital edition

All questions, answers and annotations have been written by the author. In examinations, the way marks are awarded may be different.

CONTENTS

INTRODUCTION

This Exam Preparation Guide for IB Physics supports you through the whole course. It provides concise and straightforward coverage of the syllabus, alongside specially designed guidance to help you to apply your knowledge when you get into the exams.

It is intended to be used alongside a Coursebook throughout the IB Physics course; in particular as you prepare to sit your exams at the end of course.

In addition to the concise coverage of the full syllabus, the Exam Preparation Guide also includes annotated exemplar answers, model answers and hints as well as lots of questions to test your understanding and track your progress.

Preparing for the examination

When preparing for the examination, make sure that you have seen as many past papers and mark schemes as possible. It is important that you use mark schemes as well, so that you can understand how the examiners will mark your paper. Past papers and mark schemes are not confidential information and should be made readily available to you by your school.

Sitting the examination

During the examination itself, you must pay attention to the following points for each of the three papers. For a start, make sure you have a ruler with you! It is a vital tool and will make life easier for you.

Paper 1

It is very important to really read the question carefully! Some of the questions are tricky and play on the meaning of words.

On many occasions most, if not all, of the four possible answers may be correct statements. However, you must choose the option that answers the question and not one that is merely a correct statement. When compressing a gas, the molecules collide more frequently with each other, but that does not explain why the pressure goes up.

Do not spend too much time on any one question. Remember that these are supposed to be questions that you can answer quickly. Long algebraic or arithmetical calculations are unnecessary in this paper and if you cannot get the answer quickly, the chances are that you will not get it even if you spend a long time on the question.

If you cannot choose the correct answer, see if you can eliminate those that are obviously wrong.

Sometimes you can choose the correct answer just by checking the units of the answer. If you are looking for a force, your chosen answer must have units of force!

Do not leave any blank answers. Guess if necessary!

Papers 2 and 3

Read the questions carefully and answer what is being asked. Perfect answers to questions that are not being asked do not gain you any points.

Pay attention to the command terms. This determines the amount of detail required in the answer. If the question says ‘state’, a simple sentence will do. An essay type answer is not required. If the question says ‘explain’ or ‘discuss’, a lot of detail is required. Feel free to answer the question in your own way and in your own words. However, don’t overdo it by including extra and irrelevant information. The examiner may deduct points if you say something that is incorrect or contradictory to things mentioned elsewhere.

Pay attention to the number of lines allotted to each question, which also determines to a large extent the quantity to be provided in the answer.

Watch your significant figures (round numbers at the very end and not in the intermediate stages of a calculation) and don’t forget to include units for your final answer.

Know your calculator well. The examination room is not the place to learn how to use a calculator.

Know the definitions of key terms well. If you have to use an equation in lieu of a definition, do so, but remember to define all the symbols appearing in the equation.

Section A of paper 3 contains questions based on experimental work. Remember that a ‘line’ of best fit is not necessarily a straight line.

For all three papers, pay special attention to the axes of any graphs. Often, the units for a quantity are expressed with a power of 10. Thus if the x co-ordinate of a point is 2.0 but the axis is labelled / $\times 10^{-3}$ m the value you use is $x = 2.0 \times 10^{-3}$ m.

This only applies to Paper 3. Do not plan to go to the exam hoping to answer questions on an option that you have not studied or one that you have studied by yourself outside the classroom. Do the best you can with the option studied in your school even if you do not find it interesting and you would have preferred to have studied something else.

K. A. Tsokos

Acknowledgements

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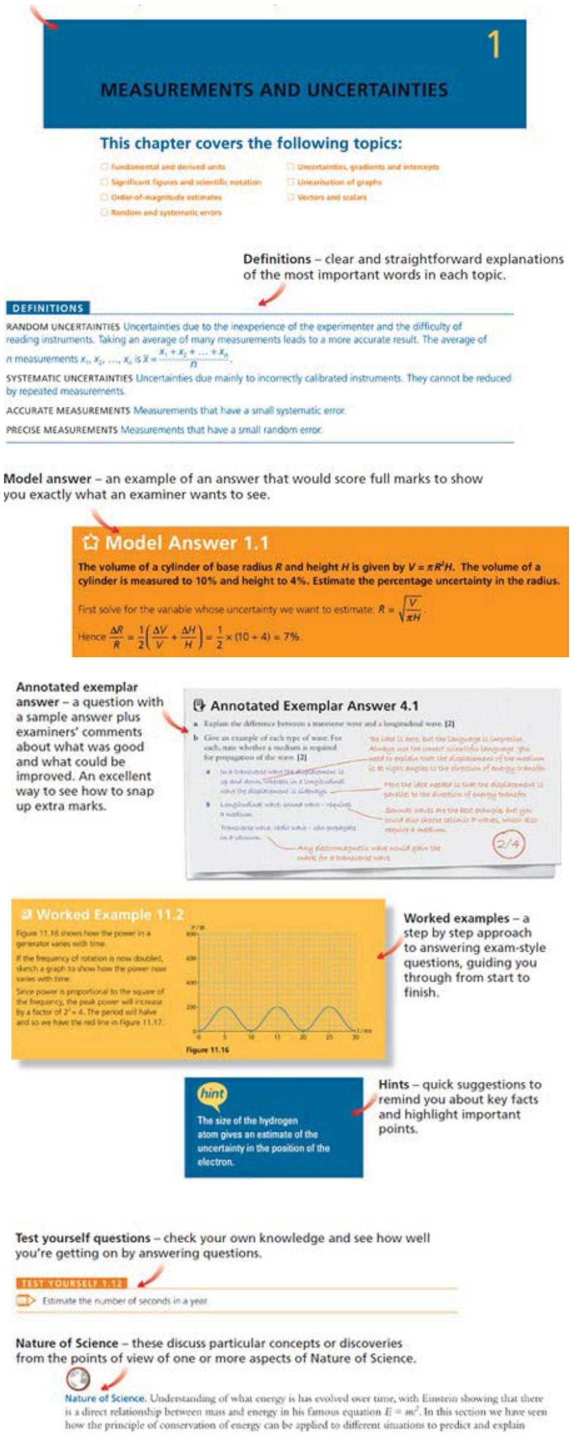
For Alexios and Alkeos and in memory of my parents

HOW TO USE THIS RESOURCE: A GUIDED TOUR

Introduction – sets the scene of each chapter, helps with navigation through the resource and gives a reminder of what's important about each topic.



1 MEASUREMENTS AND UNCERTAINTIES

This chapter covers the following topics:

- □ Fundamental and derived units
- □ Significant figures and scientific notation
- □ Order-of-magnitude estimates
- □ Random and systematic errors
- □ Uncertainties, gradients and intercepts
- □ Linearisation of graphs
- □ Vectors and scalars

1.1 Units

It is a fascinating fact that all physical quantities have units that can be expressed in terms of those for just seven **fundamental** quantities.

DEFINITIONS

FUNDAMENTAL UNITS The seven fundamental quantities in the S.I. system (the IB syllabus uses only the first six) and their units are:

• Time	second (s)
• Length	meter (m)
• Mass	kilogram (kg)
• Temperature	kelvin (K)
• Quantity of matter	mole (mol)
• Electric current	ampere (A)
• Luminous intensity	candela (cd)

DERIVED UNITS All other quantities have **derived** units, that is, combinations of the fundamental units. For example, the derived unit for force (the newton, N) is obtained using $F = ma$ to be kg ms^{-2} and that for electric potential difference (the volt, V) is obtained using $W = qV$ to be $\frac{\text{J}}{\text{C}} = \frac{\text{Nm}}{\text{As}} = \frac{\text{kgms}^{-2}\,\text{m}}{\text{As}} = \text{kgm}^2\text{s}^{-3}\text{A}^{-1}$.

SIGNIFICANT FIGURES There is a difference between stating that the measured mass of a body is 283.64 g and saying it is 283.6 g. The implication is that the uncertainty in the first measurement is ±0.01 g and that in the second is ±0.1 g. That is, the first measurement is more precise – it has more **significant figures** (s.f.). When we do operations with numbers (multiplication, division, powers and roots) we must express the result to the same number of s.f. as in the least precisely known number in the operation (Table 1.1).

Table 1.1

Number		Number of s.f.	Scientific notation
34		2	3.4×10^1
3.4		2	3.4×10^0 or just 3.4
0.0340	Zeros in front do not count but zeros at the end *in a decimal* do count.	3	3.4×10^{-2}
340	Zeros at the end *in an integer* do not count.	2	3.4×10^2

Thus the kinetic energy of a mass of 2.4 kg (2 s.f.) moving at 14.6 m s^{-1} (3 s.f.) is given as $E_K = \frac{1}{2} \times 2.4 \times 14.6^2 = 255.792\text{J} \approx 260{=}2.6{\times}10^2\,\text{J}$ (2 s.f.). Similarly, the acceleration of a body of mass 1200 kg (2 s.f.) acted upon by a net force of 5250 N (3 s.f.) is given as $\frac{5250}{1200} = 4.375 \approx 4.4\text{ms}^{-2}$ (2 s.f.).

TEST YOURSELF 1.1

The force of resistance from a fluid on a sphere of radius r is given by $F = 6\pi\eta rv$, where v is the speed of the sphere and η is a constant. What are the units of η?

TEST YOURSELF 1.2

The radius R of the fireball t seconds after the explosion of a nuclear weapon depends only on the energy E released in the explosion, the density ρ of air and the time t. Show that the quantity $\frac{Et^2}{\rho}$ has units of m^5 and hence that $R \approx \left(\frac{Et^2}{\rho}\right)^{\frac{1}{5}}$. Calculate the energy released if the radius of the fireball is 140 m after 0.025 s. (Take $\rho = 1.0\text{kgm}^{-3}$.)

1.2 Uncertainties

DEFINITIONS

RANDOM UNCERTAINTIES Uncertainties due to the inexperience of the experimenter and the difficulty of reading instruments. Taking an average of many measurements leads to a more accurate result. The average of n measurements $x_1, x_2, \ldots, x_n$ is $\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$.

SYSTEMATIC UNCERTAINTIES Uncertainties due mainly to incorrectly calibrated instruments. They cannot be reduced by repeated measurements.

ACCURATE MEASUREMENTS Measurements that have a small systematic error.

PRECISE MEASUREMENTS Measurements that have a small random error.

Nature of Science. A key part of the scientific method is recognising the errors that are present in the experimental technique being used, and working to reduce these as much as possible. In this section you have learned how to calculate errors in quantities that are combined in different ways and how to estimate errors from graphs. You have also learned how to recognise systematic and random errors.

No matter how much care is taken, scientists know that their results are uncertain. However, they need to distinguish between inaccuracy and uncertainty, and to know how confident they can be about the validity of their results. The search to gain more accurate results pushes scientists to try new ideas and refine their techniques. There is always the possibility that a new result may confirm a hypothesis for the present, or it may overturn current theory and open a new area of research. Being aware of doubt and uncertainty are key to driving science forward.

Normally we express uncertainties to just one significant figure. However, if a more sophisticated statistical analysis of the data has taken place there is some justification for keeping two significant figures.

In general, for a quantity Q we have

$$Q = \underbrace{Q_0}_{\text{measured value}} \pm \underbrace{\Delta Q}_{\text{absolute uncertainty}}, \quad \frac{\Delta Q}{Q_0} = \text{fractional uncertainty}, \quad \frac{\Delta Q}{Q_0} \times 100\% = \text{percentage uncertainty}$$

As an example, consider a measurement of the length of the side of a cube, given as 25 ± 1 mm. The 25 mm represents the **measured value** of the length and the ±1 mm represents the **absolute uncertainty** in the measured value. The ratio $\frac{1}{25} = 0.04$ is the **fractional uncertainty** in the length, and $\frac{1}{25} \times 100\% = 4\%$ is the **percentage uncertainty** in the length.

Suppose quantities a, b and c have been measured with absolute uncertainties, respectively, of Δa, Δb and Δc. If we use a, b and c to calculate another quantity Q, these uncertainties will result in an uncertainty in Q. The (approximate) rules for calculating the uncertainty ΔQ in Q are:

- If $Q = a \pm b \pm c$, then $\Delta Q = \Delta a + \Delta b + \Delta c$. That is, for addition and subtraction, **add the absolute uncertainties**.
- If $Q = \frac{ab}{c}$, then $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$. That is, for multiplication and/or division **add the fractional uncertainties**.
- If $Q = \frac{a^n}{b^m}$, then $\frac{\Delta Q}{Q} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b}$. In particular, if $Q = \sqrt{ab}$ or $Q = \sqrt{\frac{a}{b}}$, then $\frac{\Delta Q}{Q} = \frac{1}{2}\frac{\Delta a}{a} + \frac{1}{2}\frac{\Delta b}{b}$.

Model Answer 1.1

The volume of a cylinder of base radius R and height H is given by $V = \pi R^2 H$. The volume of a cylinder is measured to 10% and height to 4%. Estimate the percentage uncertainty in the radius.

First solve for the variable whose uncertainty we want to estimate: $R = \sqrt{\frac{V}{\pi H}}$

Hence $\frac{\Delta R}{R} = \frac{1}{2}\left(\frac{\Delta V}{V} + \frac{\Delta H}{H}\right) = \frac{1}{2} \times (10 + 4) = 7\%$

TEST YOURSELF 1.3

The resistance of a lamp is given by $R = \frac{V}{I}$. The uncertainty in the voltage is 4% and the uncertainty in the current is 6%. What is the absolute uncertainty in a calculated resistance value of 24Ω?

TEST YOURSELF 1.4

Each side of a cube is measured with a fractional uncertainty of 0.02. Estimate the percentage uncertainty in the volume of the cube.

TEST YOURSELF 1.5

The period of oscillation of a mass m at the end of a spring of spring constant k is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

What is the percentage uncertainty in the period if m is measured with a percentage uncertainty of 4% and the k with a percentage uncertainty of 6%?

hint

Avoid the common mistake of saying that the uncertainty is

$$\sqrt{(4\% + 6\%)} \approx 3\%.$$

Error bars

Suppose that we want to plot the point (3.0 ± 0.1, 5.0 ± 0.2) on a set of x and y axes. First we plot the point with coordinates (3.0, 5.0) and then show the uncertainties as error bars (Figure 1.1). The horizontal error bar will have length 2 × 0.1 = 0.2 and the vertical will have length 2 × 0.2 = 0.4.

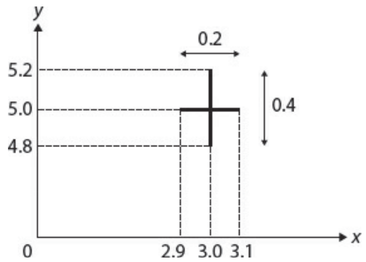


Figure 1.1

DEFINITIONS

BEST-FIT LINE The curve or straight line that goes through all the error bars; an example is shown in Figure 1.2. Note that a ‘line’ may be straight or curved.

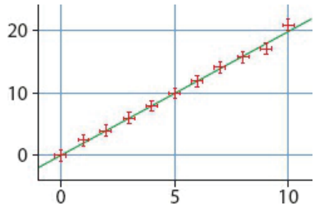


Figure 1.2

Finding slopes

To find the slope (or gradient) of a curve at a particular point (here at $x = 1.0 \times 10^{-2}$ m), draw the tangent to the curve at that point (Figure 1.3).

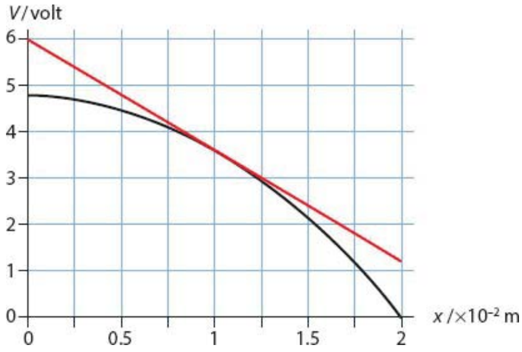


Figure 1.3

Choose two points *on the tangent* that are as far apart as possible. Note in this case that the units on the horizontal axis are 10^{-2} m, and that the slope has a negative value.

$$\text{Slope}=\frac{6.0-1.2}{0.0-2.0\times10^{-2}}\frac{\text{volt}}{\text{m}}$$
$$=-2.4\times10^{2}\,\text{Vm}^{-1}$$

Estimating areas under curves

To estimate the area under the black curve in Figure 1.4, draw a straight line (red) from the point (0, 6) to the point (4, 1.5).

Figure 1.4

It is easy to calculate the area of the trapezium under the straight line, as $\frac{(6+1.5)}{2}\times4=15.0.$.

Now estimate the number of small squares in the space between the straight line and the curve, and subtract this from the total, to give the area under the curve. There are about 53 squares.

Each one has area of 0.25 × 0.25 = 0.0625 square units, so the area between the curve and the straight line is about 53 0.0625 = 3.3.

So the area under the curve is about 15.0 − 3.3 = 11.7 square units.

Getting straight-line graphs

If we know the relationship between two variables we can usually arrange to plot the data in such a way that we get a straight line. Bear in mind that the standard equation of a straight line is

$$Y=\underset{\text{gradient}}{m}\,x+\underset{\text{vertical int ercept}}{c}$$

where we plot the variable *y* on the vertical axis and the variable *x* on the horizontal axis.

If the straight line goes through the origin (*c* = 0), we say that *y* is **proportional** to *x*.

If the best-fit line is not straight or if it does not go through the origin, then *either* of these reasons is sufficient to claim that *y* is *not* proportional to *x*.

As an example, consider the relationship $T=2\pi\sqrt{\frac{m}{k}}$ for the period *T* of a mass *m* undergoing oscillations at the end of a spring of spring constant *k*. Compare this equation and the general straight-line equation:

$$\begin{array}{ccc} T= & \frac{2\pi}{\sqrt{k}}\times & \sqrt{m} \\ \downarrow & & \downarrow \\ Y= & \underset{\text{constants}}{\frac{2\pi}{\sqrt{k}}}\times & x \end{array}$$

By identifying $T\leftrightarrow y$ and $\sqrt{m}\leftrightarrow x$ we get the equation of a straight line, $Y=\frac{2\pi}{\sqrt{k}}x$. So if we plot *T* on the vertical axis and $\sqrt{m}$ on the horizontal axis we should get a straight line whose gradient is $\frac{2\pi}{\sqrt{k}}$. Alternatively, we may write:

$$\begin{array}{ccc} T^2 = & \frac{4\pi^2}{k} \times & m \\ \downarrow & & \downarrow \\ Y = & \underbrace{\frac{4\pi^2}{k}}_{\text{constants}} \times & x \end{array}$$

By identifying $T^2 \leftrightarrow y$ and $m \leftrightarrow x$ we get the equation of a straight line, $Y = \frac{4\pi^2}{k}x$. So if we plot T^2 on the vertical axis and m on the horizontal axis we should get a straight line whose gradient is $\frac{4\pi^2}{k}$

A different procedure must be followed if the variables are related through a power relation such as $F = kr^n$, where the constants k and n are unknown. Taking natural logs (or logs to any other base), we have:

$$\begin{array}{lcr} \ln F & = \ln k + n \times & \ln r \\ \downarrow & & \downarrow \\ Y & = \ln k + n \times & x \end{array}$$

and so plotting ln F versus ln r should give a straight line with gradient n and vertical intercept ln k.

A variation of this is used for an exponential equation such as $A = A_0 e^{-\lambda t}$, where A_0 and λ are constants. Here we can take the logs of both sides to get $\ln A = \ln A_0 - \lambda t$, and so:

$$\begin{array}{lcr} \ln A & = \ln A_0 - \lambda \times & t \\ \downarrow & & \downarrow \\ Y & = \ln A_0 - \lambda \times & x \end{array}$$

Plotting ln A on the vertical axis and t on the horizontal then gives a straight line with gradient $-\lambda$ and vertical intercept ln A_0.

TEST YOURSELF 1.6

Copy Table 1.2 and fill in the blank entries.

Table 1.2

Equation	Constants	Variables to be plotted to give straight line	Gradient	Vertical intercept
$P = kT$	k			
$v = u + at$	u, a			
$v^2 = 2as$	a			
$F = \frac{kq_1q_2}{r^2}$	k, q_1, q_2			
$a = -\omega^2 x$	ω^2			
$V = \frac{kq}{r}$	k, q			
$T^2 = \frac{4\pi^2}{GM}R^3$	G, M			
$I = I_0 e^{-aT}$	I_0, a			
$\lambda = \frac{h}{\sqrt{2mqV}}$	h, m, q			
$F = av + bv^2$	a, b			
$E = \frac{1}{2}m\omega^2\sqrt{A^2 - x^2}$	m,ω^2, A			
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	f			

TEST YOURSELF 1.7

State what variables must be plotted so that we get a straight line for the relation $d = ch^{0.8}$, where c is a constant.

Estimating uncertainties in measured quantities

Useful simple rules are for estimating the uncertainty in a measured quantity is:

For *analogue meters*, use half of the smallest scale division. For example, for an ordinary meter rule the smallest scale division is 1 mm and so the uncertainty is ±0.5 mm. If this is used, for example, to measure the length of a rod, this uncertainty applies to the position of each end of the rod, for a total uncertainty of ±1 mm in the rod's length.

For *digital meters*, use the smallest division. For example, with a digital voltmeter that can read to the nearest hundredth of a volt, take the uncertainty as ±0.01 V. For an ammeter that can read to the nearest tenth of an ampere, take the uncertainty as ±0.1A.

TEST YOURSELF 1.8

Estimate the reading and the uncertainty in each of the instruments in Figure 1.5.

Figure 1.5

Annotated Exemplar Answer 1.1

The period of a pendulum is measured to be $T = (2.20 \pm 0.05)$ s. Calculate the value of T^2, including its uncertainty. **[3]**

$T^2 = 2.20^2 = 4.84\ s^2$ — The value of T^2 is correct, and with the correct units.

$\Delta T^2 = 0.05^2 = 0.0025\ s^2$ — The value of ΔT is 0.05 s, but you cannot square the uncertainty in T to find ΔT^2, the uncertainty in T^2. Make sure you know how to find fractional uncertainties when there are powers, and remember the protocol for the number of significant figures in the uncertainty.

So $T^2 = (4.84 \pm 0.0025)\ s^2$.

The final answer gains no marks because the wrong method was used to find ΔT. One way to spot errors like this is to ask if the final answer is sensible. Here the uncertainty is given to 2 s.f. in the third and fourth decimal places, when the value has two decimal places, so the answer must be wrong. The correct answer is $T^2 = (4.8 \pm 0.2)\ s^2$.

Here the fractional uncertainty in T is $\frac{\Delta T}{T} = \frac{0.05}{2.20} = 0.022 \approx 0.02$ (rounded to 1 s.f. as the uncertainty is greater than 2%). The power in T^2 is 2, so multiply the fractional uncertainty in T by 2 to find the fractional uncertainty in T^2, that is, $0.02 \times 2 = 0.04$. So $\Delta T^2 = 0.04 \times 4.84 \approx 0.2\ s^2$.

1/3

Uncertainty in the measured value of a gradient (slope)

To find the uncertainty in the gradient of the (straight) best-fit line, draw lines of maximum and minimum gradient. You must judge these by eye, taking into account *all* error bars, not just those of the first and last data points. Calculate these two gradients, m_{max} and m_{min}. A simple estimate of the uncertainty in the gradient is then $\frac{m_{max} - m_{min}}{2}$

TEST YOURSELF 1.9

Electrons that have been accelerated through a potential difference V enter a region of magnetic field B, where they are bent into a circular path of radius r. Theory suggests that $r^2 = \frac{2m}{qB^2}V$, where q is the electron's charge and m is its mass. Table 1.3 shows values of the potential difference V and radius r obtained in an experiment.

Table 1.3

Radius r / cm ±0.1 cm	Potential difference V / V	r^2 / cm^2
4.5	500	±
4.9	600	±
5.3	700	±
5.7	800	±

6.0	900	±

a Explain why a graph of r^2 against V will result in a straight line.
b State the slope of the straight line in **a** in terms of the symbols m, q, B.
c Copy Table 1.3 and in the right-hand column insert values of the radius squared, including its uncertainty.

Figure 1.6 shows the data points plotted on a set of axes.

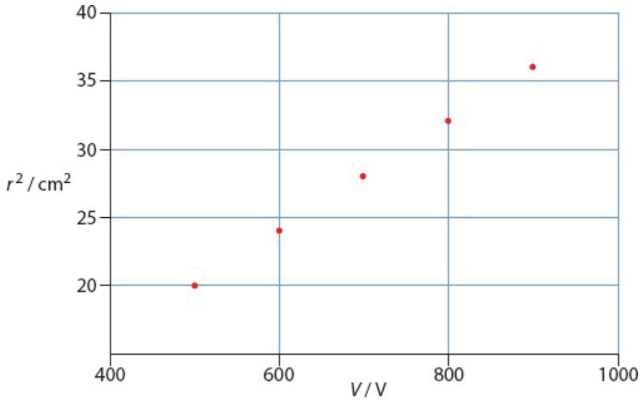


Figure 1.6

d Draw error bars for the all the data points.
e Draw a best-fit line for these data points.
f Calculate the gradient of the best-fit line, including its uncertainty.

The magnetic field used in this experiment was $B = 1.80 \times 10^{-3}$ T.

g Calculate the value that this experiment gives for the charge-to-mass ratio $\left(\frac{q}{m}\right)$ of the electron. Include the uncertainty in the calculated value.

< Back to chapter

Annotated Exemplar Answer 1.1

The period of a pendulum is measured to be $T = (2.20 \pm 0.05)$ s. Calculate the value of T^2, including its uncertainty. **[3]**

$T^2 = 2.20^2 = 4.84\ s^2$

$\Delta T^2 = 0.05^2 = 0.0025\ s^2$

So $T^2 = (4.84 \pm 0.0025)\ s^2$.

The final answer gains no marks because the wrong method was used to find ΔT. One way to spot errors like this is to ask if the final answer is sensible. Here the uncertainty is given to 2 s.f. in the third and fourth decimal places, when the value has two decimal places, so the answer must be wrong. The correct answer is $T^2 = (4.8 \pm 0.2)\ s^2$.

The value of T^2 *is correct, and with the correct units.*

The value of ΔT is 0.05 s, but you cannot square the uncertainty in T to find ΔT^2*, the uncertainty in* T^2*. Make sure you know how to find fractional uncertainties when there are powers, and remember the protocol for the number of significant figures in the uncertainty. Here the fractional uncertainty in T is* $\frac{\Delta T}{T} = \frac{0.05}{2.20} = 0.022 \approx 0.02$ *(rounded to 1 s.f. as the uncertainty is greater than 2%). The power in* T^2 *is 2, so multiply the fractional uncertainty in T by 2 to find the fractional uncertainty in* T^2*, that is, 0.02 × 2 = 0.04. So* $\Delta T^2 = 0.04 \times 4.84 \approx 0.2\ s^2$.

$$\Delta T^2 = 0.04 \times 4.84 \approx 0.2 s^2$$

1/3

1.3 Vectors and scalar quantities

DEFINITIONS

VECTOR A physical quantity that has both magnitude and direction. It is represented by arrows. The length of the arrow gives the magnitude of the vector. The direction of the arrow is the direction of the vector. Examples of vectors are displacement, velocity, acceleration, force, momentum and electric/gravitational/magnetic fields.

SCALARS A physical quantity with magnitude but not direction. A scalar can be positive or negative. Examples are distance, speed, mass, time, work/energy, electric/gravitational potential and temperature.

Adding vectors: have ***a*** and ***b*** start at the same point, O (Figure 1.7a). Draw the parallelogram whose two sides are ***a*** and ***b***. Draw the diagonal starting at O.

Subtracting vectors: have ***a*** and ***b*** start at the same point, O (Figure 1.7b). To find ***b*** − ***a*** draw the vector from the tip of ***a*** to the tip of ***b***.

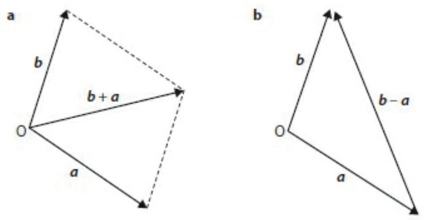


Figure 1.7 a Addition; **b** subtraction.

Components of vectors

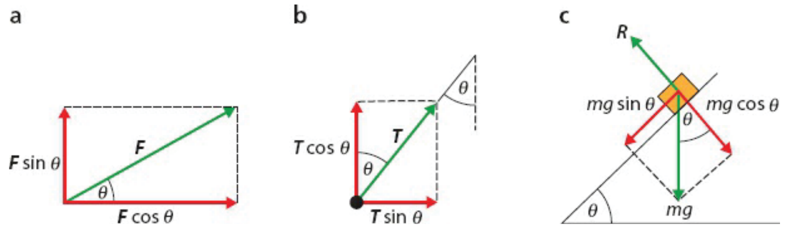


Figure 1.8 Components of vectors.

The component adjacent to the angle θ involves cos θ and that opposite to θ involves sin θ.

- Draw the forces.
- Put axes.
- Get components.
- Choose as one of your axis the direction in which the body moves or would move if it could.

TEST YOURSELF 1.10

A river is 16 m wide. A boat can travel at 4.0 ms^{-1} with respect to the water and the current has a speed of 3.0 ms^{-1} with respect to the shore, directed to the right (Figure 1.9). The boat is rowed in such a way as to arrive at the opposite shore directly across from where it started. Calculate the time taken for the trip.

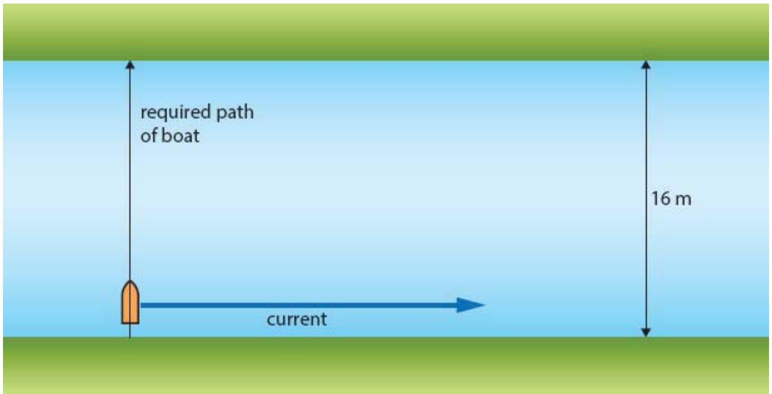


Figure 1.9

1.4 Order-of-magnitude estimates

Tables 1.4, 1.5 and 1.6 give typical values for various distances, masses and times. You are not expected to know these by heart but you must have a *general* idea of such sizes, masses and durations.

Table 1.4

Radius of observable universe	Length / m
Radius of observable universe	10^{26}
Distance to the Andromeda galaxy	10^{22}
Diameter of the Milky Way galaxy	10^{21}
Distance to Proxima Centauri (star)	10^{16}
Diameter of solar system	10^{13}
Distance to the Sun	10^{11}
Radius of the Earth	10^{7}
Size of a cell	10^{-5}
Size of a hydrogen atom	10^{-10}
Size of an average nucleus	10^{-15}
Planck length	10^{-35}

Table 1.5

The universe	Length / m
The universe	10^{53}
The Milky Way galaxy	10^{41}
The Sun	10^{30}
The Earth	10^{24}
Boeing 747 (empty)	10^{5}
An apple	0.2
A raindrop	10^{-6}
A bacterium	10^{-15}
Mass of smallest virus	10^{-21}
A hydrogen atom	10^{-27}
An electron	10^{-30}

Table 1.6

	Time / s
Age of the universe	10^{17}
Time of travel for light from nearby star (Proxima Centauri)	10^{8}
One year	10^{7}
One day	10^{5}
Period of a heartbeat	1
Period of red light	10^{-15}
Time of passage of light across an average nucleus	10^{-23}
Planck time	10^{-43}

TEST YOURSELF 1.11

Estimate the weight of an apple.

TEST YOURSELF 1.12

Estimate the number of seconds in a year.

TEST YOURSELF 1.13

Estimate the time taken by light to travel across a nucleus.

TEST YOURSELF 1.14

Estimate the time in between two of your heartbeats.

TEST YOURSELF 1.15

Estimate how many grains of sand can fit in the volume of the Earth.

TEST YOURSELF 1.16

Estimate the number of water molecules in a glass of water.

TEST YOURSELF 1.17

If the temperature of the Sun were to increase by 2% and the distance between the Earth and the Sun were to decrease by 1%, by how much would the intensity of the radiation received on Earth change?

hint

Use propagation of uncertainties ideas.

Checklist

After studying this chapter you should be able to:

- ☐ use fundamental and derived units
- ☐ use scientific notation
- ☐ determine the correct number of significant figures in a calculation
- ☐ perform order-of-magnitude estimates
- ☐ distinguish between random and systematic errors
- ☐ estimate absolute and percentage uncertainties
- ☐ determine the variables to be plotted in order to get a straight-line graph
- ☐ distinguish between vectors and scalars
- ☐ add and subtract vectors
- ☐ find the components of a vector, and reconstruct a vector from its components.

2 MECHANICS

This chapter covers the following topics:

- □ Kinematical quantities and equations
- □ Analysis of motion graphs
- □ Projectile motion
- □ Forces and equilibrium
- □ Newton's laws of motion
- □ Momentum, impulse and momentum conservation
- □ Work, energy and power

2.1 Kinematics

Kinematics is the study of motion using equations and graphs. Refer to Figure 2.1 for key terms.

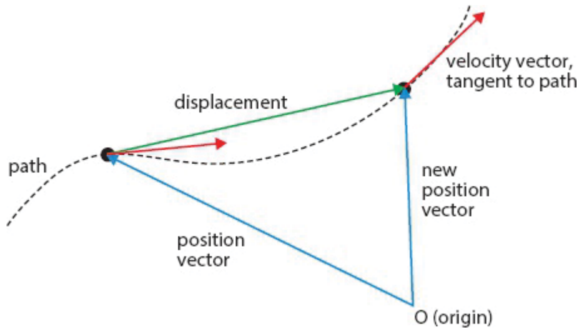


Figure 2.1

DEFINITIONS

POSITION VECTOR The vector ***r*** from some arbitrary fixed point (called the origin) to the position of a particle – that is, the distance in a given direction.

DISPLACEMENT The change $\Delta \boldsymbol{r}$ in the position vector. It is also a vector.

DISTANCE The length of the path travelled. This is a scalar quantity.

AVERAGE VELOCITY The ratio $\frac{\Delta r}{\Delta t}$ of the displacement to the time taken. This vector has the same direction as $\Delta \boldsymbol{r}$.

AVERAGE SPEED The ratio $\frac{d}{t}$ of the **total** distance travelled d to the **total** time t taken. This is a scalar. Note that the average speed is not, in general, related to the magnitude of the average velocity.

INSTANTANEOUS VELOCITY $v = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$, the rate of change with time of the displacement. Instantaneous velocity is a vector that is tangent to the path.

INSTANTANEOUS SPEED The rate of change with time of the distance travelled: $\text{speed} = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$. It is equal to the magnitude of the instantaneous velocity vector.

AVERAGE ACCELERATION The ratio $\frac{\Delta v}{\Delta t}$ of the change of the velocity vector to the time taken. This vector is in the same direction as $\Delta \boldsymbol{v}$.

INSTANTANEOUS ACCELERATION $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$, the rate of change with time of the velocity vector.

TEST YOURSELF 2.1

A particle moves on a circle with constant speed. The position vector of the particle is measured from point O, the position of the particle at $t = 0$; see Figure 2.2.

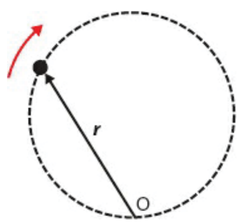


Figure 2.2

Draw a graph to show how the magnitude r of the position vector varies with time for one complete revolution.

> *hint*
>
> You only need a simple sketch, showing how the magnitude of the position vector changes as the particle moves. No calculations are needed.

TEST YOURSELF 2.2

A particle takes 5.0s to move along a semicircle of radius 5.0 m; see Figure 2.3.

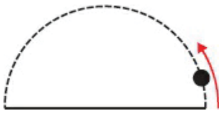


Figure 2.3

Calculate the average velocity and average speed.

> hint
> Be sure you understand the difference between average speed and average velocity.

Motion in a straight line: formulas and graphs

The basic formulas for motion in a straight line with constant acceleration are:

$$\begin{aligned} v &= u + at \\ \Delta s &= ut + \tfrac{1}{2}at^2 \\ v^2 &= u^2 + 2a\Delta s \\ \Delta s &= \left(\tfrac{u+v}{2}\right) t \end{aligned}$$

Here, u is the initial velocity, v is the velocity after a time t, Δs is the displacement and t is the elapsed time. Remember that these formulas apply only to straight-line motion with constant acceleration. If the acceleration is not constant, we must rely on the analysis of graphs.

Graphs

Information that may be extracted from a graph includes:

- the slope of a displacement–time graph, giving the velocity
- the slope of a velocity–time graph, giving the acceleration
- the area under a velocity–time graph, giving the displacement
- the area under an acceleration–time graph, giving the change in velocity.

TEST YOURSELF 2.3

An object starts from rest and moves off with constant acceleration. It covers a distance of 20m in 5.0s. What is its speed after 5.0s?

a 2.0 ms^{-1}

b 4.0 ms^{-1}

c 8.0 ms^{-1}

d 10.0 ms^{-1}

> hint
> You need a formula without acceleration.

TEST YOURSELF 2.4

Figure 2.4 is a graph of the variation with time of the velocity of a particle.

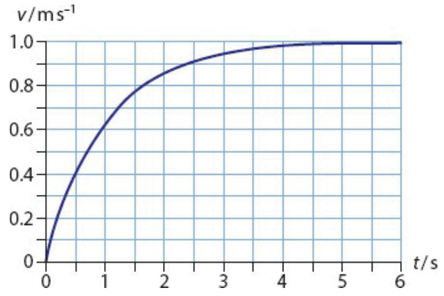


Figure 2.4

a Calculate the instantaneous acceleration at 2.0 s.
b Draw a sketch graph to show the variation with time of the acceleration of the particle.
c Estimate the displacement at 6.0s.

hint

This is not simply the velocity at 2.0 s divided by the time.

TEST YOURSELF 2.5

Figure 2.5 shows the variation with time of the acceleration of a particle. The particle starts from rest. Determine its velocity after 5.0s.

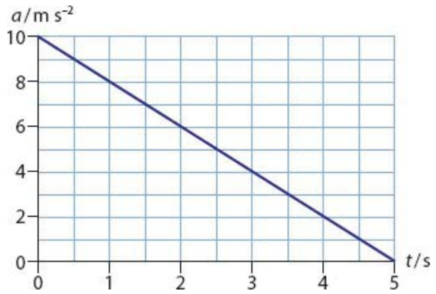


Figure 2.5

hint

The area under the curve is the change in velocity.

Projectile motion

A projectile launched with speed *u* at some angle to the horizontal, as in Figure 2.6, will follow a parabolic path in the absence of air resistance. We treat the horizontal and vertical components of the motion independently; see Figure 2.7.

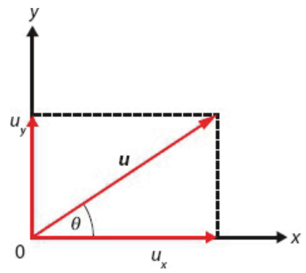


Figure 2.6

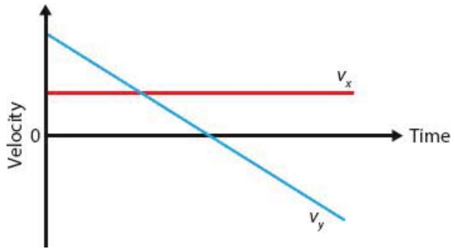


Figure 2.7

Horizontal motion: There is no acceleration in this direction and so:

$$v_x = u\cos\theta \qquad x = ut\cos\theta$$

Vertical motion: Here we have acceleration of magnitude *g* (free fall) in the vertically downward direction and so:

$$v_\gamma = u\sin\theta - gt \qquad \gamma = ut\sin\theta - \frac{1}{2}gt^2$$

TEST YOURSELF 2.6

A ball is kicked horizontally with a speed of 15 ms^{-1} from a table that is 0.80 m above the floor. Calculate:

a the time it takes the ball to hit the ground

b the horizontal distance travelled

c the angle the velocity of the ball makes with the horizontal as the ball lands on the floor.

hint

Apply the kinematic equations separately for the *x* and *y* directions. Consider the vertical motion to get the time during which the ball is in the air.

Energy considerations

Many problems in projectile motion can be solved faster using energy conservation than with the full kinematic equations, as Worked example 2.1 shows.

Worked Example 2.1

A stone is thrown off the edge of a cliff with a speed u = 12 ms^{-1} at an angle of 25° above the horizontal. The cliff is h = 30 m above the sea. Determine:

a the velocity with which the stone hits the water (take g = 9.8 ms^{-2})

b the angle at impact.

a At the point where the stone is thrown, the total energy of the stone is $E = \frac{1}{2}mu^2 + mgh$. As it hits the sea, the total energy is $E = \frac{1}{2}mv^2$. Equating the two energies gives $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$, or $v = \sqrt{u^2 + 2gh}$, so $v = 27.1 \approx 27\text{ms}^{-1}$

b The horizontal component of velocity is 12 × cos25° = 10.9 ms^{-1}. Since this remains constant, this is also the horizontal component at impact. Hence the vertical component is $\sqrt{27.1^2 - 10.9^2} = 24.8 \approx 25\text{ms}^{-1}$. The angle at impact is then $\tan^{-1}\frac{24.8}{10.9} \approx 66°$ below the horizontal.

Air resistance

As shown in Figure 2.8, the effect of air resistance on the path of a thrown object (red circles) is to reduce both the maximum height and the maximum range, making the angle of descent steeper, and distorting the shape away from parabolic (blue circles).

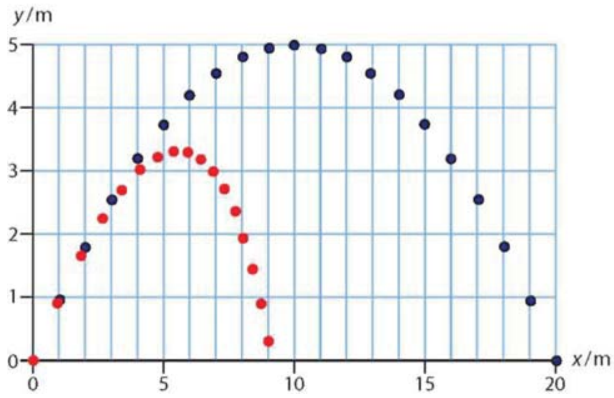


Figure 2.8

2.2 Forces and Newton's first law

DEFINITIONS

REPRESENTATION OF FORCES Forces, being vectors, are represented by arrows whose length shows the magnitude of the force. The direction of the arrow gives the direction of the force.

NET (RESULTANT) FORCE The single force whose effect is the same as the combined effect of all the individual forces on the body. This force is found by *vector addition*. The magnitude of the resultant of two forces of, say, magnitude 5 N and 7 N is at most 12 N (if the forces are in the same direction) and no less than 2 N (if they are in opposite directions); in any other case it is always between these extremes.

NEWTON'S FIRST LAW OF MOTION When the net force on a body is zero, the body moves with *constant velocity*. It is impossible to do an experiment in a box that is closed (i.e. you cannot look outside) and moving at constant velocity that will determine what that velocity is.

EQUILIBRIUM The state when the net (resultant) force on a body or system is zero.

Worked Example 2.2

Figure 2.9a shows a block of weight 100 N that is suspended by two strings of unequal length.

Estimate the tension in each string. The downward arrow represents the tension in the vertical string.

The two tensions must have a resultant that is equal and opposite to the weight of the block. This is shown by the vertical upward arrow in Figure 2.9b. Then, drawing dashed lines parallel to the strings, we find the tensions in each of them. The scale is 1 square = 20N. Measuring each with a ruler and comparing to the length of the weight, we find $\boldsymbol{T}_1$ = 80N and $\boldsymbol{T}_2$ = 56N.

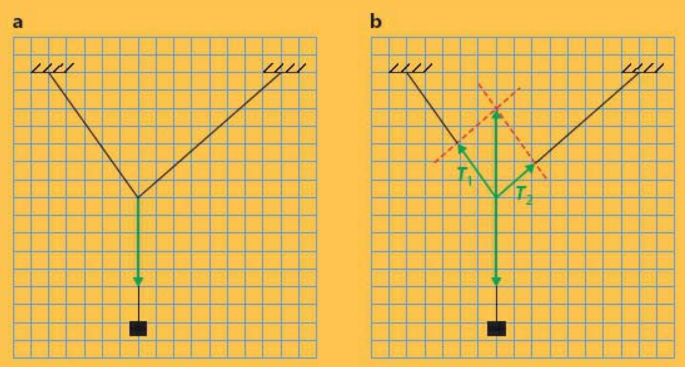


Figure 2.9

hint

Do you know the tension in the vertical string?

TEST YOURSELF 2.7

An elevator is moving downward at constant velocity. A person drops a ball from rest. Will the time it takes the ball to hit the elevator floor be less than, equal to or larger than the time the ball would have taken in an elevator at rest?

hint

You can solve the problem either by looking at things from the point of view of the observer inside the elevator or from the point of view of an observer outside. Choose one.

Friction

A frictional force occurs when a body slides over another body, in which case we speak of **dynamic friction.** The frictional force is directed opposite to the velocity and its magnitude is given by and $f = \mu_d N$, where N is the magnitude of the normal force acting on the body and μ_d is the coefficient of dynamic friction.

A frictional force can also occur when a force acts on a body but the body does not move, in which case we speak of **static friction.** There is no formula for the static frictional force, but there is one for the *largest possible* frictional force that can develop between two bodies. This is $f_{max} = \mu_s N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal reaction force between the bodies. Note that this formula gives the maximum possible static frictional force, not the frictional force in general.

Worked Example 2.3

A block of weight 3.0N rests on an inclined plane. The angle of the incline is 30 °. The coefficient of dynamic friction between the block and the plane is 0.58.

a Calculate the static frictional force between the plane and the block.

b The angle of the incline is slowly increased. When the angle barely exceeds 42°, the body begins to slide down the plane. Determine **i** the static coefficient of friction between the body and the plane, and **ii** the net force on the block as it slides down the plane.

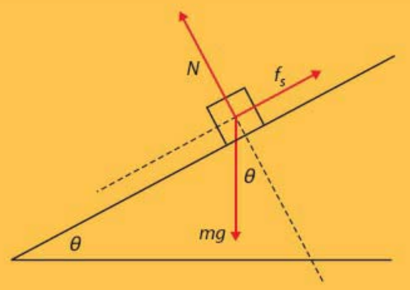


Figure 2.10

Figure 2.10 shows the forces on the block.

a Analysing the weight into components along and normal to the plane, we find that, at equilibrium:
$mg \sin\theta = f_s$

$$N = mg\cos\theta$$

Hence, $f_s = mg \sin\theta = 3.0 \times \sin 30° = 1.5$N.
(Note that the expression $f_s = \mu_s N$ gives the **maximum possible** static frictional force. The actual static frictional force is not always as large as this.)

b i Because the largest angle at which equilibrium occurs is 42°, the frictional force in this case must be the largest possible and so is given by $f_{max} = \mu_s N$. So we have that:
$mg \sin 42° = \mu_s N$
$N = mg \cos 42°$
This leads to $mg \sin 42° = \mu_s mg \cos 42°$ and so $\mu_s = \tan 42° = 0.90$.

ii The frictional force is now the dynamic frictional force since we have sliding. The angle of the incline is just barely larger than 42°. Since $N = mg \cos 42° = 3.0 \times \cos 42° = 2.23$N, the frictional force is $f_d = \mu_d N = 0.58 \times 2.23 = 1.29 \approx 1.3$N.
The net force is then $mg \sin 42° - f_d = 3.0 \times \sin 42° - 1.29 = 0.717 \approx 0.72$N.

2.3 Newton’s second law

Newton’s second law states that the net force on a body of constant mass is equal to the product of the mass and its acceleration: $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$.

Note that $\boldsymbol{F}$ and $\boldsymbol{a}$ are vector quantities and are in the same direction.

In solving mechanics problems, the following steps are helpful:

- Draw a diagram of the situation.
- Show all the forces acting on each body (free-body diagram).
- Find the net force acting on each body.
- Apply Newton’s second law *separately* to each body.

In some cases it may be convenient to treat all the bodies as one, as Test yourself 2.8 shows.

TEST YOURSELF 2.8

A block of mass M is on a horizontal, frictionless table and is connected by a string to a smaller mass m that hangs from the string; see Figure 2.11. If m is released, find the acceleration and the tension in the string.

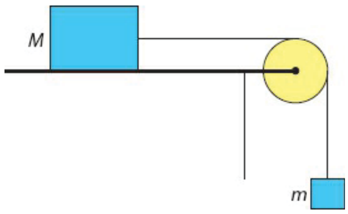


Figure 2.11

hint

Consider the two bodies as a single one of mass $M + m$. In this case, what is the net force on the whole system? Is the tension relevant if you consider the two bodies as one? If you must find the tension, as in this example, find the acceleration by first treating the bodies as one and then treating them as separate.

TEST YOURSELF 2.9

Figure 2.12 shows five identical blocks, connected by four strings labelled A, B, C and D. A force $\boldsymbol{F}$ acts on the right-hand block. Assume that friction is negligible.

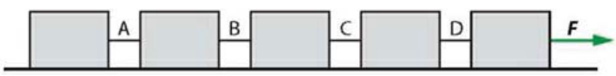


Figure 2.12

In which string is the tension the largest?

hint

Which string pulls the largest mass?

TEST YOURSELF 2.10

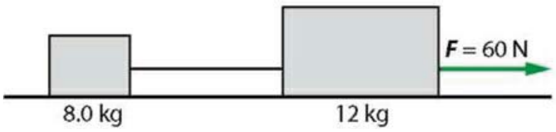


Figure 2.13

Figure 2.13 shows a force of 60N applied to a body of mass 12 kg. A string joins this body to another body of mass 8.0 kg.

> **hint**
>
> This is the case when motion just fails to take place, so we have equilibrium, and the frictional forces have their maximum values.

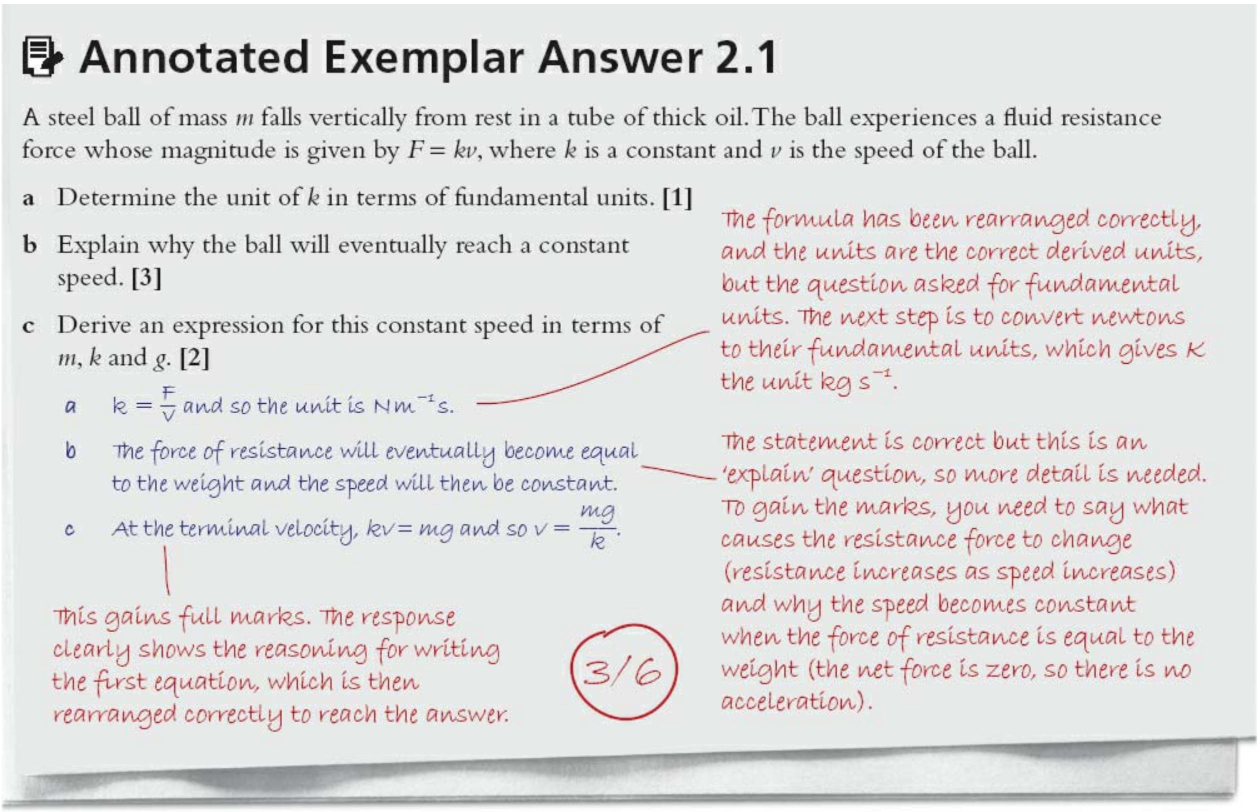
Calculate:

a the acceleration of the masses

b the tension in the string (friction is to be ignored).

c How would your answers to **a** and **b** change (if at all) if the order of the blocks were reversed?
We now assume that there are frictional forces between both blocks and the ground. The coefficient of static friction between each block and the ground is 0.30. The pulling force of 60 N is the largest force that can be applied without the blocks moving.

d Calculate the tension in the string.



Annotated Exemplar Answer 2.1

A steel ball of mass m falls vertically from rest in a tube of thick oil. The ball experiences a fluid resistance force whose magnitude is given by $F = kv$, where k is a constant and v is the speed of the ball.

a Determine the unit of k in terms of fundamental units. **[1]**

b Explain why the ball will eventually reach a constant speed. **[3]**

c Derive an expression for this constant speed in terms of m, k and g. **[2]**

a $k = \frac{F}{v}$ and so the unit is $\text{N m}^{-1}\text{s}$.

b The force of resistance will eventually become equal to the weight and the speed will then be constant.

c At the terminal velocity, $kv = mg$ and so $v = \frac{mg}{k}$.

The formula has been rearranged correctly, and the units are the correct derived units, but the question asked for fundamental units. The next step is to convert newtons to their fundamental units, which gives k the unit kg s^{-1}.

The statement is correct but this is an 'explain' question, so more detail is needed. To gain the marks, you need to say what causes the resistance force to change (resistance increases as speed increases) and why the speed becomes constant when the force of resistance is equal to the weight (the net force is zero, so there is no acceleration).

This gains full marks. The response clearly shows the reasoning for writing the first equation, which is then rearranged correctly to reach the answer.

3/6

< Back to chapter

Annotated Exemplar Answer 2.1

A steel ball of mass *m* falls vertically from rest in a tube of thick oil. The ball experiences a fluid resistance force whose magnitude is given by $F = kv$, where *k* is a constant and *v* is the speed of the ball.

a Determine the unit of *k* in terms of fundamental units. **[1]**

b Explain why the ball will eventually reach a constant speed. **[3]**

c Derive an expression for this constant speed in terms of m, *k* and *g*. **[2]**

a $\mathbf{k} = \frac{\mathbf{F}}{\mathbf{v}}$ and so the unit is $\text{N m}^{-1}\text{ s}$.

The formula has been rearranged correctly, and the units are the correct derived units, but the question asked for fundamental units. The next step is to convert newtons to their fundamental units, which gives K the unit kg s^{-1}.

b The force of resistance will eventually become equal to the weight and the speed will then be constant.

The statement is correct but this is an ‘explain’ question, so more detail is needed. To gain the marks, you need to say what causes the resistance force to change (resistance increases as speed increases) and why the speed becomes constant when the force of resistance is equal to the weight (the net force is zero, so there is no acceleration).

c At the *terminal* velocity, $v = \frac{mg}{k}$.

This gains full marks. The response clearly shows the reasoning for writing the first equation, which is then rearranged correctly to reach the answer.

2.4 Newton's third law

Newton's third law states that if a body A exerts a force on body B, then B will exert on A an equal and opposite force. (This is often remembered as 'for every action there is an equal and opposite reaction', but you should *not* use this in an exam!) Following are some examples of the third law:

- A tennis racket comes into contact with a tennis ball and exerts a force on the ball. The ball will exert an equal and opposite force on the racket.
- The weight of a body is the force that the Earth exerts on the body. The force is directed vertically downwards. Therefore the body exerts at the centre of the Earth an equal and opposite force – that is, upwards.
- A rocket exerts a force on the exhaust gases, pushing them backward. The gases exert an equal force on the rocket, pushing it forward.
- A helicopter rotor exerts a force on the air, pushing the air downwards. The air exerts an equal upward force on the rotor.

Worked Example 2.4

Figure 2.14 shows a block resting on another block that itself rests on a table.

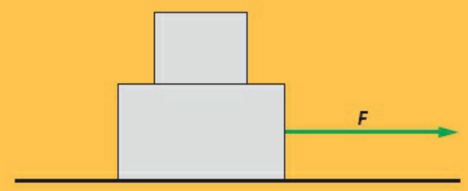


Figure 2.14

A force ***F*** is applied to the lower block and the two blocks move together. There is no friction between the table and the lower block but there is friction between the blocks. Copy Figure 2.15 and label the forces on each block, identifying action–reaction pairs. Draw action–reaction pairs in the same colour.

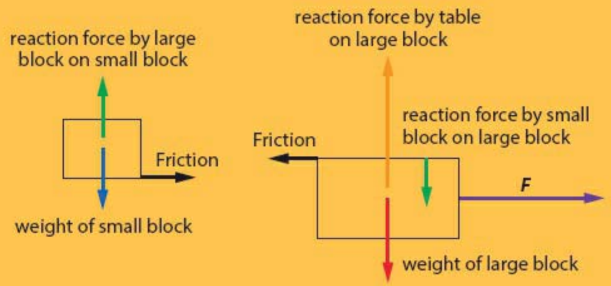


Figure 2.15

TEST YOURSELF 2.11

A girl stands on a weighing scale inside an elevator that is accelerating vertically upwards. The forces on the girl are her weight ***W*** and the reaction force ***R*** on her by the scale. What is the reading of the scale?

a $R + W$
b W
c R
d $R - W$

hint

The scale reads the force exerted on it.

hint

Two forces that are equal and opposite are *not necessarily* the 'action–reaction' pair of the law. The forces mentioned in the third law act on *different* bodies.

TEST YOURSELF 2.12

A block is put on a wedge of angle θ to the horizontal, as shown in Figure 2.16.

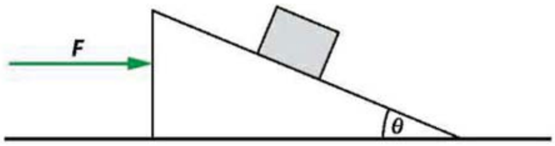


Figure 2.16

A horizontal force of magnitude F is applied to the wedge, accelerating it forward. The block does not move relative to the wedge. Show that $\tan\theta = \frac{F}{W}$, where W is the combined weight of the wedge and the block. (Assume that there is no friction between the block and the wedge or between the wedge and the ground.)

> **hint**
>
> It is helpful to choose the direction of motion as one of the axes. Why is the angle between N and the vertical also θ.

TEST YOURSELF 2.13

A body of mass m is on the floor of an elevator that is moving upwards with acceleration a (Figure 2.17).

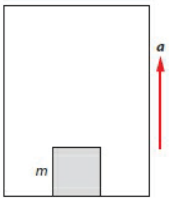


Figure 2.17

Derive an expression for the reaction force from the floor in terms of m, g and a.

> **hint**
>
> Is the reaction force equal to the weight? Imagine that the body is resting on a scale. Then the scale exerts a force of magnitude R on the body. By Newton's third law the force the body exerts on the scale is also R in magnitude. The scale reads the force acting on it, which is R. So if the elevator is accelerating, the scale does *not* read the weight of the body.

2.5 Momentum, impulse and momentum conservation

DEFINITIONS

MOMENTUM The product of the mass and the velocity of a body, $\boldsymbol{p} = m\boldsymbol{v}$. This is a vector with the same direction as that of velocity.

IMPULSE The product $\boldsymbol{F}\Delta t$ of the average force on the body times the time during which the force acts. Impulse is the *change in momentum* of a body, and is a vector. The magnitude of the impulse is the area under the curve in a graph of $\boldsymbol{F}$ against t.

NEWTON'S SECOND LAW IN TERMS OF MOMENTUM $F_{\text{net}} = \frac{\Delta p}{\Delta t}$. This form of the second law is useful when the mass varies. In cases where the mass of a body (or system) is constant, this becomes the usual $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$, because, with a constant mass,

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma.$$

CONSERVATION OF MOMENTUM When the resultant (net) external force on a system is zero, the total momentum of the system stays constant.

Proof of momentum conservation

If a system has total momentum $\boldsymbol{p}$, then by Newton's second law $F_{\text{net}} = \frac{\Delta p}{\Delta t}$, where $\boldsymbol{F}_{\text{net}}$ is the net external force on the system. If $\boldsymbol{F}_{\text{net}} = 0$, it follows that $\Delta\boldsymbol{p} = 0$ – that is, the total momentum stays constant.

TEST YOURSELF 2.14

A ball of mass 0.20 kg moving at 4.0 ms^{-1} collides with a vertical wall and rebounds with a speed of 2.50 ms^{-1} (Figure 2.18).

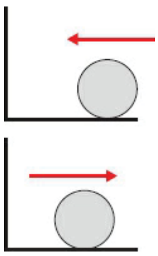


Figure 2.18

The ball stays in contact with the wall for 0.14s.

hint

Remember that momentum is a vector.

hint

It depends on what you take the system to be.

hint

Don't forget the weight. Remember that $\frac{\Delta p}{\Delta t}$ gives the **net** force.

a Calculate the magnitude and direction of the change in momentum of the ball.
b Calculate the magnitude of the average force on the ball.
c Discuss whether momentum conservation applies to this situation.
d The same ball now falls vertically on the floor. The impact and rebound speeds are as

before. State and explain whether the reaction force from the floor on the ball is different from the answer in **b.**

TEST YOURSELF 2.15

A rocket of mass 120 kg contains an additional 80 kg of fuel. Gases from the burnt fuel leave the rocket with a speed of $u = 3.0 \times 10^2$ ms^{-1} *relative to the rocket*, at a rate of μ = 2.2 kgs^{-1}. The rocket is initially at rest in outer space.

a Explain why the magnitude of the force exerted by the rocket on the gases is given by μu.

> *hint*
>
> What is the force exerted on the gases?

b Calculate the magnitude of this force.

c Explain why the rocket accelerates.

d Calculate the *initial* acceleration of the rocket.

TEST YOURSELF 2.16

A projectile moving horizontally collides elastically with a vertical steel plate. During contact with the plate, the force on the projectile varies with time according to the graph in Figure 2.19.

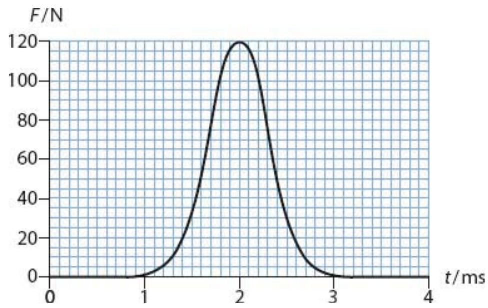


Figure 2.19

> *hint*
>
> What does the area under the curve tell you? Notice the units on the axes.

Estimate

a the duration of the contact

b the impulse delivered to the ball and

c the average force on the projectile.

d Given that the speed of the projectile before and after the collision was 18 ms^{-1}, calculate its mass.

2.6 Work, Energy and Power

DEFINITIONS

WORK DONE For a constant force, and motion along a straight line, the work done by this force is the product of its magnitude and the distance travelled in the direction of the force, $W = Fd \cos\theta$, where θ is the angle between the force and the direction of motion. Work is a scalar quantity. (Note that if the force is *not* constant, you cannot use this formula. In this case the work is the area under a graph of force versus distance.)

ENERGY The ability to do work. Energy is a scalar quantity. In mechanics we deal with the following forms of **mechanical energy**:

- KINETIC ENERGY $E_K = \frac{1}{2}mv^2$ (another formula for kinetic energy in terms of momentum is $E_K = \frac{p^2}{2m}$).
- GRAVITATIONAL POTENTIAL ENERGY $E_P = mgh$, where the height may be measured from any horizontal level.
- ELASTIC POTENTIAL ENERGY OF A SPRING $E_E = \frac{1}{2}kx^2$ (k is the spring constant).
- TOTAL MECHANICAL ENERGY The sum $E_T = E_K + E_P + E_E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$.

CONSERVATION OF ENERGY Total energy, mechanical and otherwise, cannot be created or destroyed. It only gets transformed from one form to another. In a system where no frictional or other dissipative forces act, the total **mechanical** energy stays the same. If such forces do act, then mechanical energy gets *transformed* into other forms of non-mechanical energy, for example **thermal** energy. The work done by these forces is the change in the total mechanical energy of the system.

ELASTIC COLLISION A collision in which the total kinetic energy before and after the collision are the same.

POWER The rate at which a force does work (or the rate at which energy is consumed, produced or dissipated) – that is, $P = \frac{\Delta W}{\Delta t}$. Power is measured in joules per second or watt (W). (If a constant force F acts on a body moving at speed v along a straight line, then the instantaneous power developed is $P = \frac{\Delta W}{\Delta t} = \frac{F \times \Delta s}{\Delta t} = Fv$. You should know how to derive this in an exam.)

WORK–KINETIC ENERGY RELATION The work done by the **net** force on a body is equal to the change in the body's kinetic energy, $W_{net} = \Delta E_K$. (Notice that this refers to the **net** force. This is an important relation in mechanics, with many applications.)

TEST YOURSELF 2.17

A body with an initial kinetic energy of 48J moves along a horizontal straight line. It is brought to rest by a constant frictional force of 6.0N. Calculate the distance travelled until the body stops.

hint

Rather than trying to find initial speed and acceleration and using kinematic equations, look directly at the work–kinetic energy relation.

TEST YOURSELF 2.18

A force of magnitude 20N pushes a body along a horizontal circle of radius 5.0 m at constant speed. The direction of the force is always tangent to the circle. Calculate the work done by this force as the body moves through one full circle.

hint

What angle does the force make with the direction of motion? Don't be misled by the fact that the net displacement is zero – does the definition of work involve net displacement or total distance moved?

TEST YOURSELF 2.19

The work done in extending a spring from its natural length to an extension e is W. The work done in extending the same spring from an extension e to an extension $2e$ is

a W

b $2W$

c $3W$

d $4W$

hint

How does the work done depend on extension?

TEST YOURSELF 2.20

A block of mass 5.0 kg and speed 3.0 m s^{-1} collides head-on with a stationary block of mass 7.0 kg. The two blocks stick together.

a Calculate the common speed of the two blocks after the collision.

b Why is this collision not elastic?

TEST YOURSELF 2.21

A car of weight 1.4×10^4N climbs an incline at a constant speed of 8.0 ms^{-1}. The incline makes an angle of 6.0° with the horizontal. A constant frictional force of 520N opposes the motion of the car.

a Calculate power dissipated by the frictional force.

b Calculate the rate of increase of the car's potential energy.

c The power developed by the engine is 45 kW. Estimate the efficiency of the car.

hint

The engine must provide enough power to cover both the power dissipated by friction and the power needed to increase the gravitational potential energy of the car as it climbs higher.

Model Answer 2.1

An object is dropped from rest from a large height above the surface of the Earth. Air resistance is not negligible. Describe the energy transformations taking place.

Gravitational potential energy is being transformed into kinetic energy and thermal energy in the body and the surrounding air.

The body will eventually reach terminal speed, in which case gravitational potential energy is being transformed entirely into thermal energy.

The origin of conservation principles

Nature of Science. Understanding of what energy is has evolved over time, with Einstein showing that there is a direct relationship between mass and energy in his famous equation $E = mc^2$. In this section we have seen how the principle of conservation of energy can be applied to different situations to predict and explain what will happen. Scientists have been able to use the theory to predict the outcome of previously unknown interactions in particle physics.

The principle of conservation of energy is perhaps the best known example of a conservation principle. But where does it come from? It turns out that all conservation principles are consequences of symmetry. In the case of energy, the symmetry is that of ‘time translation invariance’.

This means that when describing motion (or anything else) it does not matter when you started the stopwatch. So a block of mass 1 kg on a table 1 m above the floor will have a potential energy of 10J according to both an observer who starts his stopwatch ‘now’ and another who started it 10 seconds ago. The principle of conservation of momentum, is also the result of a symmetry. The symmetry this time is ‘space translation invariance’, which means that in measuring the position of events it does not matter where you place the origin of your ruler.

Checklist

After studying this chapter you should be able to:

- □ **apply the equations of kinematics**
- □ **analyse graphs of motion**
- □ **solve problems of projectile motion**
- □ **identify forces acting on bodies and solve problems of equilibrium**
- □ **apply Newton’s laws of motion to various situations**
- □ **know when to use $F_{net} = ma$ and when to use $F_{net} = \frac{\Delta p}{\Delta t}$**
- □ **apply the principles of momentum conservation and energy conservation**
- □ **calculate work done by a force, and the power developed.**

3 THERMAL PHYSICS

This chapter covers the following topics:

- □ Molecular theory of solids, liquids and gases
- □ Temperature scales
- □ Specific heat capacity and specific latent heats
- □ Phase changes
- □ Equation of state of an ideal gas
- □ Moles, molar mass and the Avogadro constant
- □ Differences between real and ideal gases

3.1 Thermal concepts

DEFINITIONS

ABSOLUTE (KELVIN) TEMPERATURE SCALE A temperature scale on which the lowest possible temperature is zero degrees. The absolute and Celsius scales are related by T(K) = t(°C) + 273.

AVERAGE RANDOM KINETIC ENERGY OF MOLECULES The average random kinetic energy of the molecules of a substance is proportional to the absolute temperature of the substance: $\frac{1}{2}mv^2 = \frac{3}{2}kT$, where k is the Boltzmann constant. (At T = 0 K, the random kinetic energy of the molecules is therefore zero.)

INTERNAL ENERGY, U The total random kinetic energy of the molecules of a system plus its total intermolecular potential energy. (It is customary to call this part of the internal energy its **thermal** energy. Internal energy also includes chemical energy and nuclear energy, but we do not consider these in this topic.)

INTERMOLECULAR FORCES There are forces, electromagnetic in origin, between any two molecules in any substance, whether solid, liquid or vapor. These forces are strongest in the solid phase and weakest in the vapor phase. (Because of these forces, increasing the separation of molecules requires that work be done; this work increases the intermolecular potential energy, and hence the internal energy of the substance. For an ideal gas, these forces are zero and so internal energy only consists of the random kinetic energy of the molecules.)

HEAT, Q 'Energy in transit' – energy that is transferred from one body to another due to a difference in temperature.

DIRECTION OF ENERGY FLOW Heat, on its own, always gets transferred from a higher-temperature to a lower-temperature region.

THERMAL EQUILIBRIUM When two bodies are at the same temperature they are said to be in thermal equilibrium, and no further net energy gets transferred between them.

IDEAL GAS A theoretical model in which there are no intermolecular forces between the particles of the gas and so no intermolecular potential energy in its internal energy. An ideal gas obeys the equation $pV = nRT$ at all pressures, volumes and temperatures. (Real gases obey the gas law approximately and only for a range of pressures, volumes and temperatures.)

MOLE The S.I. unit for quantity. One mole of any substance always contains $N_A = 6.02 \times 10^{23}$ molecules. This number is known as the **Avogadro constant.**

MOLAR MASS The mass in grams of one mole of a substance.

TEST YOURSELF 3.1

State the molar mass of **a** $^{238}_{92}\mathrm{U}$, **b** $^{4}_{2}\mathrm{He}$ and **c** H_2O.

TEST YOURSELF 3.2

The molar mass of lead is 207 g mol^{-1} and its density is 1.13×10^4 kg m^{-3}. Estimate the average separation of lead atoms.

hint

In Figure 3.1 each dashed square represents a cube with one lead atom at its centre.

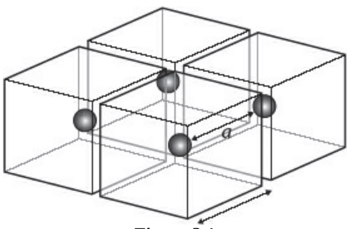


Figure 3.1

TEST YOURSELF 3.3

The temperature of a liquid changes from 22°C to 32°C.

Calculate the change in the temperature of the liquid, and its final temperature in kelvin.

> hint
>
> How are kelvins and degrees Celsius related?

3.2 Calorimetry

DEFINITIONS

SPECIFIC HEAT CAPACITY, c The energy required to change the temperature of a unit mass by one degree.

CHANGE OF PHASE Substances undergo *phase changes* at specific, *constant* temperatures. For example, ice will melt to water at 0°C (its melting point), and water will boil to steam at 100°C (its boiling point).

SPECIFIC LATENT HEAT The energy required to change the phase of a unit mass at constant temperature. (It is important to mention that this definition is for a constant temperature).

A solid can melt into a liquid and a liquid can boil into vapor. The reverse processes are a liquid freezing into a solid and vapor condensing into a liquid. These are phase changes and occur at a *constant temperature*.

When a quantity Q of heat is supplied to a body of mass m and there is no change of phase, the body's temperature will increase by $\Delta\theta$ according to $Q = mc\Delta\theta$, where the constant c is the body's specific heat capacity. A quantity Q of heat supplied to a body of mass m at its melting or boiling point is $Q = mL$, where the constant L is known as the specific latent heat of fusion (if melting) or vaporisation (if boiling).

Worked Example 3.1

Calculate the energy required to change 2.0 kg of ice at −15°C to liquid water at +15°C. Data available: for liquid water, $c_{water} = 4200\ \mathrm{Jkg^{-1}\ K^{-1}}$; for ice, $c_{ice} = 2100\ \mathrm{Jkg^{-1}\ K^{-1}}$ and the specific latent heat of fusion is $L = 334\ \mathrm{kJkg^{-1}}$.

We can view this process as happening in three stages:

1 Raise the temperature of ice from −15°C to 0°C.
2 Melt the ice at constant temperature (0°C).
3 Raise the temperature of the melted ice (which is now liquid water at 0°C) to +15°C.

1 $Q = mc\Delta\theta = 2.0 \times 2100 \times 15 \approx 63$ kJ.
2 To melt 2.0 kg of ice at 0°C to liquid water at 0°C we require $Q = mL = 2.0 \times 334 \approx 670$ kJ.
3 $Q = mc\Delta\theta = 2.0 \times 4200 \times 15 \approx 126$ kJ.

Thus we require a total of about 860 kJ.

Model Answer 3.1

State and explain the change in the internal energy of a solid substance as it melts.

During melting, the average separation of molecules increases and this implies an increase in intermolecular potential energy. The total kinetic energy of the molecules stays the same and since internal energy is the sum of the kinetic and the intermolecular energies, internal energy increases.

TEST YOURSELF 3.4

An 80 g piece of aluminium ($c = 900\mathrm{Jkg^{-1}\ K^{-1}}$) at 250 °C is dropped into 0.75 kg of water ($c = 4200\mathrm{Jkg^{-1}K^{-1}}$) at 15 °C in a calorimeter. The calorimeter has mass 200 g and is made of aluminium. Estimate the final temperature of the water.

hint

Recall that everything will have the same final temperature; call it T. What loses heat and what gains heat?

TEST YOURSELF 3.5

A sample of 120 g of solid paraffin initially at 20 °C is heated using a heater of constant power. The specific heat capacity of solid paraffin is 2500 J kg^{-1} K^{-1}. The temperature of paraffin varies with time as shown in Figure 3.2.

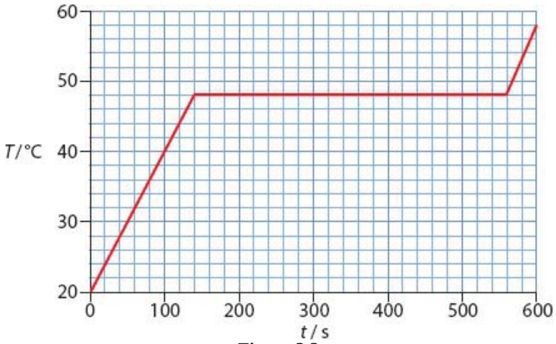


Figure 3.2

Use the graph to determine:

a the power of the heater

b the melting temperature of paraffin

c the specific latent heat of fusion of paraffin

d the specific heat capacity of paraffin in the liquid phase.

e Explain why the temperature of paraffin stays constant during melting.

hint

Recall what internal energy consists of.

3.3 The kinetic model of gases

Pressure

Imagine a force of magnitude F acting on a region of area A (as in Figure 3.3), making an angle θ with the normal to the area.

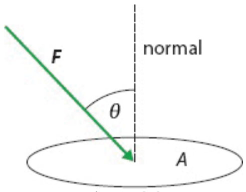


Figure 3.3

Pressure is defined as the *normal* force per unit area acting on the surface: $P = \frac{F\cos\theta}{A}$. The unit of pressure is Nm^{-2}, usually called a pascal (Pa).

A gas is a very large collection of molecules with intermolecular distances that are, on average, about 10 times larger than in the solid and liquid phases. Imagine a gas molecule colliding at right angles with the wall of its container and rebounding (Figure 3.4).

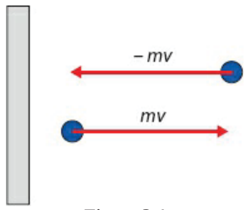


Figure 3.4

The molecule experiences a force because its momentum has changed from mv to $-mv$, that is, the magnitude of its momentum has changed by an amount $2mv$. Therefore there is also a force by the molecule on the wall. Pressure is the result of the force from the very many molecules colliding with the walls. Two factors affect the pressure of a gas – the average speed of the molecules and the frequency with which the molecules collide with the walls.

Ideal gases: An *ideal gas* is an idealised form of a gas that obeys the following assumptions:

- The molecules are hard spheres of negligible volume.
- Collisions (between molecules, and between molecules and the walls) are elastic.
- The duration of collisions is short compared to the time between collisions.
- There are no forces between molecules except during contact.
- The molecules move randomly with a range of speeds.
- The molecules obey the laws of mechanics.

Real gases: Unlike an ideal gas, a real gas can be liquefied or solidified. A real gas always has intermolecular forces and its internal energy includes intermolecular potential energy. An ideal gas is a good approximation to a real gas when the gas is at low density. You will most likely find a gas at low density under conditions of large volume, low pressure and moderate temperature.

Evaporation: A liquid *boils* at a specific temperature, and molecules from anywhere in the liquid can leave the liquid. But a liquid can *evaporate* (i.e. turn into a vapor) at any temperature. Unlike boiling, the evaporation process involves *surface molecules only*. The rate of evaporation increases with increasing temperature and surface area of the liquid.

Evaporation is accompanied by cooling because it is the faster molecules that will escape, leaving behind the slower ones. Thus the average speed and hence the average kinetic energy of the molecules left behind decreases. Since temperature is a measure of the average kinetic energy of the molecules, the temperature therefore decreases.

Model Answer 3.2

A gas is compressed at constant temperature. Explain why the pressure increases.

Since the temperature is constant, the average molecular speed stays the same. But since the volume decreases, the frequency of collisions increases because molecules have a shorter distance to cover between collisions. Hence the pressure increases.

TEST YOURSELF 3.6

A gas is heated at constant volume. Explain why the pressure will increase.

hint

Does the temperature stay constant?

TEST YOURSELF 3.7

a State two differences between boiling and evaporation.

b Explain why the temperature of a liquid decreases while the liquid is evaporating.

> **hint**
>
> What is temperature related to?

The ideal gas law

The *state* of a gas is determined when we know the pressure *p*, the volume *V*, the temperature *T* and the quantity of the gas – that is, *n*, the number of moles of it. For an ideal gas these are related by $pV = nRT$. The gas constant *R* is universal and has the value $R = 8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}$.

You can use this formula to find new values of (p, V, n, T) after a change in the state of a gas. From $pV = nRT$ we have $R = \frac{pV}{nT} = \text{constant}$. Therefore if the state of an ideal gas changes from (p_1, V_1, n_1, T_1) to (p_2, V_2, n_2, T_2), we have $\frac{p_1V_1}{n_1T_1} = \frac{p_2V_2}{n_2T_2}$, since both fractions give the gas constant.

Models must be correct but also simple

Nature of Science. Boyle thought that a gas consists of particles joined by springs. Newton thought that a gas consists of particles that exert repulsive forces on each other. Bernoulli thought that a gas is a collection of a very large number of particles that exert forces on each other only when they collide. All three could explain why a gas exerts a pressure on its container but it is Bernoulli's picture that is the simplest. We assume that the ordinary laws of mechanics apply to the individual particles making up the gas.

Even though the laws apply to each individual particle we cannot observe or analyse each particle individually since there are so many of them. By concentrating on average behaviours of the whole gas and using probability and statistics, physicists developed a new field of physics known as statistical mechanics. This has had enormous success in advancing our understanding of gases and other systems, including where the approximation to an ideal gas breaks down.

TEST YOURSELF 3.8

Calculate the number of moles in $4.0 \times 10^{-3}\ \mathrm{m^{-3}}$ of an ideal gas kept at temperature 300 K and pressure 2.0×10^5 Pa.

TEST YOURSELF 3.9

An ideal gas is heated, at constant volume, from a temperature of 27°C and pressure 3.0×10^5 Pa to a temperature of 127°C. No gas enters or leaves the container. Calculate the new pressure.

> **hint**
>
> What temperature scale must be used here?

TEST YOURSELF 3.10

An isolated container is divided into two equal volumes by a partition. In the first half there is 1 mole of an ideal gas at pressure *P* and temperature *T*. In the second half there are 2 moles of an ideal gas at pressure 2*P* and temperature *T*. The partition is removed. Which of the following is the final overall pressure, if the temperature stays the same?

a $\frac{P}{2}$ **b** P **c** $\frac{3P}{2}$ **d** $2P$

TEST YOURSELF 3.11

An ideal gas expands, at constant pressure of 4.0×10^5 Pa, from a volume of 2.0×10^{-3} m^{-3} and temperature 300 K to a volume of 5.0×10^{-3} m^{-3}. Calculate the new temperature of the gas.

TEST YOURSELF 3.12

For an ideal gas:

a Define internal energy.

b State and explain how the internal energy and the absolute temperature are related.

hint

Does an ideal gas have potential energy? What is absolute temperature related to?

TEST YOURSELF 3.13

An ideal gas in a cylinder with a piston is compressed by pushing in the piston very rapidly. Explain, using the molecular model of a gas, why the temperature of the gas will increase.

hint

We want to explain the increase in temperature, not the increase in pressure.

TEST YOURSELF 3.14

Neon $\left({}^{20}_{10}\mathrm{Ne}\right)$ and argon $\left({}^{40}_{18}\mathrm{Ar}\right)$ gases are kept at the same temperature. What is the ratio of the average speed of neon atoms to that of argon?

a 2 **b** $\sqrt{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{\sqrt{2}}$

hint

Remember that the mass of an atom in atomic mass units u is numerically equal to the nucleon (mass) number.

Annotated Exemplar Answer 3.1

a By reference to the equation of state, describe what is meant by an ideal gas. **[2]**

b A container of volume $0.20\,m^3$ contains 5.0 moles of a gas at a temperature of 27 °C. Calculate the pressure in the container. **[3]**

a *An ideal gas obeys the equation $pV = nRT$, where p is pressure, V is volume, N is the number of moles, R is the gas constant and T is the temperature.*

The equation is stated correctly and the symbols are explained, but the key point about an ideal gas is that this law is obeyed at all temperatures and pressures.

b

$$p = \frac{nRT}{V}$$

$$= \frac{5.0 \times 8.31 \times 27}{0.20}$$

$$= 5609.25 \approx 5.6\,kPa$$

A good first step in a calculation is to state the equation you will use.

The value for temperature should be in kelvin, not degrees Celsius. This is a common mistake – make sure you do not make it!

The final answer is rounded to a suitable number of decimal places, but gets no marks. At atmospheric pressure of 101 kPa and 25 °C (298 K), the molar gas volume is about 24 dm³, or 0.024 m³, so 5 mol would occupy about 0.1 m³. In a volume about double the size, the pressure would be about half, so we should expect an answer a bit larger than 50 kPa. The given answer of 5.6 kPa is much too small. The correct answer is 62 kPa.

2/5

Checklist

After studying this chapter you should be able to:

- ☐ describe the differences between solids, liquids and gases in terms of molecules
- ☐ work with the Celsius and absolute (Kelvin) temperature scales
- ☐ solve problems involving specific heat capacity and specific latent heats
- ☐ describe phase changes and account for energy changes during a phase change
- ☐ apply the equation of state of an ideal gas
- ☐ work with moles, molar mass and the Avogadro constant
- ☐ describe the differences between real and ideal gases.

< Back to chapter

Annotated Exemplar Answer 3.1

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The equation is stated correctly and the symbols are explained, but the key point about an ideal gas is that this law is obeyed at all temperatures and pressures.

b $P = \frac{nRT}{v}$

A good first step in a calculation is to state the equation you will use.

$= \frac{5.0 \times 8.31 \times 27}{0.20}$

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2/5

4 OSCILLATIONS AND WAVES

This chapter covers the following topics:

- □ Simple harmonic motion
- □ Wave characteristics: period, amplitude, frequency, wavelength and intensity
- □ Wavefronts and rays
- □ Transverse and longitudinal waves
- □ The nature of electromagnetic waves
- □ The nature of sound
- □ Reflection and refraction
- □ Diffraction and interference
- □ Superposition
- □ Polarization
- □ Standing waves

4.1 Simple harmonic motion

In simple harmonic motion (SHM), a displacement from equilibrium gives rise to a restoring force – and therefore an acceleration – that is *proportional to, and in the opposite direction to, the displacement from equilibrium.*

Mathematically, this can be written as $a = -kx$, where k is a constant. A graph of acceleration versus displacement for SHM is a straight line through the origin with a negative gradient, such as the one in Figure 4.1.

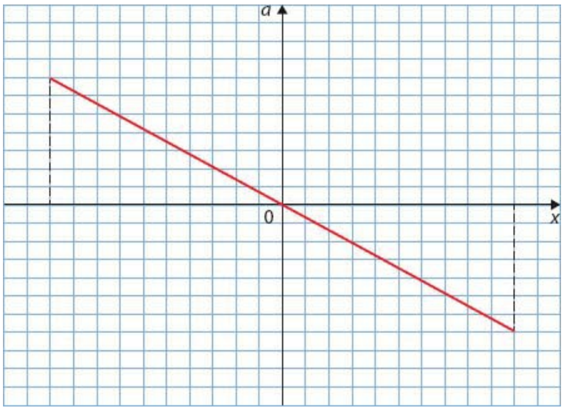


Figure 4.1

You should be able to draw graphs, such as those in Figure 4.2, showing the variation with time of the displacement, speed and acceleration for SHM. Be careful to show the relative positioning of these three graphs.

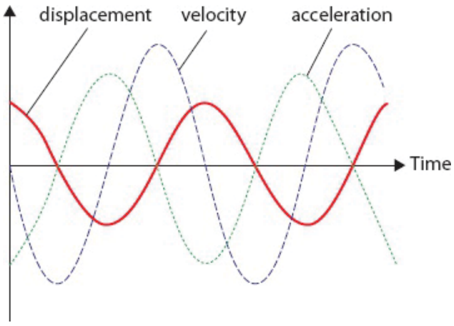


Figure 4.2

Energy in simple harmonic motion

In simple harmonic motion, two types of energy are transformed into one another as the motion progresses: kinetic energy and potential energy. For example, for a mass at the end of a horizontal spring, we have kinetic energy as the particle moves past the equilibrium position and (elastic) potential energy at the extremes of the motion. At all other points we have both kinds of energy.

TEST YOURSELF 4.1

The acceleration is related to displacement in three different cases, as follows:

a $a = -4x$, **b** $a = 4x$, **c** $a = -4^2x$. In which case(s) do we have SHM?

TEST YOURSELF 4.2

Figure 4.3 shows a graph of acceleration versus displacement of a particle.

a Use the graph to explain why the oscillations are SHM.

b State the amplitude of the oscillations.

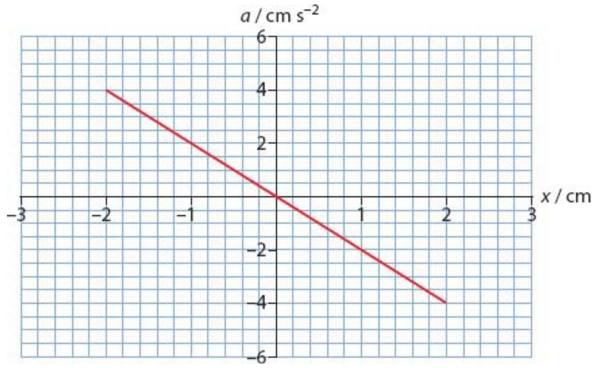


Figure 4.3

TEST YOURSELF 4.3

Figure 4.4 shows how the speed of a particle varies with time.

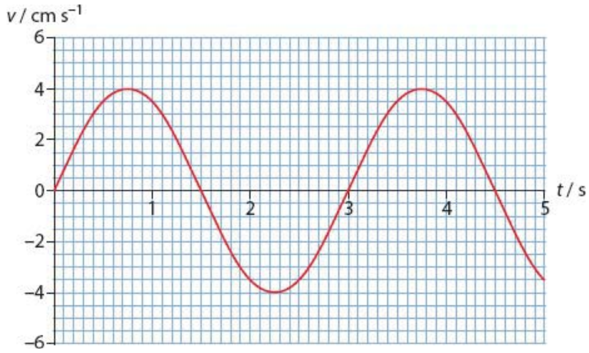


Figure 4.4

Determine:

a the period of the motion

b a time at which the particle goes through the equilibrium position

c a time at which the particle is as far from the equilibrium position as possible.

d Draw a graph to show how the displacement varies with time. (No numbers on the vertical axis are necessary.)

TEST YOURSELF 4.4

Figure 4.5 shows the kinetic energy of a 0.25 kg mass that is undergoing SHM as a function of its displacement.

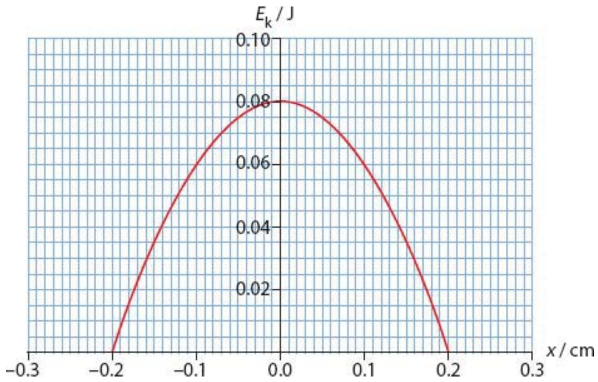


Figure 4.5

a Calculate the maximum speed.

b On a copy of this figure, draw a sketch graph to show the variation of potential energy with displacement.

c Use the graph to estimate the displacement when the potential energy is equal to the kinetic energy.

4.2 What is a wave?

A wave is the propagation of a *disturbance* through a medium. As the wave passes through the medium, the molecules of the medium are *displaced* from their equilibrium positions and are made to *oscillate* about these positions in simple harmonic motion. A wave can transfer energy and momentum.

It is important to understand that, although the molecules of the medium undergo only small displacements about an equilibrium position, the wave itself can move very large distances. A good example is the 'wave' in the stands in a football game. The fans move up and down just a bit but the wave they create can travel the entire circumference of the stadium.

Table 4.1 lists properties of various kinds of waves.

Table 4.1

Type of waves	Mechanical waves	Electromagnetic waves	Matter waves
Examples	Water waves, sound, string waves, earthquake waves	Light, radio waves, X-rays, infrared radiation, ultraviolet radiation	Waves required by the quantum behavior of matter (see Chapter 12)
Medium	Require a physical medium, cannot propagate in vacuum	Can propagate in vacuum as well as various physical media	
Wave speed	Determined by the properties of the medium.	In vacuum, all electromagnetic waves have the same speed, $c = 3 \times 10^8$ m s^{-1}.	
Disturbance	The change of a property of the medium from its equilibrium value (when there is no wave)	The oscillating values of electric and a magnetic fields (see Figure 4.6)	

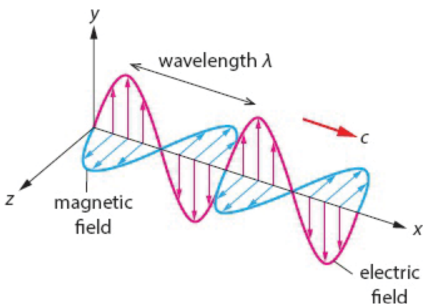


Figure 4.6 An electromagnetic wave propagating along the *x*-axis.

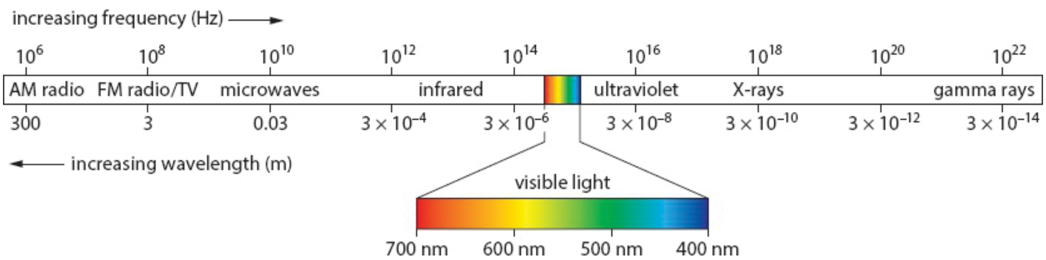


Figure 4.7 The electromagnetic spectrum.

DEFINITIONS

DISPLACEMENT A vector to the position of a point in a medium from the equilibrium position of the point.

AMPLITUDE, *A* The maximum displacement.

CREST A point of maximum positive displacement.

TROUGH A point of maximum negative displacement.

WAVELENGTH, *λ* The length of a full wave. It may be found from a displacement versus distance graph.

FREQUENCY, *f* The number of full waves produced by a source in 1s. It is measured in hertz (Hz). The frequency is the inverse of the period: $f = \frac{1}{T}$.

PERIOD, *T* The time taken for one full wave to pass by. It may be found from a displacement versus time graph. The period is the inverse of the frequency: $T = \frac{1}{f}$.

WAVE SPEED, *v* The speed at which a crest moves past an observer at rest in the medium. It equals $v = f\lambda$.

INTENSITY, *I* The power transferred per unit area by a wave. It is proportional to the square of the amplitude: $I \propto = A^2$. For a point source emitting uniformly in all directions, the intensity a distance *d* from the source intensity is proportional to $\frac{1}{d^2}$.

Transverse waves

In a transverse wave the displacement of the medium is *at right angles* to the direction of wave movement or energy transfer. A displacement–distance graph such as the one in Figure 4.8 is a 'photograph' of the medium at a particular instant of time, in this case t = 0s.

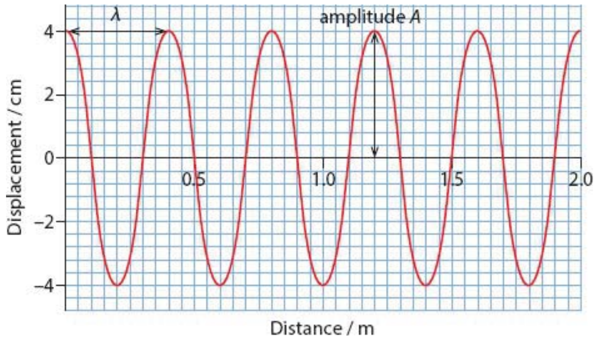


Figure 4.8

From a displacement–distance graph we can determine the amplitude (here 4.0 cm) and the wavelength (here 0.40 m).

But we may want to concentrate on a *particular point* in the medium. For example, the point at x = 0.30 m has a displacement of 0 cm at t = 0s. Immediately afterwards, the point will have a negative displacement. The displacement of this specific point with time is shown in Figure 4.9. Each point in the wave undergoes SHM with the same frequency and amplitude as all other points in the wave.

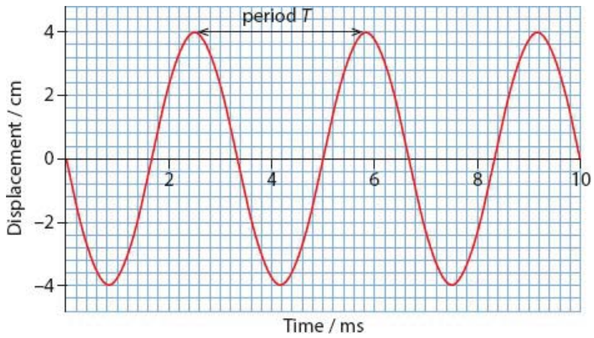


Figure 4.9

From this displacement–time graph we can see that the period is 3.4 ms. The frequency is therefore

$f = \frac{1}{3.4\times10^{-3}} \approx 294\,\text{Hz}$ and the wave speed is $v = f\lambda = 0.40 \times 294 = 118 \approx 120\text{ms}^{-1}$.

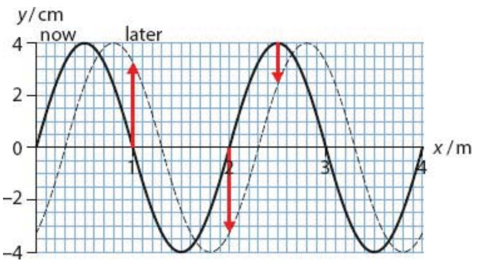


Figure 4.10

In Figure 4.10, the solid line shows a transverse wave on a string at t = 0. The wave moves to the right. The dashed line shows the same wave a time t = 1ms later. In a time of t = 1ms the crest has moved forward a distance of 0.30 m. The speed of the wave is therefore $v = \frac{0.30}{1\times10^{-3}} = 300\,\text{ms}^{-1}$.

The wavelength is 2.0 m, and the frequency is therefore $f = \frac{v}{\lambda} = \frac{300}{2} = 150\,\text{Hz}$.

We see that the crests have changed positions – this is what is expected for a travelling wave. By drawing the waves now and a bit later, we can see how the points on the string move. We see, for example, that the point at x = 1.0m moved upwards whereas the points at x = 2.0 m and x = 2.5 m moved downwards.

Longitudinal waves

In a longitudinal wave the displacement of the medium is *parallel* to the direction of the wave movement or energy transfer. Sound is a good example of a longitudinal wave. Figure 4.11 shows a longitudinal wave in a Slinky spring.

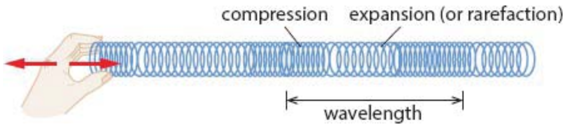


Figure 4.11

Figure 4.12 shows the displacement–distance graph for a longitudinal wave. The small red arrows represent the *displacement* of the particles of the medium (for example, air molecules in the case of sound) at a particular time.

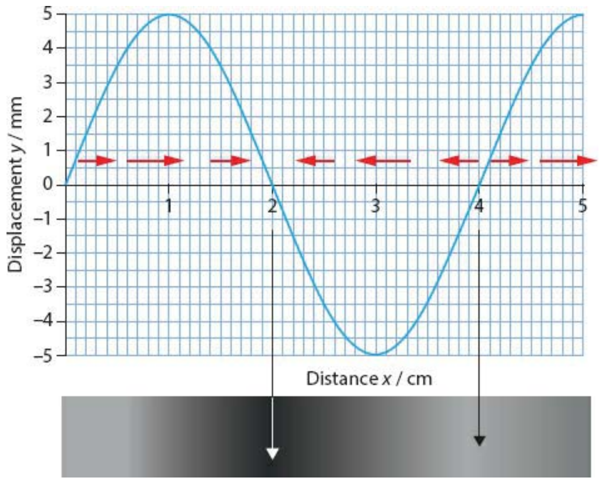


Figure 4.12

The bottom panel indicates the ‘bunching’ of the particles. Particles to the left of x = 2.0 cm move to the right, while those to the right move left, so the region at x = 2.0 is the centre of a compression. In a similar way, the region around x = 4.0 is the centre of a rarefaction.

(Note that a graph of displacement versus distance or versus time does not allow us to tell whether the wave is transverse or longitudinal.)

Annotated Exemplar Answer 4.1

a Explain the difference between a transverse wave and a longitudinal wave. **[2]**

b Give an example of each type of wave. For each, state whether a medium is required for propagation of the wave. **[2]**

a In a transverse wave the displacement is up and down, whereas in a longitudinal wave the displacement is sideways.

The idea is here, but the language is imprecise. Always use the correct scientific language. You need to explain that the displacement of the medium is at right angles to the direction of energy transfer.

Here the idea needed is that the displacement is parallel to the direction of energy transfer.

b Longitudinal wave: sound wave – requires a medium.

Sounds waves are the best example, but you could also choose seismic P-waves, which also require a medium.

Transverse wave: radio wave – can propagate in a vacuum.

Any electromagnetic wave would gain the mark for a transverse wave.

2/4

Wavefronts and rays

DEFINITIONS

RAY A line indicating the direction of energy transfer of a wave.

WAVEFRONT A surface at right angles to rays is called as wavefront. All the points on a wavefront have the same phase.

By convention we draw wavefronts through wave crests, so two neighbouring wavefronts are separated by one wavelength (Figure 4.13).

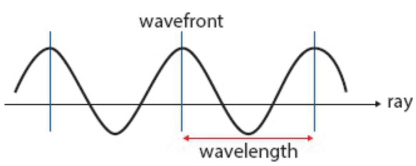


Figure 4.13

Figure 4.14 shows circular wavefronts. We would create such wavefronts if, for example, we dropped a small stone into a lake.

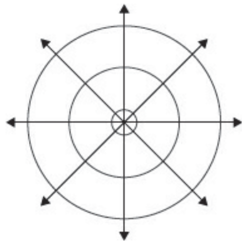


Figure 4.14

< Back to chapter

Annotated Exemplar Answer 4.1

a Explain the difference between a transverse wave and a longitudinal wave. **[2]**

b Give an example of each type of wave. For each, state whether a medium is required for propagation of the wave. **[2]**

a In a transverse wave the displacement is up and down, whereas in a longitudinal wave the displacement is sideways.

The idea is here, but the language is imprecise. Always use the correct scientific language. You need to explain that the displacement of the medium is at right angles to the direction of energy transfer.

Here the idea needed is that the displacement is parallel to the direction of energy transfer.

b Longitudinal wave: sound wave – requires a medium.

Sounds waves are the best example, but you could also choose seismic P-waves, which also require a medium.

Transverse wave: radio wave – can propagate in a vacuum.

2/4

4.3 Reflection

The law of reflection states that the angle of incidence (the angle between the incident ray and the normal to the surface) is equal to the angle of reflection (the angle between the reflected ray and the normal), and that, furthermore, the incident and reflected rays and the normal are all in the same plane (Figure 4.15).

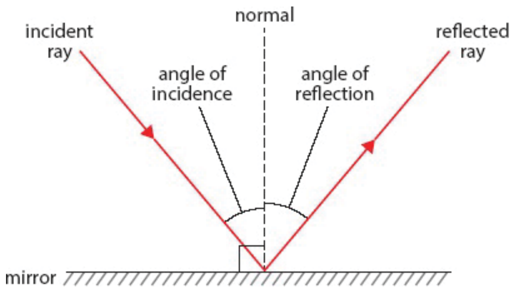


Figure 4.15

Reflection of pulses

In Figure 4.16a, the pulse reflects off a fixed end. The reflected pulse inverts, because when it reaches the fixed end it pulls upwards at the fixed end. By Newton’s third law there is downward force on the rope that creates the inverted pulse.

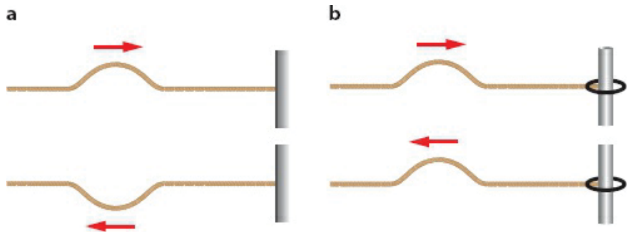


Figure 4.16 Reflection of pulse: **a** from a fixed end; **b** from a free end.

In Figure 4.16b the reflection is off a free end. The reflected pulse does not invert.

4.4 Refraction

In general, a wave will change its speed, but not its frequency, as it moves from one medium into another. This means that its wavelength will also change. Referring to Figure 4.17, the angles of incidence and refraction are related by:

$$\frac{\text{Sin } \theta_1}{v_1} = \frac{\sin\ \theta_2}{v_2} \quad \text{(Snell's law)}$$

where v_1, v_2 are the speeds of the wave in the two media.

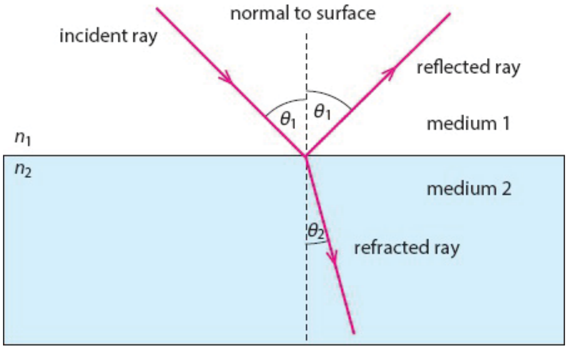


Figure 4.17

For light, we may rewrite this equation as $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where $n = \frac{c}{v}$ is the refractive index of the medium and c is the speed of light in vacuum. In Figure 4.17 the ray crosses to medium 2 which has a lower wave speed and so it bends *towards* the normal.

Similar behavior nay be seen in water waves moving from deep to shallow water (Figure 4.18). Their speed in shallow water is lower (we know this because the distance between the wavefronts in shallow water is smaller), and so we have refraction.

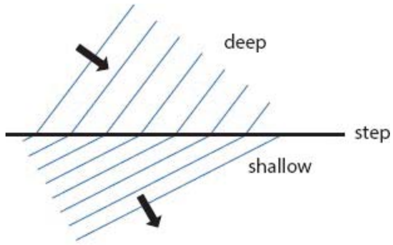


Figure 4.18

Figure 4.18

In this case, the ratio of the speeds in the deep and shallow parts is about 1.7, found from measuring the distances between wavefronts.

TEST YOURSELF 4.5

A ray of light in air of wavelength 540 nm enters water ($n = 1.33$), making an angle of 50° with the water surface. Calculate:

a the speed of light in water

b the wavelength of light in water

c the angle of the ray with the normal in water.

hint

Be careful which angles you use here.

4.5 Total internal reflection

This is a special case of refraction in which the ray moves from a medium of higher refractive index to one of lower refractive index. In this case the refracted ray bends away from the normal (Figure 4.19).

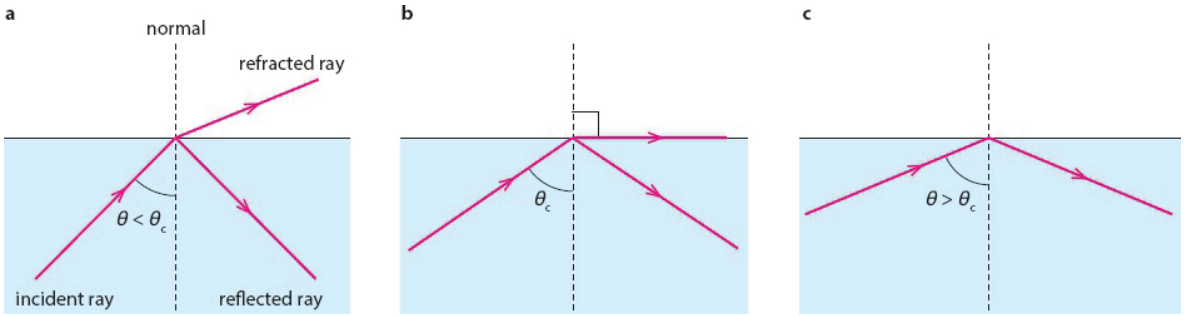


Figure 4.19

As the angle of incidence is increased, the angle of refraction will also increase, eventually becoming 90°. The angle of incidence at this point is called the **critical angle.** For any angle of incidence greater than the critical angle, no refraction takes place. The ray simply reflects back into the medium from which it came.

The critical angle can be found from Snell's law:

$$n_1 \sin \theta_C = n_2 \sin 90^\circ, \sin \theta_C = \frac{n_2}{n_1}, \theta_C = \arcsin\frac{n_2}{n_1}$$

TEST YOURSELF 4.6

Calculate the critical angle for a water–glass boundary. (Water refractive index = 1.33; glass refractive index = 1.45.)

hint

From which medium must the ray come for total internal reflection to occur?

4.6 Diffraction

Diffraction refers to the spreading of a wave as it goes through an opening (aperture) or past an obstacle.

In Figure 4.20a, the size of the opening is similar to the wavelength itself, and there is no appreciable spreading as the wave goes through. In Figure 4.20b, the opening is significantly smaller than the wavelength, and the wave spreads noticeably. This would apply, for example, to sound going through a window.

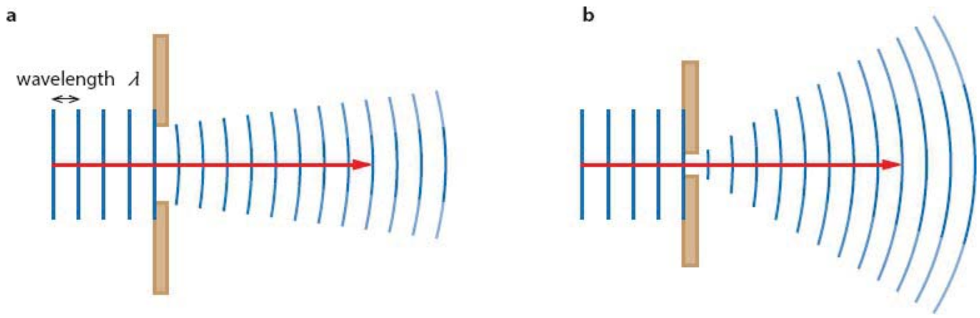


Figure 4.20

Diffraction applies to all waves but its extent depends crucially on the size of the wavelength compared to the size of the aperture or obstacle. A small wavelength relative to opening or obstacle size means little diffraction. Appreciable diffraction takes place when the wavelength is comparable to or larger than the opening size.

Figure 4.21 shows diffraction around an obstacle and an edge. Notice that eventually the wave reappears behind the obstacle.

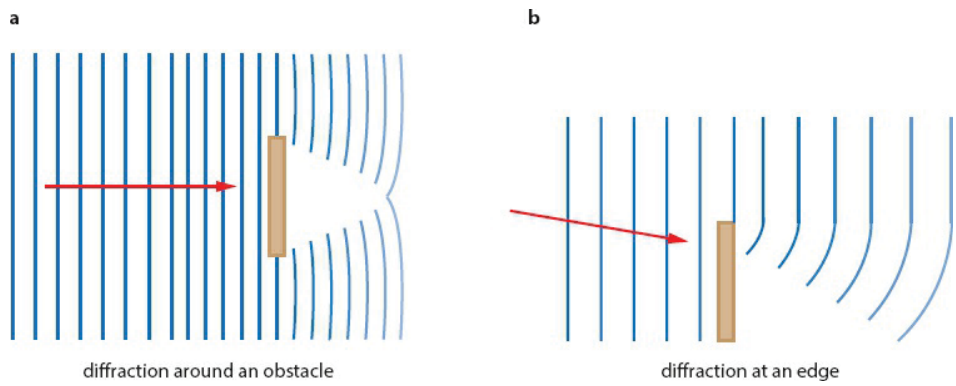


Figure 4.21 Diffraction **a** around an obstacle; **b** at an edge.

4.7 Superposition and interference

DEFINITIONS

PRINCIPLE OF SUPERPOSITION When two waves (of the same kind) meet, the resultant displacement is the sum of the individual displacements.

In Figure 4.22, two waves (red and blue) meet. The sum of the two displacements is shown by the heavy green line. For example, at $x = 0$ both waves have a displacement of 0.60 cm. The sum (green line) then has displacement 1.2 cm.

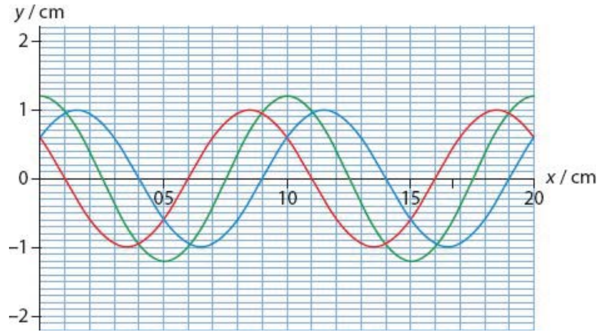


Figure 4.22

In Figure 4.23 the two waves meet exactly crest-to-crest (here one wave is shown slightly displaced for clarity). The result is a wave with double the amplitude of the individual waves. This is *constructive interference.*

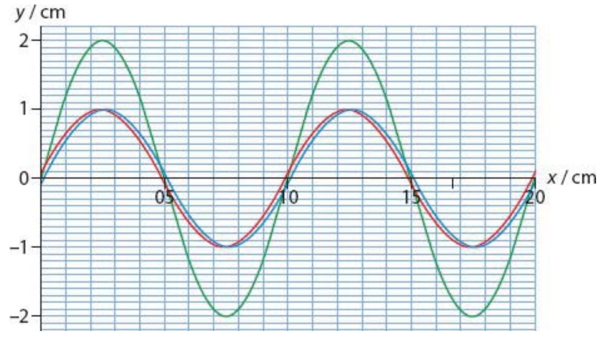


Figure 4.23

In Figure 4.24 the two waves meet crest-to-trough. The result is that the two cancel each other out. The resulting wave has zero amplitude; in other words, we do not have a wave! This is *destructive interference*. (If the two waves have different amplitude they will never completely cancel out.)

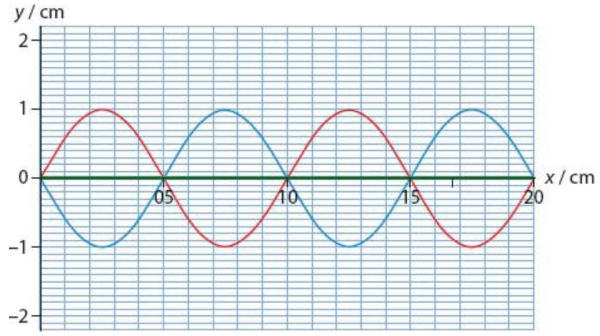


Figure 4.24

What determines whether we have constructive or destructive interference at a particular point? It depends on the difference in the path lengths to that point from each source (see Figure 4.25).

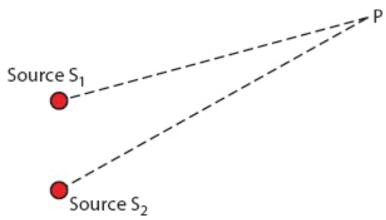


Figure 4.25

An electromagnetic wave consists of an electric and a magnetic field at right angles to each other.

We call the extra distance, $S_2P - S_1P$, the **path difference.**

$$\frac{\text{Path difference}}{\lambda} = \underset{\text{integer}}{n} \Rightarrow \text{constructive interference}$$

$$\frac{\text{Path difference}}{\lambda} = \underset{\text{half-integer}}{n + \tfrac{1}{2}} \Rightarrow \text{destructive interference}$$

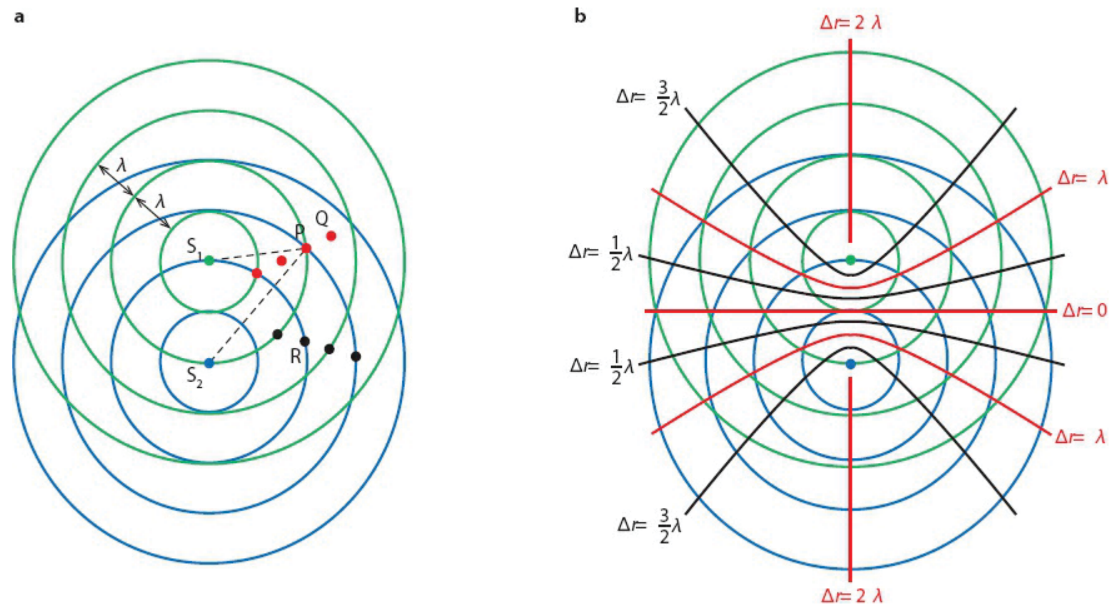


Figure 4.26 Wavefronts from two sources

Figure 4.26a shows two sources and various points at different distances from the sources.

Points in red have a path difference that is in integral multiple of the wavelength, so they correspond to points of constructive interference. Points in black have a path difference that is a half-integral multiple of the wavelength, so they correspond to points of destructive interference. Figure 4.26b shows the result of joining points with the same path difference.

TEST YOURSELF 4.7

Two sources S_1 and S_2 emit waves of wavelength $\lambda = 0.30$ m. Point P is such that $S_1P = 3.40$ m and $S_2P = 4.0$ m and point Q is such that $S_1Q = 3.20$ m and $S_2Q = 3.65$ m. Determine what is observed at both P and at Q.

Interference pattern from two sources

In Figure 4.27, light is incident on a pair of very small slits in a screen. The light diffracted through these slits acts like a pair of sources which are **coherent** – that is, their waves are exactly in step with one another. The diffracted waves superpose at a screen far away. The figure shows the variation in intensity observed on the screen as a function of the distance along the screen. (Units on the vertical axis are arbitrary.)

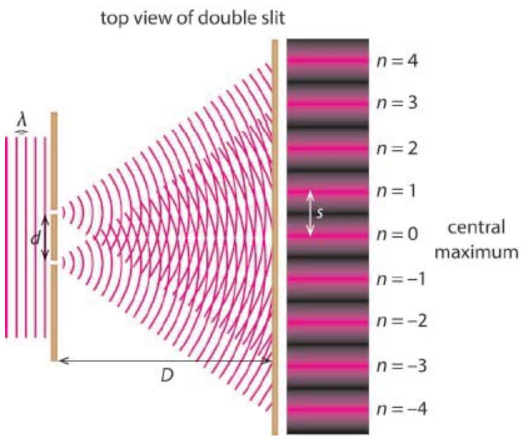


Figure 4.27

The pattern shows equally spaced maxima with equal intensities (Figure 4.28). The maxima and minima depend on the path difference from the two slits.

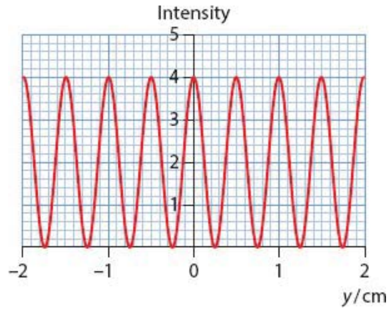
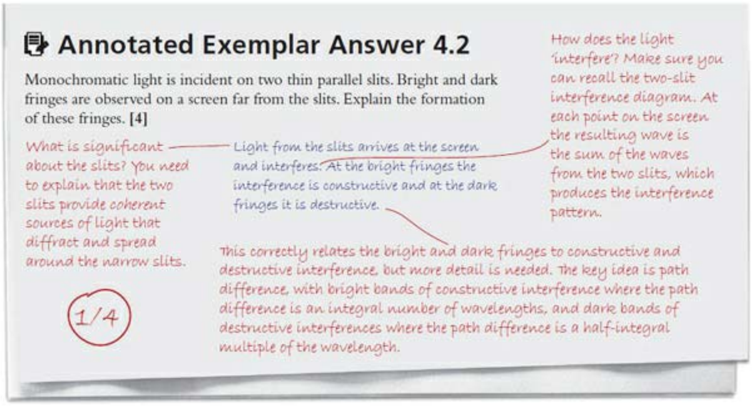


Figure 4.28

The separation Δs of two consecutive maxima (or minima) is given by $\Delta s = \frac{\lambda D}{d}$.

D = distance to screen

d = separation of slits



Annotated Exemplar Answer 4.2

Monochromatic light is incident on two thin parallel slits. Bright and dark fringes are observed on a screen far from the slits. Explain the formation of these fringes. **[4]**

Light from the slits arrives at the screen and interferes. At the bright fringes the interference is constructive and at the dark fringes it is destructive.

What is significant about the slits? You need to explain that the two slits provide coherent sources of light that diffract and spread around the narrow slits.

How does the light 'interfere'? Make sure you can recall the two-slit interference diagram. At each point on the screen the resulting wave is the sum of the waves from the two slits, which produces the interference pattern.

This correctly relates the bright and dark fringes to constructive and destructive interference, but more detail is needed. The key idea is path difference, with bright bands of constructive interference where the path difference is an integral number of wavelengths, and dark bands of destructive interferences where the path difference is a half-integral multiple of the wavelength.

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TEST YOURSELF 4.8

Light carries energy. What happens to the energy of two waves that interfere destructively?

> **hint**
>
> Remember that energy must be conserved.

Model Answer 4.1

Two coherent sources of light of equal amplitude produce an interference pattern. Describe and explain the changes to the intensity of the fringes if the amplitude of one of the sources is now reduced.

Let the amplitudes of the two waves at the point where they meet be A_1, A_2. The amplitude will be $A_1 + A_2$ at points of constructive interference and $A_1 - A_2$ at points of destructive interference. Since $A_1 \neq A_2$, at the minima there will no longer be complete cancellation, so the fringes will no longer be completely dark. At the maxima the amplitude will be less than before and so these fringes will no longer be as bright as before.

> **hint**
>
> Think carefully about what happens at the maxima and at the minima. This is an 'explain' question, so some detail is required.

Interference pattern from a single source

Figure 4.29 shows a wave moving through a small opening.

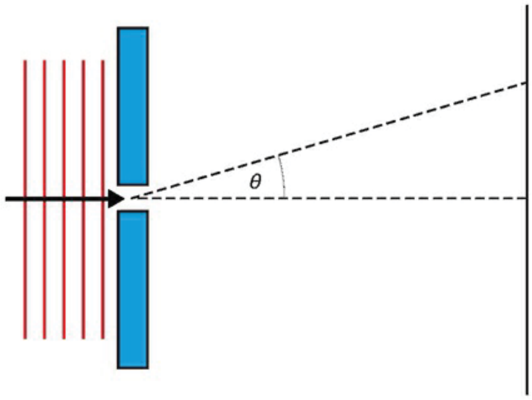

Figure 4.29

The resulting complicated intensity pattern at a screen far away is shown in Figure 4.30 as a function of the angle θ from the central maximum. The wave spreads and gives a non-zero intensity even when $\theta \neq 0$.

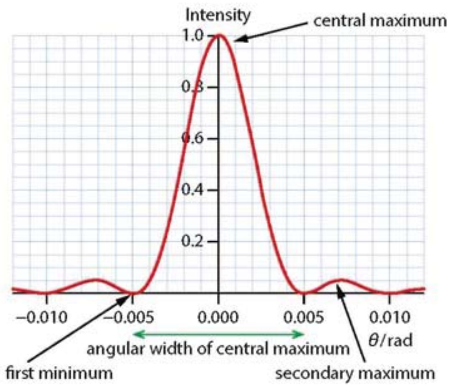

Figure 4.30

This can be described as the result of interference between rays leaving *different parts* of the opening. The pattern is typical of single-slit diffraction, and you should be familiar with its features. (Units on the vertical axis are arbitrary.)

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Annotated Exemplar Answer 4.2

Monochromatic light is incident on two thin parallel slits. Bright and dark fringes are observed on a screen far from the slits. Explain the formation of these fringes. **[4]**

What is significant about the slits? You need to explain that the two slits provide coherent sources of light that diffract and spread around the narrow slits.

Light from the slits arrives at the screen and interferes. At the bright fringes the interference is constructive and at the dark fringes it is destructive.

How does the light 'interfere'? Make sure you can recall the two-slit interference diagram. At each point on the screen the resulting wave is the sum of the waves from the two slits, which produces the interference pattern.

This correctly relates the bright and dark fringes to constructive and destructive interference, but more detail is needed. The key idea is path difference, with bright bands of constructive interference where the path difference is an integral number of wavelengths, and dark bands of destructive interferences where the path difference is a half-integral multiple of the wavelength.

1/4

4.8 Polarisation

DEFINITIONS

POLARISED WAVE A wave in which the displacement remains always in a single plane. This is a property of transverse waves only.

Light is a transverse wave, with electric and magnetic fields oscillating at right angles to one another and to the direction of energy transfer. In Figure 4.31a, the electric field is oscillating in a single plane, vertical in this case. In Figure 4.31b it is oscillating in a horizontal plane. We say the light is **polarised.**

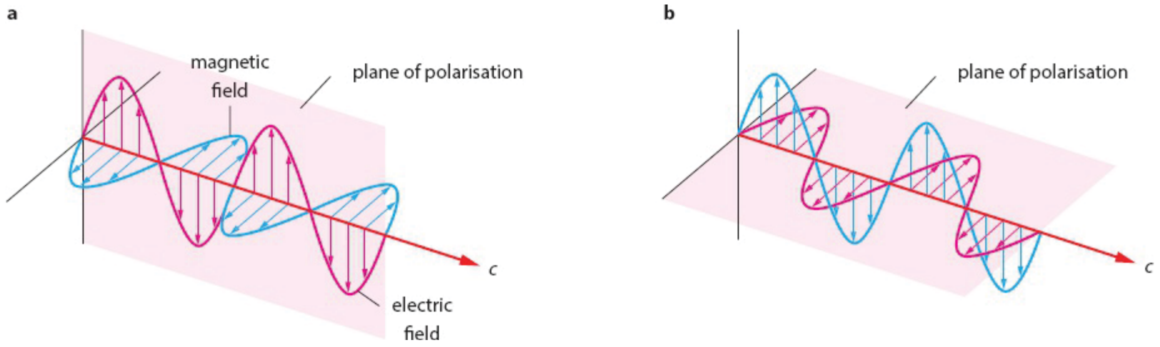


Figure 4.31 An EM wave that is **a** vertically polarised; **b** horizontally polarised.

Light from light bulbs or the Sun is unpolarised. Light can be polarised by passing it though a **polariser,** typically a piece of plastic whose long molecular structure only allows the transmission of the component of the electric field parallel to a particular direction (Figure 4.32).

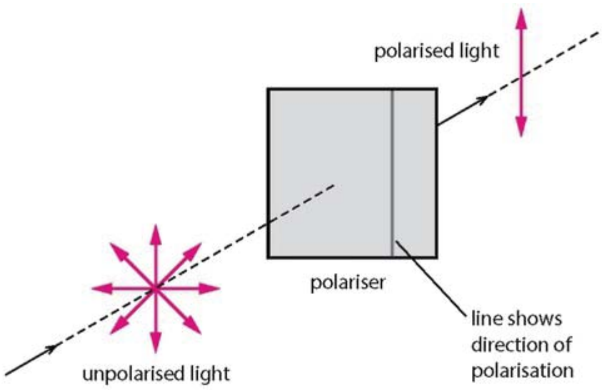


Figure 4.32

When unpolarised light is transmitted through a polariser the transmitted intensity is half the incident intensity. In Figure 4.33a, unpolarised light is transmitted through a polariser with a vertical transmission axis, and the transmitted intensity is half the incident intensity, $I = \frac{I_0}{2}$.

If this polarised light is then passed through another polariser whose transmission axis is at right angles to the direction of the electric field, no electric field is transmitted and so no light passes through (Figure 4.33b).

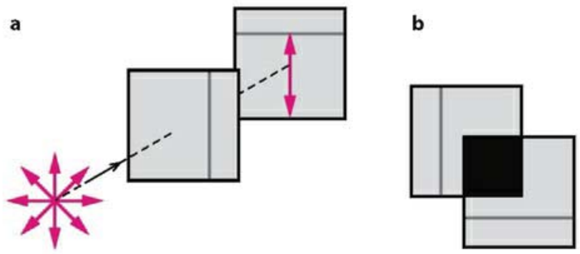


Figure 4.33

Malus' law

Consider vertically polarised light that is directed towards a polariser with a vertical transmission axis. The polariser is then rotated about the direction of the ray by an angle θ. The incident light has its electric field vertical. We know that only the component parallel to the transmission axis gets through. The component of the electric field along an axis parallel to the transmission axis is $E_0 \cos\theta$ (Figure 4.34).

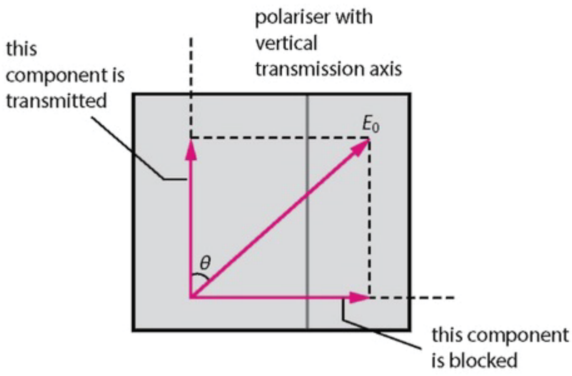


Figure 4.34

The transmitted *intensity* is therefore $I = I_0 \cos^2 \theta$, a result known as **Malus' law.** Figure 4.35 shows the transmitted intensity as the polariser is rotated from 0 to 180°.

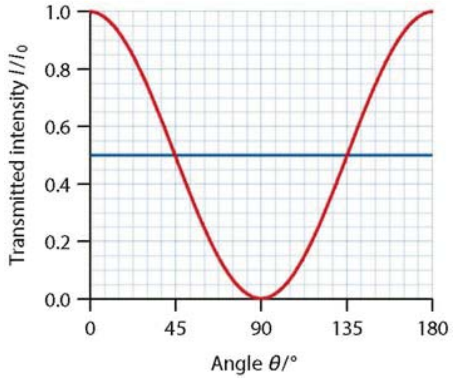


Figure 4.35

TEST YOURSELF 4.9

Unpolarised light is incident on three polarisers that are arranged one behind the other. The transmission axes of the first and third polarisers are vertical and that of the middle polariser is at 45° to the other two. Calculate the fraction of intensity that is transmitted.

Polarisation by reflection

Light will reflect off a surface. It turns out that light with electric field parallel to the surface is reflected more efficiently than light with electric field in other directions. The result is reflected light that is *mainly horizontally polarised.* This is indicated in Figure 4.36 by the smaller double-headed arrows for the electric field in the plane of the page.

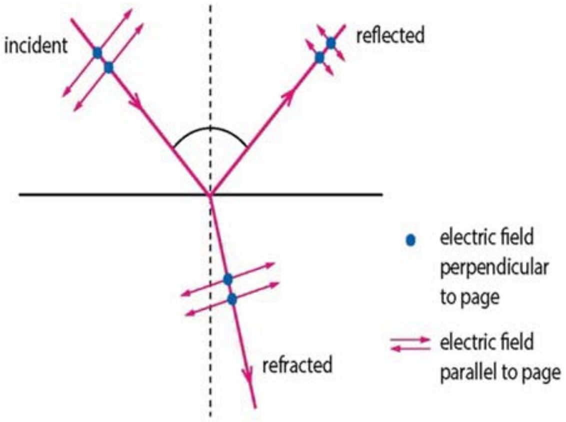


Figure 4.36

Competing theories and progress in science

Nature of Science. At the start of this section we mentioned the conflict between Newton and Huygens over the nature of light. In 1817 Augustin-Jean Fresnel published a new wave theory of light. The mathematician Siméon Poisson favoured the particle theory of light, and worked out that Fresnel's theory predicted the presence of a bright spot in the shadow of a circular object, which he believed was impossible. François Arago, a supporter of Fresnel, was able to show there was indeed a bright spot in the centre of the shadow. In further support of his theory, Fresnel was able to show that the polarisation of light could only be explained if light was a transverse wave. The wave theory then took precedence, until new evidence showed that light could behave as both a wave and a particle (but unfortunately not the type of particle Newton had in mind!).

4.9 Standing waves

A **standing wave** is a wave formed when two identical travelling waves moving in opposite directions meet and superpose.

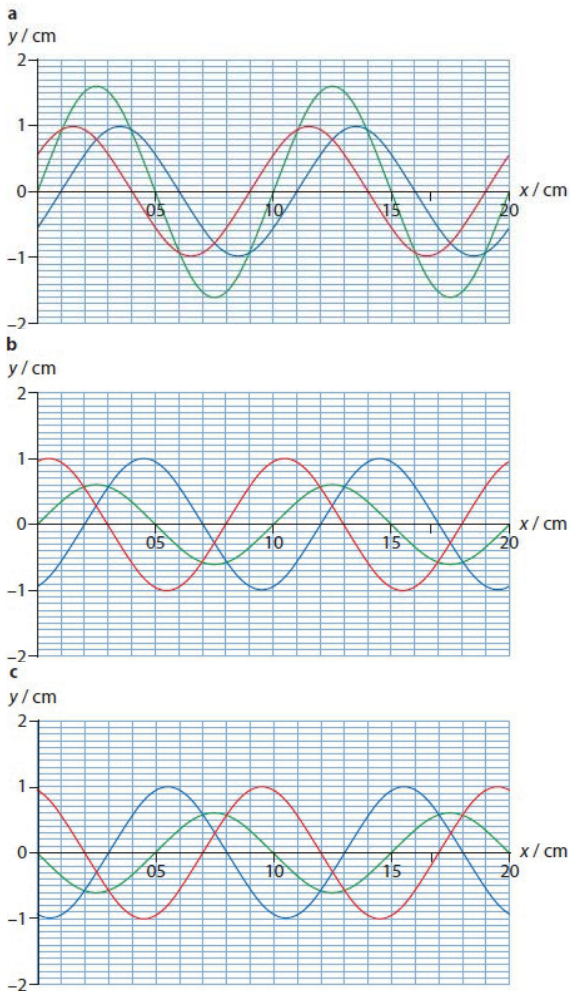


Figure 4.37

Figure 4.37 shows, at three consecutive times:

- a travelling wave on a string, moving *to the right* (blue line)
- a second *identical* wave travelling *to the left* on the same string (red line)
- the *superposition* of the two travelling waves (green line).

We can see that the superposition (green line) stays in the same place; *only the amplitude changes.* This is why this is called a standing wave: the pattern does not move to the right or left.

Certain points on the string (x = 0, 0.5, 1.0, 1.5 and 2.0 m) always have zero displacement: these points are called **nodes.** Note that the distance between consecutive nodes is half a wavelength.

Standing waves on strings

If a string is fixed at both ends, these ends are both nodes of the standing wave. Figure 4.38 shows examples of several standing waves on a string with fixed ends.

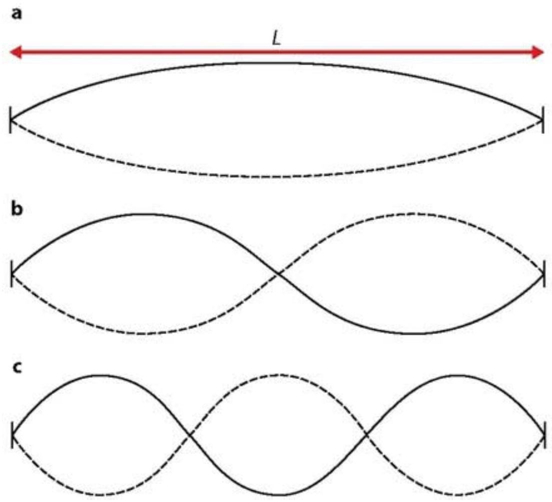


Figure 4.38

In Figure 4.38a the entire string length is occupied by one half-wave. This standing wave is called the **first harmonic:** $L = 1 \times \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$.

In Figure 4.38b the entire length is occupied by two half-waves. This is called the **second harmonic:**

$L = 2 \times \frac{\lambda_2}{2} \Rightarrow \lambda_2 = L$.

In Figure 4.38c the string length is occupied by three half-waves. This is called the **third harmonic:**

$L = 3 \times \frac{\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2L}{3}$.

Each of these modes has a unique frequency. To create a particular harmonic, we vibrate the string up and down (for example by attaching one end to an oscillator) with its associated frequency. This creates a travelling wave on the string, which, upon reflection from the fixed end, creates a second travelling wave in the opposite direction.

TEST YOURSELF 4.10

A string has both ends fixed. Calculate the ratio of the frequency of the second to the first harmonic.

Standing waves in pipes

Standing waves can also be produced within pipes.

Open–open pipes

Consider a pipe of length L that is open at both ends. A travelling wave sent down the pipe will reflect from the ends (even though they are open), so we have conditions for the formation of a standing wave.

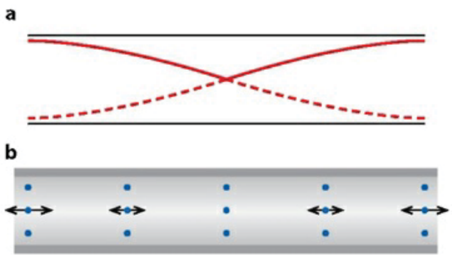


Figure 4.39

Figure 4.39 represents the **first harmonic** in a pipe with *both ends open.* Panel (a) shows the *mathematical* shape of the wave, while panel (b) illustrates how the air molecules oscillate in the pipe. The molecules at the ends oscillate the most: they are at antinodes. The molecules in the middle do not oscillate at all: they are at a node. In this case, $L = 1 \times \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$.

Figure 4.40 represents the **second harmonic** in a pipe with *both ends open.* Here $L = 2 \times \frac{\lambda_2}{2} \Rightarrow \lambda_2 = L$.

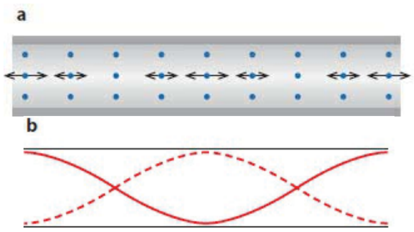


Figure 4.40

Closed–open pipes

Pipes can have open or closed ends. If the end is closed the standing wave established in the pipe will have a node at this end. If the end is open the standing wave will have an antinode.

In Figure 4.41 one quarter-wavelength fills the pipe; this the **first harmonic** of a pipe with *one end open:*

$L = 1 \times \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L.$

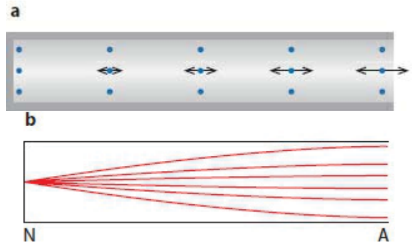


Figure 4.41

In a similar way, in Figure 4.42 three quarter-wavelengths fill the pipe; this is the **third harmonic** of a pipe with *one end open:* $L = 3 \times \frac{\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4L}{3}$.

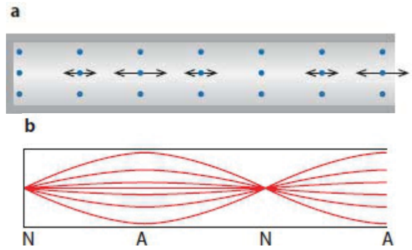


Figure 4.42

Note that *only odd-numbered harmonics are possible in a pipe with one end open.*

Differences between travelling and standing waves

Travelling waves transfer energy, standing waves do not. (Standing waves do have energy but do not transfer it.)

Travelling waves have constant amplitude, standing waves do not.

Different points on a standing wave oscillate with different amplitudes. In travelling waves the phase changes as we move along the length of the medium. In standing waves points within two consecutive nodes are in phase. Points within the next pair nodes are also in phase but differ by a phase of π from the first set of points.

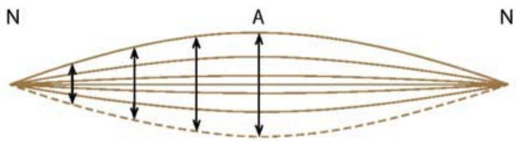


Figure 4.43

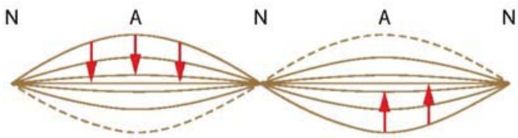


Figure 4.44

TEST YOURSELF 4.11

Two tubes have the same length. One (X) is open at both ends and the other (Y) is closed at one end and open at the other. Calculate the ratio of the frequencies $\frac{f_X}{f_Y}$ of the first harmonics.

TEST YOURSELF 4.12

A tuning fork of frequency 560 Hz is sounded over a tube that is filled with water (Figure 4.45).

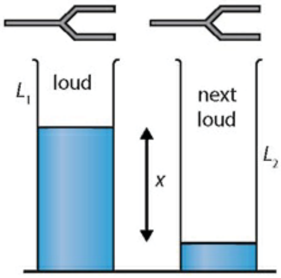


Figure 4.45

hint

The tuning fork is the same so the wavelength of sound is the same in both cases. So we must have the first harmonic in the first case and the third harmonic in the other with equal wavelengths.

The water is slowly removed and, when the length of the air column is L_1, a loud sound is heard from the tube for the first time. The level of the water is slowly reduced further, and when a loud sound is again heard, the length of the air column is L_2 such that $L_2 - L_1 = x = 30$ cm. Calculate the speed of sound.

hint

How is x related to the wavelength?

Checklist

After studying this chapter you should be able to:

- □ describe simple harmonic oscillations and identify the conditions that lead to them
- □ understand the basic wave characteristics of period, amplitude, frequency, wavelength, wavefronts, rays and intensity
- □ describe the difference between transverse and longitudinal waves
- □ describe the nature of electromagnetic waves
- □ describe the nature of sound
- □ apply the principle of superposition
- □ understand polarisation
- □ apply the laws of reflection and refraction including total internal reflection
- □ understand the phenomena of diffraction and interference
- □ understand the formation of standing waves and solve problems with standing waves.

5 ELECTRICITY AND MAGNETISM

This chapter covers the following topics:

- □ Coulomb's law
- □ Electric fields
- □ Electric current and electric resistance
- □ Electric circuits
- □ EMF and power
- □ Primary and secondary cells
- □ Magnetic fields and magnetic forces

5.1 Coulomb's law and electric fields

DEFINITIONS

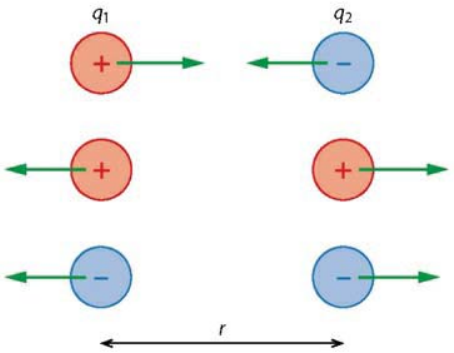


Figure 5.1

COULOMB'S LAW Two point charges q_1, q_2 separated by a distance r exert a force on each other whose magnitude is $F = k\frac{q_1 q_2}{r^2}$. The constant k, known as Coulomb's constant, is $k = 8.99 \times 10^9$ N m^2 C^{-2}. The force is directed along the line joining the two point charges. It is attractive for charges of opposite sign and repulsive for charges of the same sign (Figure 5.1).

ELECTRIC FIELD, E The electric force per unit charge exerted on a point positive charge q:

$$E = \frac{F}{q} \Leftrightarrow F = qE$$

A positive point charge q placed a distance r from the centre of another point charge Q will experience a force

$F = k\frac{Qq}{r^2}$, so the electric field created by Q is $E = \frac{F}{q} = \frac{\frac{kQq}{r^2}}{q} = \frac{kQ}{r^2}$ (see Figure 5.2).

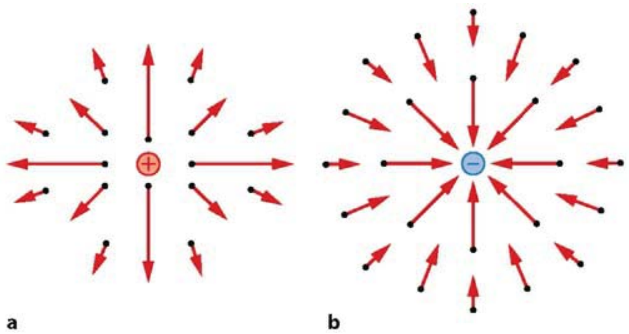


Figure 5.2 The electric field at various positions near **a** a positive and **b** a negative point charge.

TEST YOURSELF 5.1

The force between two positively charged point particles X and Y is F. The charge on X is doubled. Which entry below now describes the forces on each charge?

	Force on X	Force on Y
a	2F	F
b	2F	2F
c	F	F
d	F	2F

TEST YOURSELF 5.2

Draw the electric field around a negatively charged conducting sphere.

hint

What is the direction of the field? At what angle do the field lines meet the surface of the sphere? What is the electric field inside a conductor?

TEST YOURSELF 5.3

An electrically neutral conducting sphere is suspended vertically from an insulating thread. A point charge of magnitude Q is brought near the sphere. The electric force between the point charge and the sphere:

hint

The sphere is conducting, so its charge can move around under the influence of the point charge.

a depends on whether Q is positive or negative.
b is always zero.
c is always repulsive.
d is always attractive.

TEST YOURSELF 5.4

Four point charges of equal magnitude are fixed at the corners of a square (Figure 5.3).

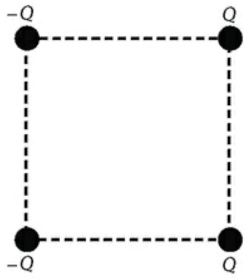


Figure 5.3

Which arrow shows the direction of the resultant electric field strength at the centre of the square due to the four point charges?

hint

Imagine a positive charge at the centre of the square and find the direction of the force on this charge from each of the four charges separately. Then use vector addition.

5.2 Electric current and resistance

DEFINITIONS

ELECTRIC CURRENT, *I* The charge that moves past a cross-section of a conductor per unit time: $I = \frac{\Delta Q}{\Delta t}$. The unit of current is the ampere; 1A = Cs^{-1}. (The definition of the **ampere** is given in terms of the magnetic force between parallel currents.)

POTENTIAL DIFFERENCE (PD) BETWEEN TWO POINTS The work done per unit charge in moving charge between the two points: $V = \frac{W}{q}$. The unit of potential difference is the volt (V); 1V = 1 JC^{-1}. Note that the work done to move a charge *q* through a potential difference *V* is then $W = qV$.

ELECTRON VOLT (eV) 1 eV is the energy required to move a charge of *e* through a potential difference of 1 V. Thus an electron moved through a potential difference of 6 V requires 6 eV, and an alpha particle (charge 2e) requires 12 eV. In S.I. units, 1 eV = 1.6×10^{-19} C × 1V = 1.6×10^{-19} J.

ELECTRICAL RESISTANCE The ratio of the potential difference *across* a conductor to the current *through* it: $R = \frac{V}{I}$.

The unit of resistance is the ohm: 1Ω = 1 V A^{-1}. A current of 2.0 A in a resistor of 25Ω implies a potential difference across the resistor of $V = RI = 50$ V.

POWER The rate at which energy is dissipated or produced.

OHM'S LAW At constant temperature, many metallic conductors have the property that the current through them is proportional to the potential difference across them: $I \propto V$. This is known as Ohm's law. It implies that the resistance is constant. A graph of current through the conductor versus the voltage across it then gives a straight line through the origin. Materials with such a *I*–*V* relationship are called **ohmic**.

CONDUCTOR A material, such as a metal, with lots of 'free' electrons – electrons dispersed throughout the metal volume which do not belong to any particular atom (Figure 5.4).

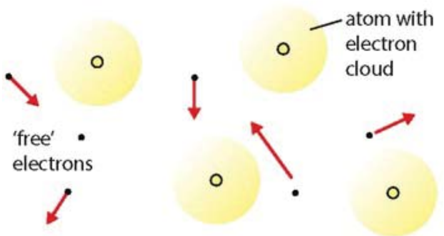


Figure 5.4

When an electric field is applied within a conductor, its free electrons acquire a small **drift velocity** opposite to the electric field (Figure 5.5). (This diagram is not to scale. Drift velocity is very much smaller than the random velocity of the electrons, typically $v_{drift} = 10^{-3}$ m s^{-1}, whereas $v_{random} = 10^{5}$ m s^{-1}.)

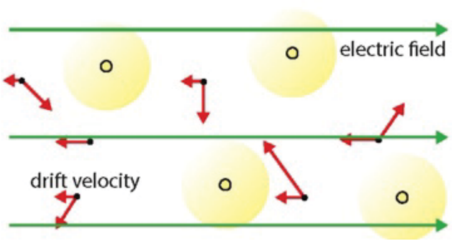


Figure 5.5

This means that the electrons have a net motion, opposite to the electric field, so electric charge is transferred in a direction opposite to the electric field. This is called **electric current**.

INSULATOR A material with very few or no free electrons.

The microscopic–macroscopic connection

Nature of Science. If you are plumber, do you need to know the molecular structure of water? The flow of water in pipes is a macroscopic phenomenon whereas the detailed molecular structure of water is microscopic. We have a vast difference in scales of length in the two cases. In very many phenomena the presence of two different scales means that the detailed physics operating at one scale does not affect the physics at the other.

This is also the case with current: it was possible to give detailed descriptions of the behaviour of current in circuits long before it was discovered that current is electrons moving in the same direction. (However, the most complicated problems in physics are those in which the physics at one length scale does affect the physics at the other scale.)

TEST YOURSELF 5.5

Calculate the resistance of a conductor whose current varies with applied voltage as shown in Figure 5.6 when the voltage is 0.30 V.

> hint
>
> It’s not the slope or inverse slope that you need to find.

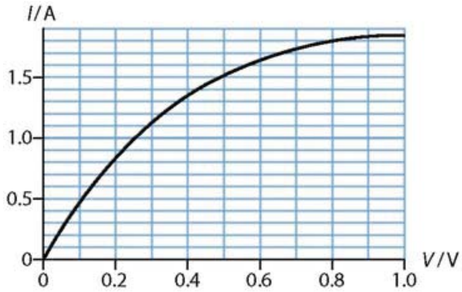


Figure 5.6

Formulas for power: Since current is defined as $I = \frac{\Delta Q}{\Delta t}$, in a time Δt an amount of charge $\Delta Q = I\,\Delta t$ gets transferred through a conductor (Figure 5.7).

If the potential difference across the conductor is V, the work required to move the electric charge is $W = (\Delta Q)\,V = (I\Delta t)\,V$. The power, which is work per unit time, is $P = \frac{I\Delta t V}{\Delta t}$, thus $P = VI$.

Using the definition of resistance, $R = \frac{V}{I}$, we have the equivalent forms $P = VI = RI^2 = \frac{V^2}{R}$.

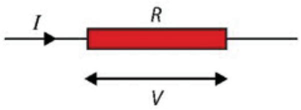


Figure 5.7

Formula for resistance of a wire: For a conductor of uniform cross-sectional area (Figure 5.8), $R = \rho\frac{L}{A}$, where L is the length and A the cross-sectional area of the conductor. The constant ρ is called the **resistivity**, and depends on temperature and the conductor material.

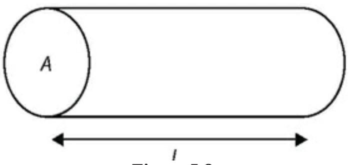


Figure 5.8

Formula for current: Imagine a conductor of cross-sectional area A. Let there be n free electrons per unit volume, and suppose that these electrons have a drift velocity v (Figure 5.9).

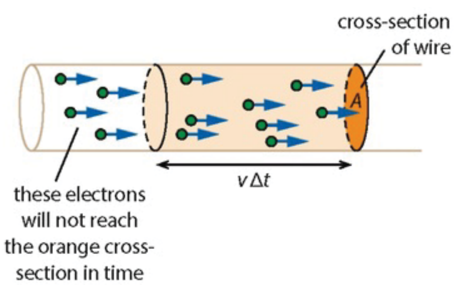


Figure 5.9

In time Δt the electrons that will cross the orange cross-sectional area are those within a distance $v\Delta t$ of that cross-sectional area – that is, within the shaded volume $(v\Delta t)\,A$. The total number of electrons within the shaded volume is $n(v\Delta t)A$, so the charge within the volume is $\Delta Q = qn(v\Delta t)A$ (here q stands for the charge on one electron). So the current is $I = \frac{\Delta Q}{\Delta t} = \frac{qn(v\Delta t)A}{\Delta t} = qnvA$.

TEST YOURSELF 5.6

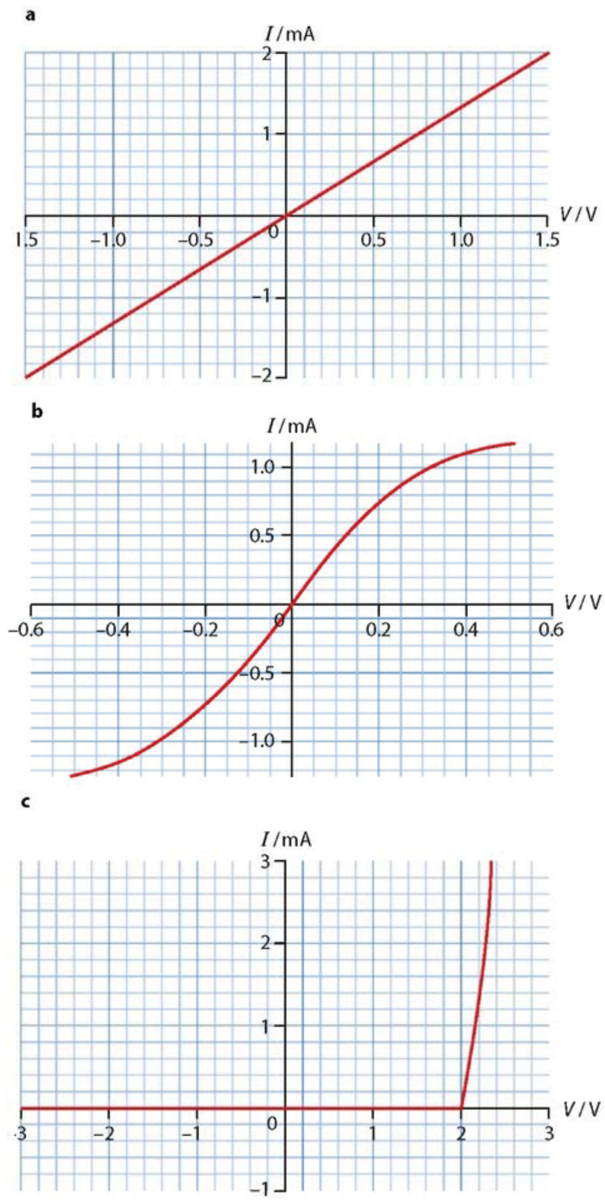
Figure 5.10 shows a conductor of variable cross-sectional area. The cross-sectional area at Y is four times that at X.

Figure 5.10

Which of the following is correct about the ratio of currents and drift speeds at X and Y?

	$\frac{I_X}{I_Y}$	$\frac{V_X}{V_Y}$
a	1	$\frac{1}{4}$
b	1	4
c	4	$\frac{1}{4}$
d	4	4

Figure 5.11 shows the *I*–*V* characteristics of various devices. Only **a** is ohmic. In **b** the resistance increases as *V* increases, as happens in a filament lamp, for example. Graph **c** represents a diode and **d** a thermistor. (In exams, the voltage is sometimes plotted on the vertical axis, so be careful.)



d

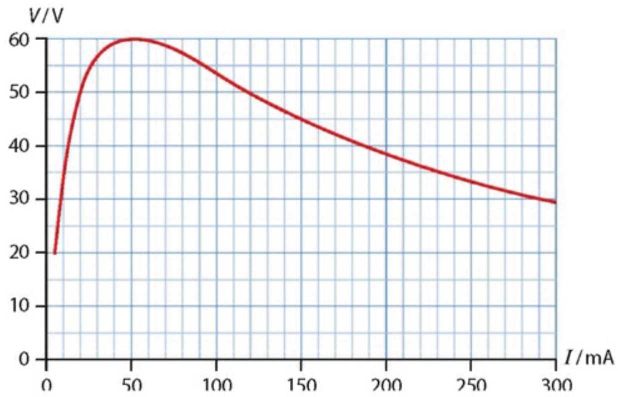


Figure 5.11

TEST YOURSELF 5.7

Determine the resistance of a lamp rated as 60 W at 240 V. Assuming that the resistance stays constant, what would the power of the lamp be if it were connected across a voltage of 120 V?

TEST YOURSELF 5.8

a The length and the radius of a cylindrical conductor of resistance *R* are both doubled at constant temperature. Calculate the new resistance of the conductor.

b A cylindrical conductor has length *L* and resistance *R*. The conductor is cut into two equal parts, each of length *L*/2. The two parts are connected in parallel. Determine the resistance of the combination.

Annotated Exemplar Answer C.1

a By reference to diffraction, state one advantage of using high-frequency ultrasound in medical diagnosis. **[2]**

b Explain why, in medical diagnosis using ultrasound, a gel-like substance is placed on the patient's skin between the skin and the probe. **[2]**

a High-frequency ultrasound has a small wavelength (about 0.15 mm). This allows smaller structures to be detected than with ultrasound of a lower frequency, which would have a longer wavelength.

The answer shows that you understand that the higher the frequency, the shorter the wavelength, and that the size of the object that can be imaged depends on the wavelength. But you lose a mark for not making the link to diffraction. Always check that you have answered the question.

b The gel has impedance close to that of tissue and so most of the ultrasound gets transmitted into the body. The ultrasound that enters the body gets reflected off organs and can also exit the body so it can be picked up by a detector.

You need to mention what happens if the impedances do not match. Without the gel, most of the ultrasound would be reflected at the interface between air and skin. Make sure you include all relevant detail in your written answer.

2/4

< Back to chapter

Annotated Exemplar Answer 5.1

The resistivity of copper is 1.69×10^{-8} Ω m.

a Using copper as an example, explain what is meant by ‘resistivity’. **[2]**

b A copper wire of diameter 1.00 mm and length *L* m has a resistance of 0.20 Ω. What diameter of copper wire of the same length would have a resistance of 0.80 Ω? **[3]**

a $\rho = \frac{RA}{L}$

The equation for resistivity is in the data booklet, so stating the equation is easy to do and can help you to recall how the variables are linked. Make sure you know what is in the booklet and how to use it.

The equation is not enough on its own – you need this additional text to ‘explain

The resistivity ρ depends on the material of the conductor and its temperature.

This clearly links the value given for copper to a theoretical conducting wire.

It is a measure of how easy it is for current to flow in a material. For copper a wire 1 m long with cross-sectional area 1 m^2 would have a resistance of 1.69×10^{-8} Ω.

b $R \propto \frac{1}{A}$

Good – the essential relationship is clearly stated.

The inverse proportion between resistance and cross-sectional area is correct.

The resistance is 4 times as great so the area is $\frac{1}{4}$ the size.

$Diameter = \frac{1.00}{4} = 0.25\,mm.$

A slip at the end means the answer is wrong. The area is proportional to diameter^2, so the diameter is half the size, 0.50 mm.

4/5

5.3 EMF, cells and circuits

DEFINITIONS

EMF The total work done per unit charge in moving charge from one terminal of a cell to the other:

$$\text{EMF}, \varepsilon = \frac{W}{Q}$$

EMF is associated with conversions of various forms of energy into electrical energy. In the case of a battery chemical energy gets converted into electrical energy. EMF is measured in joules per coulomb, and this combination is called the volt.

PRIMARY CELL A battery or cell that runs out of energy and must therefore be discarded when it runs out of energy.

SECONDARY CELL A battery or cell that can be recharged and used again.

TERMINAL VOLTAGE The voltage V at the terminals of a battery or cell. It is given by $V = \varepsilon - Ir$, where ε is the battery's EMF and r is its internal resistance (see Figure 5.12).

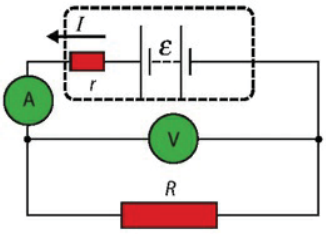


Figure 5.12

The power dissipated in a resistance R is VI. The power dissipated in the battery's internal resistance r is rI^2. Hence $\varepsilon I = rI^2 + VI \Rightarrow V = \varepsilon - Ir$ (energy conservation). So the potential difference across the battery in Figure 5.12 is exactly equal to the EMF *only when the current in the circuit is zero.*

EMF and power

Since

$$\text{EMF}, \varepsilon = \frac{W}{Q}$$

dividing numerator and denominator by time gives

$$\text{EMF}, \varepsilon = \frac{\frac{W}{t}}{\frac{Q}{t}} = \frac{\text{total power}}{\text{total current}}$$

In other words, we have the useful result that the total power delivered by a battery is εI.

Series circuits

Here are two important facts about a series circuit – that is, one in which resistances are connected one after the other so the same current runs though each (Figure 5.13).

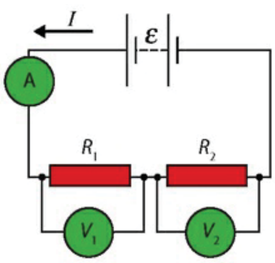


Figure 5.13

The total power delivered by the battery is εI. The power used in R_1 is R_1I^2. The power used in R_2 is R_2I^2.

Hence, $\varepsilon I = R_1 I^2 + R_2 I^2 \Rightarrow I = \frac{\varepsilon}{R_1 + R_2}$.

So **the effective (total) resistance of two resistors in series is the sum of the two.**

For the same reason, we also have that $\varepsilon I = V_1 I + V_2 I \Rightarrow \varepsilon = V_1 + V_2$.

So **the sum of the potential differences across resistors in series is equal to the EMF of the battery**.

In the circuit in Figure 5.13, if ε = 12 V, R_1 = 10 kΩ and R_2 = 50 kΩ, we must have that $V_1 + V_2$ = 12 V and $V_2 = 5 \times V_1$ (since both have the same current and R_2 is $5R_1$). We therefore find V_1 = 2.0 V and V_2 = 10 V

TEST YOURSELF 5.9

A battery of internal resistance 0.80 Ω sends a current of 1.4 A through an external resistance. The work required to push one electron through the external resistor is 7.2×10^{-19} J. Calculate the EMF of the battery.

Parallel circuits

A parallel circuit is one in which resistances are connected in parallel – that is, sharing the same potential difference – as in Figure 5.14.

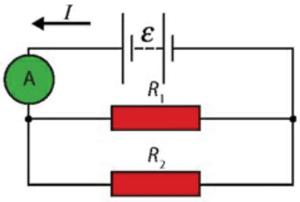


Figure 5.14

The total current delivered by the battery is $\frac{\varepsilon}{R_T}$, where R_T is the total resistance of the parallel combination of the two resistors R_1 and R_2. The current in R_1 is $\frac{\varepsilon}{R_1}$. The current in R_2 is $\frac{\varepsilon}{R_2}$. Therefore, by conservation of electric charge:

$$\underbrace{\frac{\varepsilon}{R_T}}_{\text{total } I} = \underbrace{\frac{\varepsilon}{R_1}}_{\text{current in } R_1} + \underbrace{\frac{\varepsilon}{R_2}}_{\text{current in } R_2} \quad \text{hence} \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}.$$

TEST YOURSELF 5.10

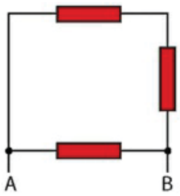


Figure 5.15

a Calculate the total resistance between A and B in Figure 5.15.
Each resistor has a resistance of 60 Ω.

b The resistance between A and B is measured to be 120 Ω.
Which resistor has gone faulty, and how?

TEST YOURSELF 5.11

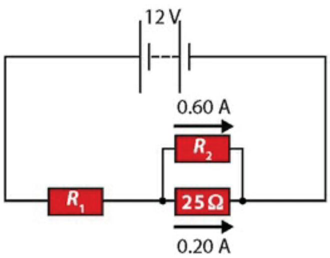


Figure 5.16

In the circuit In Figure 5.16 the battery has negligible internal resistance and an EMF of 12 V.

Calculate the value of **a** resistance R_2 and **b** resistance R_1.

TEST YOURSELF 5.12

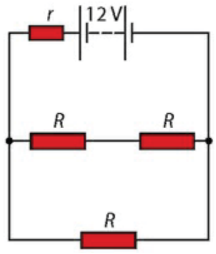


Figure 5.17

A battery of EMF 12 V and internal resistance r = 2.0 Ω is connected to three resistors, each of resistance R = 6.0 Ω, as shown in Figure 5.17.

a Calculate the total current in the circuit.

b The bottom 6.0 Ω resistor burns out (i.e. its resistance becomes infinite). Calculate the total resistance of the circuit.

TEST YOURSELF 5.13

Two lamps, A and B, are connected in series to a battery of negligible internal resistance. Lamp A is brighter than lamp B.

a Determine which lamp has the greater resistance.

b The lamps are now connected to the same battery in parallel. State and explain which lamp is the brighter of the two.

hint

Brightness is related to power. Which form of the power formula is convenient here?

Model Answer 5.1

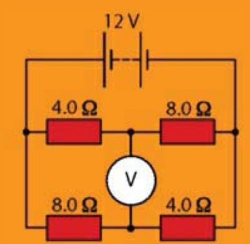


Figure 5.18

In the circuit in Figure 5.18, the battery has EMF 12 V and negligible internal resistance. The voltmeter is ideal. Calculate the magnitude of the reading of the voltmeter.

hint

You now need a different version of the power formula.

The total resistance of the circuit is $\frac{1}{12}+\frac{1}{12}=\frac{1}{6} \Rightarrow R_T=6.0\Omega$ and so the current leaving the battery is $I=\frac{12}{6.0}=2.0\text{A}$. The current in the top 4.0 Ω resistor is then 1.0 A and the potential difference across it is 4.0 × 1.0 = 4.0 V. So the potential at the top end of the voltmeter is 12 − 4.0 = 8.0 V. Similarly, the current in the bottom 8.0 Ω resistor is 1.0 A and the potential difference across it is 8.0 × 1.0 = 8.0 V. So the potential at the bottom end of the voltmeter is 12 − 8.0 = 4.0 V. The potential difference across the voltmeter is then 4.0 V.

The potential divider

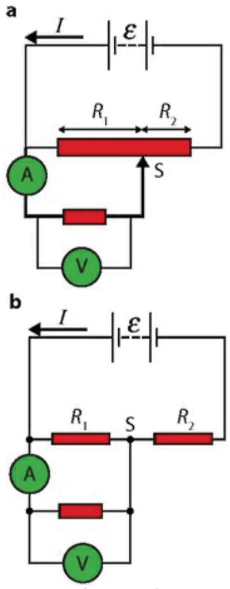


Figure 5.19

The circuit in Figure 5.19a is called a **potential divider** circuit. It looks complicated but is in fact completely equivalent to the conventional circuit in Figure 5.19b. The point where the slider S touches the resistor determines how the resistance R splits into R_1 and R_2.

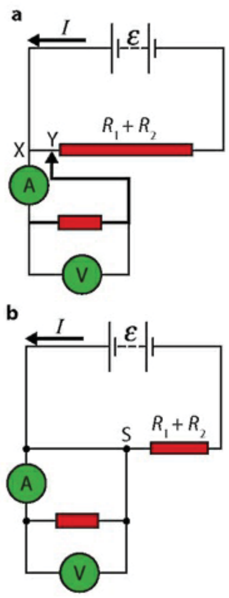


Figure 5.20

Two extreme positions of the slider and their equivalent conventional circuits are shown below. In Figure 5.20 the voltmeter will show 0 V (even if the battery has an internal resistance) because its ends (X and Y) are at the same potential.

In Figure 5.21 the voltmeter will show its greatest reading. This will be ε if the battery has zero internal resistance. It will be $V = \varepsilon - Ir$ otherwise.

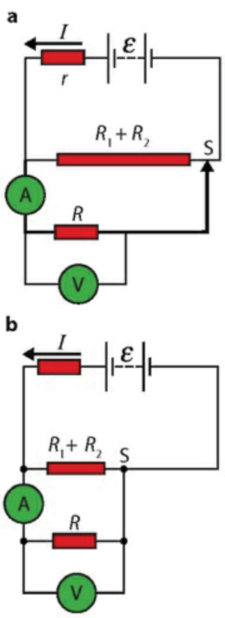


Figure 5.21

TEST YOURSELF 5.14

Calculate the maximum reading of the voltmeter in the potential divider circuit in Figure 5.21 if $\varepsilon = 12$ V, $R_1 + R_2 = 100\ \Omega$, $R = 5.0\ \Omega$ and $r = 2.0\ \Omega$.

TEST YOURSELF 5.15

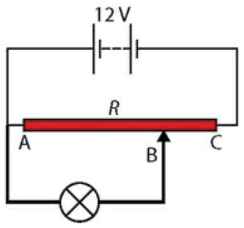


Figure 5.22

Figure 5.22 shows a potential divider circuit. The battery has negligible internal resistance and AC is a wire whose resistance increases uniformly with length.

The lamp (shown as a cross in a circle) is rated as 12 W at 6.0 V at normal operation. The lamp operates normally when the moveable contact is attached to point B such that the length AB is double that of BC. Calculate

a the resistance of the light bulb

b the current through the light bulb

c the resistance *R* of the wire AC.

DEFINITIONS

THERMISTOR A resistor in which the resistance decreases as the temperature increases.

LIGHT-DEPENDENT RESISTOR (LDR) A resistor in which the resistance decreases as the incident intensity of light increases.

TEST YOURSELF 5.16

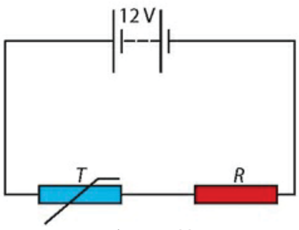


Figure 5.23

a The thermistor *T* in Figure 5.23 has resistance *R* at room temperature. What will happen to the potential difference across *T* when the temperature is increased? Assume zero internal resistance.

b The thermistor is now replaced by an LDR.
What will happen to the voltage across it if the light intensity is increased?s

hint

What happens to the resistance of a thermistor when the temperature increases? What happens to the resistance of an LDR when the light intensity is increased?

Kirchoff’s laws

These two laws can be used to solve complicated multi-loop circuits with more than one cell (and they can be used in simple circuits as well).

The **current law** states that the total current into any junction equals the total current out of the junction:

$$\sum_{\text{in}} I = \sum_{\text{out}} I$$

This is a consequence of charge conservation.

The **loop law** states that, in each closed loop, the sum of the EMFs equals the sum of the voltages across all resistors in the loop:

$$\sum \text{EMF} = \sum V$$

This is a consequence of energy conservation.

In summing EMFs and voltages we use the rules shown in Figure 5.24.

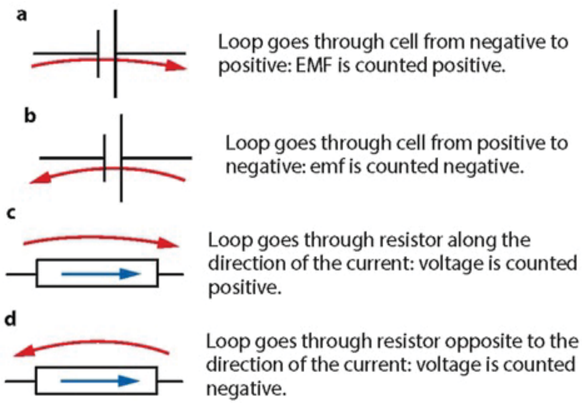


Figure 5.24

Let us apply these laws to the circuit in Figure 5.25.

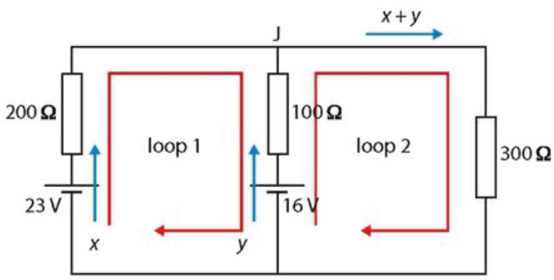


Figure 5.25

1 Draw lines to indicate the direction to go around each loop (red lines).

2 Draw arrows to indicate the direction of current through each cell (blue lines). Name the currents x and y. (We do not know the actual current directions but it will not matter.)

3 Apply the current law to junction J to deduce that the current at the top of loop 2 is $x + y$. This is simpler than calling this current by a third variable such as z. This limits the unknowns to just 2 (x, y) as opposed to 3 (x, y and z).

4 Make sure you label the current in each resistor, and apply the loop law to loop 1 to write:

$$+\underbrace{23}_{\substack{\text{loop through cell 'right' way}\\ \text{so positive sign}}} - \underbrace{16}_{\substack{\text{loop through cell 'wrong' way}\\ \text{so negative sign}}} = +\underbrace{200x}_{\substack{\text{loop same way as current}\\ \text{so positive sign}}} - \underbrace{100y}_{\substack{\text{loop opposite to current}\\ \text{so negative sign}}}.$$

5 Do the same for loop 2 to get: $+16 = +100\,y + 300(x + y)$.

Note that there are several ways you could proceed with this problem – for example, for the second loop you could take the one indicated in Figure 5.26.

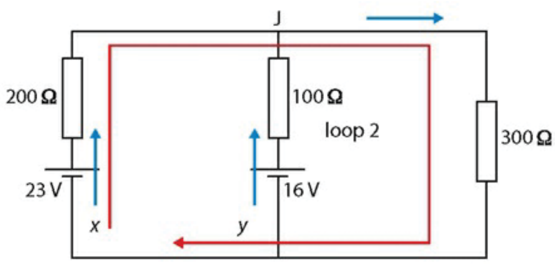


Figure 5.26

(You can use your calculator to solve these equations very quickly.)

$$200x - 100y = 7$$
$$300x - 400y = 16$$

Multiply the first equation by 4 to get

$$800x - 400y = 28$$
$$300x - 400y = 16$$

Add together to get:

$$1100x = 44 \Rightarrow x = \frac{44}{1100} = 0.040\text{A}$$

Hence

$$200 \times 0.040 - 100y = 7 \Rightarrow 100y = 200 \times 0.040 - 7 = 1 \Rightarrow y = 0.010\text{A}$$

Both currents have come out positive. This means that our original guess for their direction was correct. If a current came out negative it would mean that its direction is opposite to what we had assumed.

TEST YOURSELF 5.17

a Copy the circuit in Figure 5.27, and use it to determine the current in each cell (magnitude and direction).

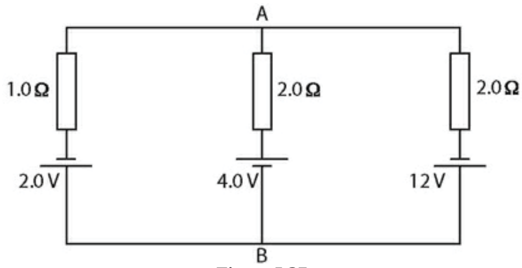


Figure 5.27

b Comment on your answer to **a**.
c Find the potential difference between points A and B.
d Find the power generated and the power dissipated in the circuit and verify that energy is conserved.

hint

You must give the directions of all currents to get full marks.

hint

Look for something unusual or unexpected, rather than an obvious observation such as the fact that the three cells have different currents.

hint

For full marks you should show that power generated and power dissipated are equal.

Discharge of a cell

The discharge time of a cell depends on the rate at which current passes through it. Figure 5.28 shows how the terminal voltage at the cell varies with time.

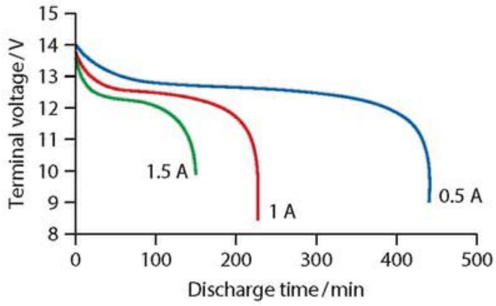

Terminal voltage/V
15
14
13
12
11
10
9
8
1.5 A
1 A
0.5 A
0
100
200
300
400
500
Discharge time/min

Figure 5.28

5.4 Magnetic fields

An electric current in wires of various shapes will produce a *magnetic field*.

The magnetic field is a vector and its direction is tangent to the magnetic field lines, just as electric fields are tangent to electric field lines.

Figure 5.29a shows the magnetic field lines produced by a *long straight* wire. The lines are circles around the wire.

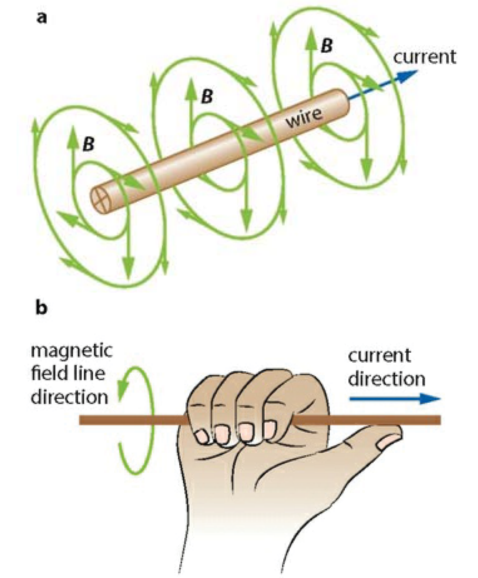


Figure 5.29

The magnetic field itself is tangent to these circles in a direction found from the right-hand grip rule (Figure 5.29b).

A magnetic field is also produced by a current in a solenoid (coil), as pictured in Figure 5.30a.

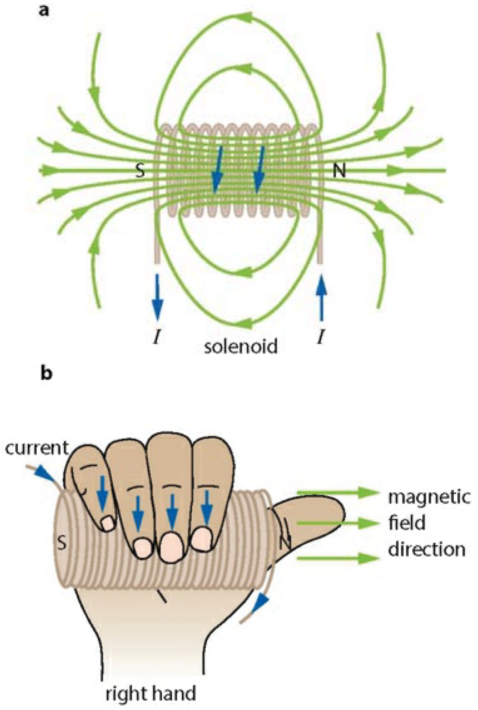


Figure 5.30

A second right-hand grip rule gives the direction of the magnetic field given the direction of the current in the solenoid (Figure 5.30b).

The magnetic field outside the solenoid is similar to that of a bar magnet (Figure 5.31). The magnetic field of the bar magnet is also caused by tiny microscopic currents due to the motion of electrons in the atoms of the magnet.

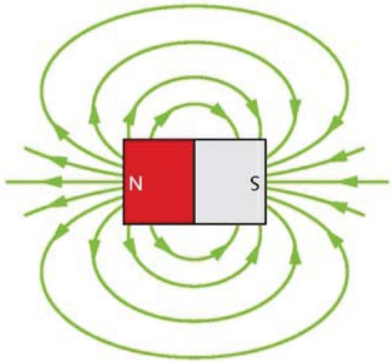


Figure 5.31

Motion of a charged particle in electric and magnetic fields

DEFINITION

MAGNETIC FORCE A magnetic field B exerts a force on moving charges ($F = qvB \sin \theta$) and electric currents ($F = BIL \sin \theta$). The direction of the force is given by the rules illustrated in Figure 5.32. (The hand is a right hand.)

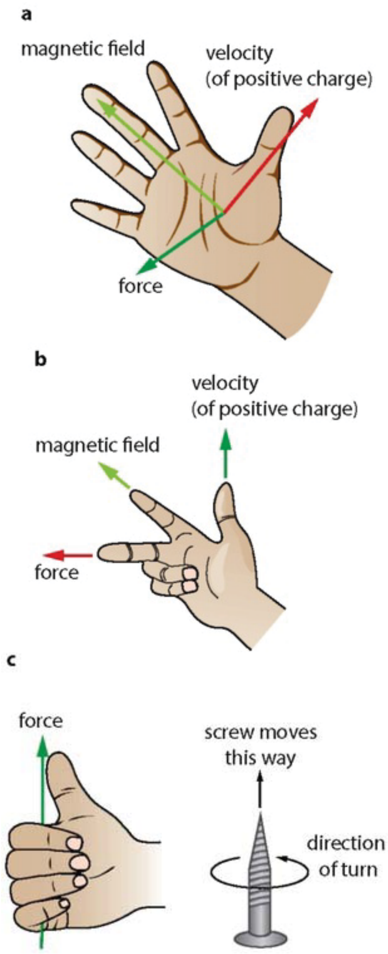


Figure 5.32

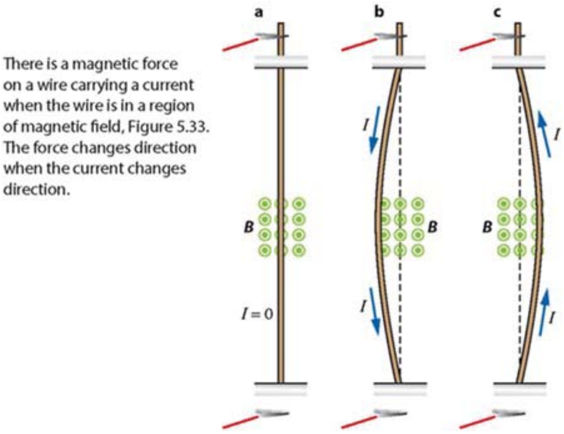


Figure 5.33

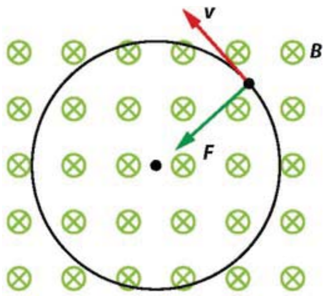


Figure 5.34

A charged particle of mass *m* and charge *q* that enters a region of magnetic field ***B*** at right angles to the magnetic field (Figure 5.34) will move in a circular path of radius *R* given by:

$$\underbrace{m\frac{v^2}{R}}_{\text{centripetal force}} = \underbrace{qvB}_{\text{magnetic force}} \Rightarrow R = \frac{mv}{qB}.$$

The magnetic force does zero work on the charged particle because the force is at right angles to its velocity. Therefore the magnetic force cannot change the speed of the particle (only the direction of the velocity).

Parallel wires carrying current will attract or repel each other. This is because each wire produces magnetic field at the position of the other wire and so a magnetic force is exerted. Parallel currents attract and anti-parallel currents repel (Figure 5.35). The forces (on equal lengths of wire) are equal and opposite even if the currents are different.

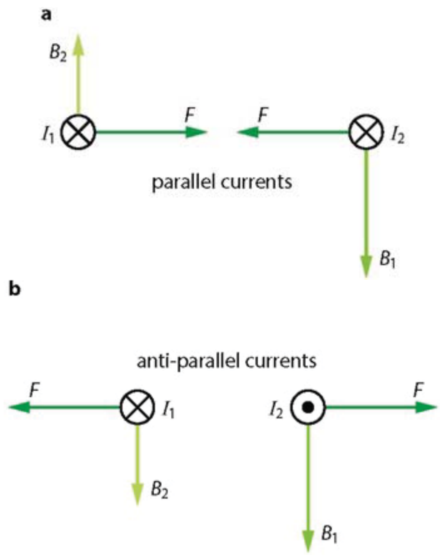


Figure 5.35

DEFINITION

AMPERE Two parallel conductors 1 m apart have a current of 1 A in them if the force on 1 m of their length is exactly 2×10^{-7} N.

Crossed electric and magnetic fields

An electron enters a region of electric and magnetic fields as shown in Figure 5.36.

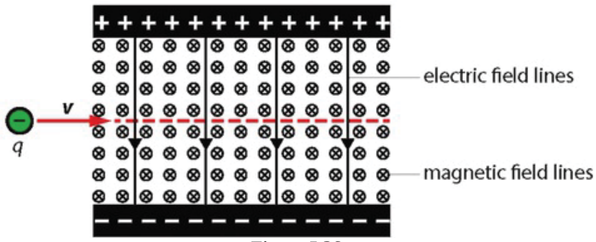


Figure 5.36

An electron is negatively charged, so the *electric* force on it is opposite in direction to the electric field – upwards in this case. The right-hand rule gives a downwards direction for the *magnetic* force on the electron.

Suppose in this case that the path of the electron is undeflected. This means that the upwards electric force on the electron has been cancelled by the downwards magnetic force, so $qE = qvB$, thus $E = vB$.

Note that a positively charged particle – for example a proton or an alpha particle – would also be undeflected if it entered with the same velocity. Think about how the electric and magnetic forces on these particles compare to those on the electron.

A very large number of problems are solved using the result from **electricity** that the work done in moving a charge q across a potential difference ΔV is $W = q\Delta V$. Combining this with the result from **mechanics** that the work done is also the change in kinetic energy $W = \Delta E_K$ gives:

$q\Delta V = \Delta E_K$

TEST YOURSELF 5.18

Two wires, X and Y, carry equal currents into the page, as shown in Figure 5.37. Point Z is at the same distance from X and Y.

> **hint**
>
> Find the magnetic field at Z from each wire separately, and use vector addition.

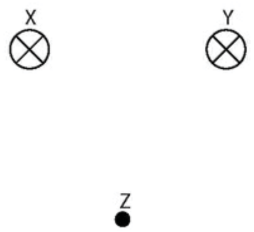
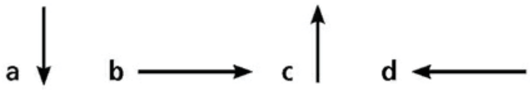
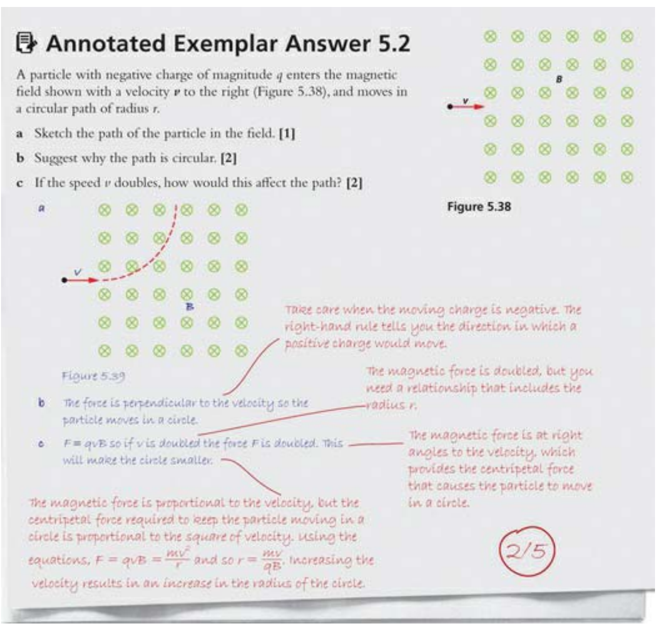


Figure 5.37

What is the direction of the magnetic field at Z?





Annotated Exemplar Answer 5.2

A particle with negative charge of magnitude q enters the magnetic field shown with a velocity v to the right (Figure 5.38), and moves in a circular path of radius r.

a Sketch the path of the particle in the field. **[1]**

b Suggest why the path is circular. **[2]**

c If the speed v doubles, how would this affect the path? **[2]**

Figure 5.38

a

Figure 5.39

b The force is perpendicular to the velocity so the particle moves in a circle.

c $F = qvB$ so if v is doubled the force F is doubled. This will make the circle smaller.

Take care when the moving charge is negative. The right-hand rule tells you the direction in which a positive charge would move.

The magnetic force is doubled, but you need a relationship that includes the radius r.

The magnetic force is at right angles to the velocity, which provides the centripetal force that causes the particle to move in a circle.

The magnetic force is proportional to the velocity, but the centripetal force required to keep the particle moving in a circle is proportional to the square of velocity. Using the equations, $F = qvB = \frac{mv^2}{r}$ and so $r = \frac{mv}{qB}$. Increasing the velocity results in an increase in the radius of the circle.

2/5

Checklist

After studying this chapter you should be able to:

- □ apply Coulomb's law
- □ work with electric fields
- □ solve problems with electric current and electric resistance
- □ understand electric circuits
- □ define EMF and power
- □ understand the difference between primary and secondary cells
- □ work with magnetic fields and magnetic forces in various situations.

< Back to chapter

Annotated Exemplar Answer 5.2

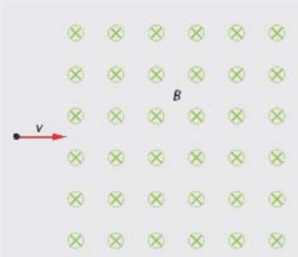


Figure 5.38

A particle with negative charge of magnitude q enters the magnetic field shown with a velocity $\boldsymbol{v}$ to the right (Figure 5.38), and moves in a circular path of radius r.

a Sketch the path of the particle in the field. **[1]**
b Suggest why the path is circular. **[2]**
c If the speed v doubles, how would this affect the path? **[2]**

a

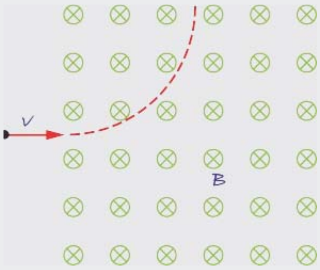


Figure 5.39

Take care when the moving charge is negative. The right-hand rule tells you the direction in which a positive charge would move.

b *The force is perpendicular to the velocity so the particle moves in a circle.*

The magnetic force is doubled, but you need a relationship that includes the radius r.

c *F = qvB so if v is doubled the force F is doubled. This will make the circle smaller.*

The magnetic force is at right angles to the velocity, which provides the centripetal force that causes the particle to move in a circle.

The magnetic force is proportional to the velocity, but the centripetal force required to keep the particle moving in a circle is proportional to the square of velocity. Using the equations, $F = qvB = \frac{mv^2}{r}$ *and so* $r = \frac{mv}{qB}$. *Increasing the velocity results in an increase in the radius of the circle.*

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6 CIRCULAR MOTION AND GRAVITATION

This chapter covers the following topics:

- □ Rotation and angular velocity
- □ Centripetal acceleration and centripetal forces
- □ Newton's law of gravitation
- □ Gravitational field strength
- □ Orbital motion

6.1 Circular motion

In circular motion, a particle moves along a circle or a part of a circle (see Figure 6.1).

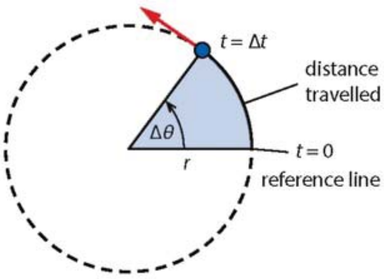

Figure 6.1

DEFINITIONS

PERIOD The time to make one full revolution. Since the speed v is constant and the object covers a distance of $2\pi r$ in a time of T seconds, it follows that $v = \frac{2\pi r}{T}$.

FREQUENCY The number of revolutions per second. It is the reciprocal of the period.

ANGULAR SPEED Angular speed ω is defined as $\omega = \frac{\text{angle swept out}}{\text{time taken}} = \frac{\Delta\theta}{\Delta t}$. For a complete revolution, $\Delta\theta = 2\pi$ and $\Delta f = T$, so we also have $\omega = \frac{2\pi}{T}$. Angular speed has units of radians per second, and since radians are unitless, this can be written s^{-1}.

Angular speed and linear speed are related: from $\omega = \frac{2\pi}{T}$ and $v = \frac{2\pi r}{T}$ it follows that $v = \omega r$.

Similarly, angular speed and frequency are related: $\omega = \frac{2\pi}{T} = 2\pi f$.

TEST YOURSELF 6.1

A horizontal disc rotates about a vertical axis through its centre. Point X is a distance d from the centre and point Y a distance $2d$. What are the ratios $\frac{\omega_Y}{\omega_X}$ and $\frac{v_Y}{v_X}$?

	$\frac{\omega_Y}{\omega_X}$	$\frac{v_Y}{v_X}$
a	1	1
b	1	2
c	2	1
d	2	2

Centripetal acceleration

DEFINITION

CENTRIPETAL ACCELERATION The acceleration of circular motion, in which only the velocity direction is changing.

Even if the speed along the circle is constant, we still have acceleration because the direction of the velocity keeps changing. (Note that this is a very common examination topic.)

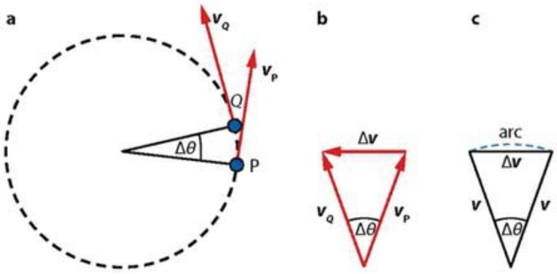

Figure 6.2

The direction of the acceleration is the direction of the *change in velocity*. In Figure 6.2a the velocity vector ***v*** changes direction as the particle moves from point P to point Q. The change in the velocity vector from P to Q is given by Δ***v*** (Figure 6.2b). If the angle $\Delta\theta$ is very small, the arc length and the length of the chord are the same (Figure 6.2c).

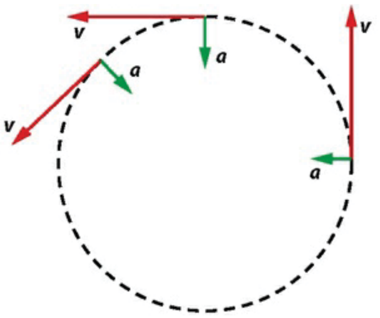

Figure 6.3

The acceleration has magnitude $\frac{v^2}{r}$ and is directed towards the centre of the circle (shown by the green arrows in Figure 6.3).

Alternative formulas for centripetal acceleration

In circular motion the time for a complete revolution is the period *T*. The reciprocal of the period is the frequency $f : f = \frac{1}{T}$. Because $v = \frac{2\pi r}{T}$, we can write:

$$a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

Another formula for centripetal acceleration is $a = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$.

Centripetal force

Newton's second law states that $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$. This means that if we have acceleration then there is a force *in the same direction as the acceleration*. In circular motion the centripetal acceleration is towards the centre of the circular path, so we must also have a net force towards the centre. This is referred to as the **centripetal force.**

Here are some examples of centripetal forces:

- A car takes a circular bend: the centripetal force is provided by friction between the road and the tyres; Figure 6.4 shows a view from above.

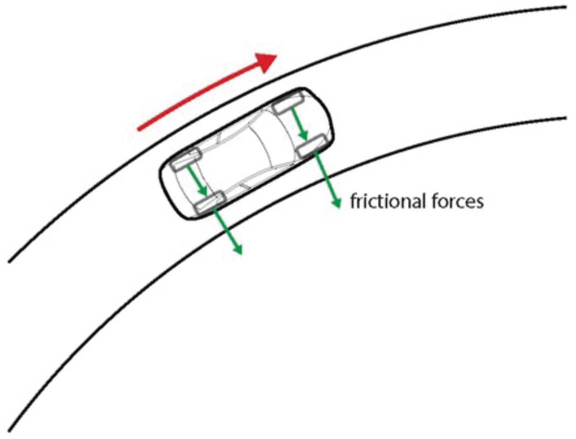

Figure 6.4

- The Earth revolves around the Sun: the centripetal force on the Earth is its gravitational attraction to the Sun (Figure 6.5).

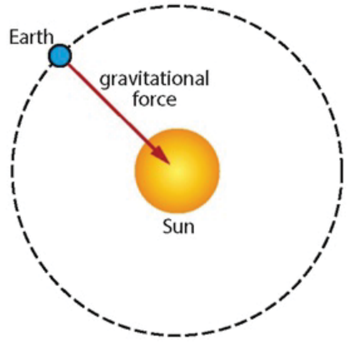

Figure 6.5

- A car drives over the crest of a hill whose curvature is circular (Figure 6.6). The centripetal force on the car is $mg - N$, the combination of the upward normal force exerted by the road and the downward force of the car's weight.

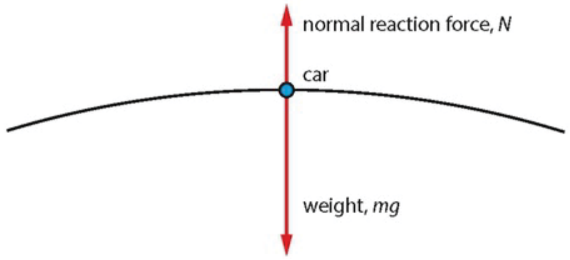

Figure 6.6

TEST YOURSELF 6.2

A car goes around a horizontal circular bend of radius 45 m. The coefficient of static friction between the tyres and the road is 0.60.

a Determine the maximum speed at which the car can go around the bend.

b Suggest whether the maximum speed found in **a** could be increased if the road were banked rather than horizontal.

hint

It is insufficient to simply state without justification that the speed will be greater.

TEST YOURSELF 6.3

A car is going over the crest of a hill whose curvature is circular with a radius of R. At the highest point its speed is v. Show that the maximum speed at which the car does not lose contact with the road is given by $v = \sqrt{gR}$.

hint

In a 'show' question like this, all steps should be shown in detail. In terms of what you know about forces, what does it mean to say that a car loses contact with the road?

TEST YOURSELF 6.4

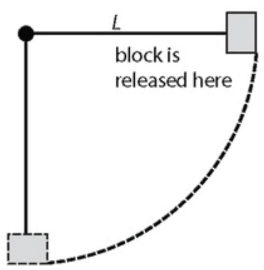

Figure 6.7

A block of mass m, at the end of a string of length L which is fixed at the other end, is held so that the string is horizontal, as shown in Figure 6.7, and then released. Write an expression, in terms of m and L, for the tension in the string as it moves past the vertical position.

hint

Is the block in equilibrium as it moves past the vertical position? Is it moving in a straight line or in a circular path? Use Newton’s second law (since you are looking for a force), but you will also need to apply conservation of energy.

6.2 Gravitation

Newton's law of gravitation

Two *point* masses m_1 and m_2 separated by a distance *r* *attract* each other with a force whose magnitude is $F = G\frac{m_1 m_2}{r^2}$. The force is directed along the line joining the two masses.

This formula can also be used for two uniform spherical masses (Figure 6.8), in which case the separation *r* is their centre-to-centre separation.

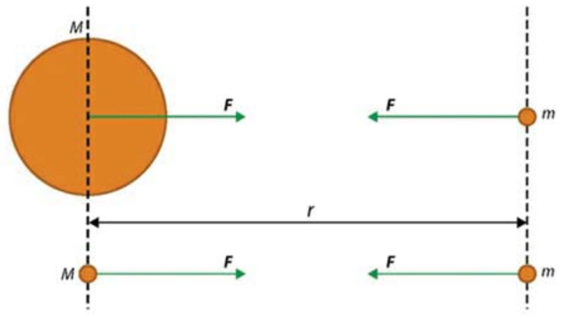

Figure 6.8

G is the universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

TEST YOURSELF 6.5

The weight of a body is 20 N on the surface of a particular planet. What is its weight at a height above the surface which is equal to the planet's radius *R*?

hint

Remember that weight is gravitational force.

Model Answer 6.1

The Earth orbits the Sun in a circular orbit of radius 1.5×10^{11} m. Calculate the mass of the Sun.

The gravitational force on the Earth is $F = G\frac{Mm}{r^2}$, where *m* is the Sun's mass, *m* is the Earth's mass and *r* is the radius of the Earth's orbit around the Sun. This is also the centripetal force acting on the Earth, so $G\frac{Mm}{r^2} = \frac{mv^2}{r} \Rightarrow M = \frac{v^2 r}{G}$. In one year the Earth moves a distance $2\pi r$,

so $v = \frac{2\pi r}{T} = \frac{2\pi \times 1.5 \times 10^{11}\ \text{m}}{365 \times 24 \times 60 \times 60\ \text{s}} = 2.99 \times 10^4\ \text{ms}^{-1}$.

Hence $M = \frac{v^2 r}{G} = \frac{(2.99 \times 10^4)^2 \times 1.5 \times 10^{11}}{6.67 \times 10^{-11}} = 2.0 \times 10^{30}\ \text{kg}$.

The concept of a field

How does a force propagate from one body to another? To ask the question anthropomorphically, how does a mass 'know' that another mass is attracting it? In other words, if the Sun were to magically disappear, after how much time would its force on Earth become zero?

In an attempt to answer this question, physicists introduced the idea of a **field.** We say that a mass *M* is the source of a **gravitational field** around it. Other masses respond to this field by experiencing a gravitational force. To 'feel' the gravitational field you must have mass. In the case of gravity, we may say that a gravitational field exists in a region of space around a mass, and exerts a force on any other mass that is present in that region of space. The field is a property of the mass creating the field (the 'source') and (usually) of the distance from it.

Gravitational field *g*

DEFINITION

GRAVITATIONAL FIELD STRENGTH The gravitational force per unit mass exerted on a small, point mass *m*:

$$g = \frac{F}{m} \Leftrightarrow F = mg.$$

Recall that the field is a property of the 'source' mass. But in the definition above, it appears that the affected mass *m* also enters the definition. But a field measures the force per unit mass, so the dependence on *m* must somehow cancel out.

Consider the gravitational field created by a spherical mass *M*. A point mass *m* placed a distance *r* from the centre of *M* will experience a force $F = G\frac{Mm}{r^2}$ and so the gravitational field strength is

$$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m} = \frac{GM}{r^2}.$$

This formula only applies if *M* is a point mass or a spherical mass.

TEST YOURSELF 6.6

The gravitational field strength at the surface of Earth is g. What is the gravitational field strength at the surface of a planet of twice the mass and twice the radius of the Earth?

a $\frac{g}{8}$ **b** $\frac{g}{4}$ **c** $\frac{g}{2}$ **d** g

hint

There are several ways to approach this 'comparison' type of question.

Because the gravitational force ***F*** is a vector quantity, ***g*** is a vector quantity too. Its magnitude is given by $\frac{F}{m}$ and its direction is the same as that of the force.

TEST YOURSELF 6.7

The mass of the Earth is 81 times that of the Moon. Determine the distance (from the centre of the Earth) of the point along the line joining the two bodies where the combined gravitational field of the Earth and the Moon is zero. The Earth–Moon distance is 3.8×10^8 m.

TEST YOURSELF 6.8

Two spherical bodies of masses *M* and *m* (*M* > *m*) are separated by a distance *d* (Figure 6.9).

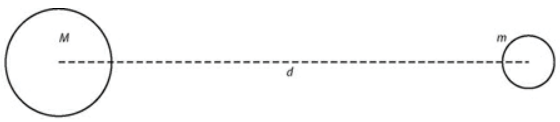

Figure 6.9

Which graph in Figure 6.10 shows the correct variation, as a function of the distance *r* from the centre of the larger mass, of the strength of the combined gravitational field of the two bodies? Negative *g* means the field is directed to the left.

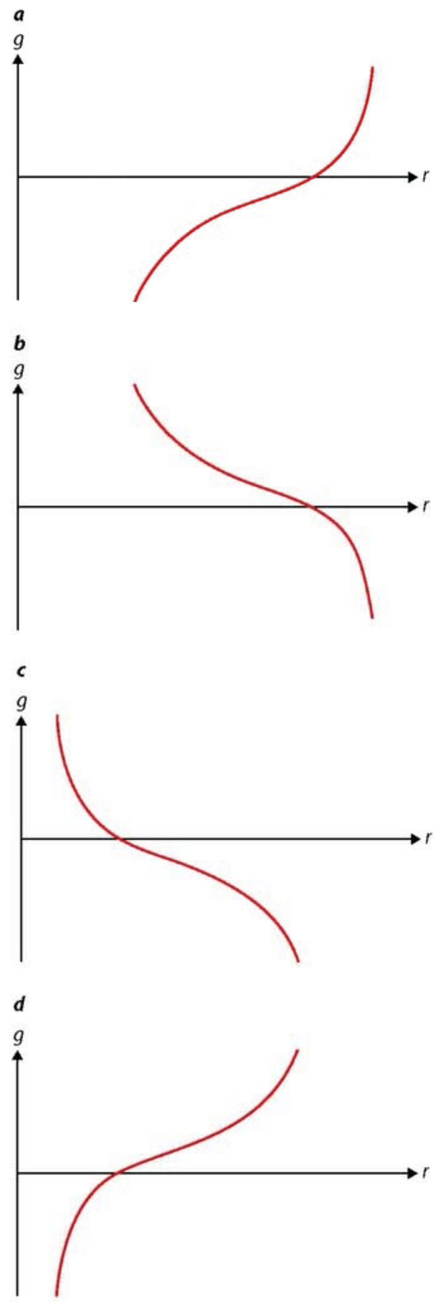

Figure 6.10

Field patterns

Figure 6.11

The gravitational field is a vector. Figure 6.11 shows gravitational field lines for a spherical planet: the lines are radial. At any point in this diagram the direction of the gravitational field strength vector is given by the tangent to the gravitational field line at that point, and in the direction of the arrow.

Close to the surface of the planet, the surface looks flat and the gravitational field lines are approximately parallel lines (Figure 6.12).

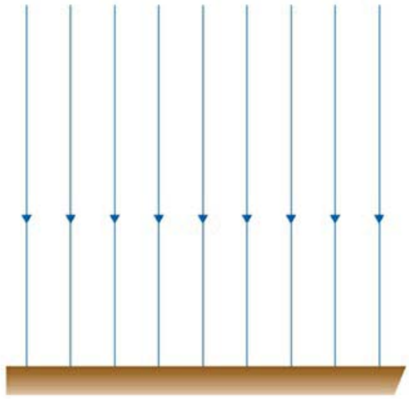

Figure 6.12

Orbital motion

A classic application of Newton's law of gravitation is the calculation of the period of motion of a satellite around the Earth or of a planet around the Sun.

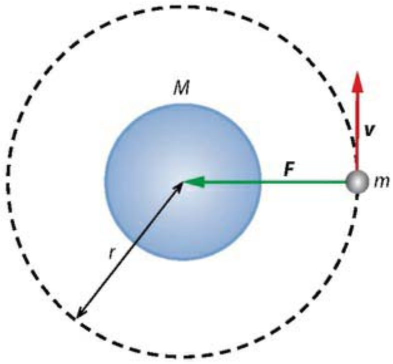

Figure 6.13

Figure 6.13 shows a particle of mass *m* orbiting a larger body of mass *M* in a circular orbit of radius *r*.

The only force on the particle is the force of gravitation, $F = \frac{GMm}{r^2}$, and so this force provides the centripetal force on the particle. Therefore $\frac{mv^2}{r} = \frac{GMm}{r^2}$. Cancelling the mass *m* and a factor of *r* leads to $v = \sqrt{\frac{GM}{r}}$. (You should be able reproduce this derivation in an exam.)

This gives the speed in a circular orbit of radius *r*. Squaring $v = \frac{2\pi r}{T}$, we deduce that $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3 \Rightarrow T = \frac{2\pi}{\sqrt{GM}} r^{\frac{3}{2}}$.

This shows that the period of a planet going around the Sun is proportional to the $\frac{3}{2}$ power of its orbital radius.

Annotated Exemplar Answer 6.1

The International Space Station orbits the Earth in a circular orbit once every 92 minutes.

a Show that the period T of the orbit is given by $T^2 = \dfrac{4\pi^2 r^3}{GM}$, where M is the mass of the Earth and r is the radius of the orbit. **[3]**

b Calculate the height of the orbit above the Earth's surface. Take the mass of the Earth to be 6.0×10^{24} kg and its radius to be 6.4×10^6 m. **[5]**

a $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2} \Rightarrow v^2 = \dfrac{GM}{r}$

The equations $\frac{mv^2}{r}$ and $\frac{GMm}{r^2}$ are both in the data booklet. You should explain that you have equated them because they both represent the centripetal force that keeps the body in orbit.

$v = \dfrac{2\pi r}{T}$, so $\dfrac{4\pi^2 r^2}{T^2} = \dfrac{GM}{r}$

Circular motion questions often require you to make this connection – the circumference divided by the period is the velocity.

Rearranging, $T^2 = \dfrac{4\pi^2 r^3}{GM}$

a Period = 92 minutes = 92×60 s = 5520 s

Good – you have remembered to change to SI units.

$r^3 = \dfrac{GMT^2}{4\pi^2}$

$r^3 = 3.0888 \times 10^{20}$

It would be better to include an extra line where you show all the values substituted into the equation. Then if you make an error in the calculation, you can still gain some marks.

$r = 6.7598 \times 10^6$ m ≈ 6800 km

The calculation of the orbit is correct, but the question asks for the height above the Earth's surface. Subtracting the radius of the Earth gives the answer, 400 km.

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Predictions versus understanding

Nature of Science. Combining the laws of mechanics with the law of gravitation enables scientists to predict with great accuracy the orbits of spacecraft, planets and comets. But to what degree do they enable an understanding of why planets, for example, move the way they do? In ancient times, Ptolemy was also able to predict the motion of planets with exceptional precision.

In what sense is the Newtonian approach 'better'? Ptolemy's approach was specific to planets and could not be generalised to other examples of motion, whereas the Newtonian approach can. Ptolemy's method gives no explanation of the observed motions whereas Newton 'explains' the motion in terms of one single universal concept, that of a gravitational force that depends in a specific way on mass and separation. In this sense the Newtonian approach is superior and represents progress in science. But there are limits to the degree to which one demands 'understanding':

- the obvious question for Newton would be, 'Why is there a force between two masses?'.
- Newton could not answer this question – and no-one has been able to since.

There is more in Option **A** on relativity about Einstein's attempt to answer this question.

Checklist

After studying this chapter you should be able to:

- ☐ **work with linear and angular displacements and velocities**
- ☐ **understand the meaning of centripetal acceleration**
- ☐ **apply Newton's second law to motion along a circular path**
- ☐ **apply Newton's law of gravitation**
- ☐ **work with gravitational field strength**
- ☐ **apply Newton's laws of mechanics and gravitation to orbital motion.**

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Annotated Exemplar Answer 6.1

The International Space Station orbits the Earth in a circular orbit once every 92 minutes.

a Show that the period T of the orbit is given by $T^2 = \frac{4\pi^2 r^3}{GM}$, where M is the mass of the Earth and r is the radius of the orbit. **[3]**

b Calculate the height of the orbit above the Earth's surface. Take the mass of the Earth to be 6.0×10^{24} kg and its radius to be 6.4×10^6 m. **[5]**

Answer	Comment
a $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r}$	*The equations $\frac{mv^2}{r}$ and $\frac{GMm}{r^2}$ are both in the data booklet. You should explain that you have equated them because they both represent the centripetal force that keeps the body in orbit.*
$v = \frac{2\pi r}{T}$, so $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$ *Rearranging,* $T^2 = \frac{4\pi^2 r^2}{GM}$	*Circular motion questions often require you to make this connection - the circumference divided by the period is the velocity.*
a *Period = 92 minutes = 92 × 60 s = 5520 s*	*Good - you have remembered to change to SI units.*
$r^3 = \frac{GMT^2}{4\pi^2}$ $r^3 = 3.0888 \times 10^{20}$	*It would be better to include an extra line where you show all the values substituted into the equation. Then if you make an error in the calculation, you can still gain some marks.*
$r = 6.7598 \times 10^6 m$ $\approx 6800\ km$	*The calculation of the orbit is correct, but the question asks for the height above the Earth's surface. Subtracting the radius of the Earth gives the answer, 400 km.*

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7 ATOMIC, NUCLEAR AND PARTICLE PHYSICS

This chapter covers the following topics:

- □ Discrete energy levels
- □ Atomic spectra
- □ Radioactive decay
- □ Nuclear reactions and energy released
- □ Mass defect and binding energy
- □ Quantum numbers and conservation laws
- □ Quarks, leptons and exchange particles
- □ Feynman diagrams

7.1 Energy levels

DEFINITIONS

EMISSION SPECTRUM The set of wavelengths of light emitted by a gas. Gases emit light when heated to a high temperature or when exposed to a high electric field. The light emitted consists of a discrete set of *specific* wavelengths. Figure 7.1 shows the emission spectrum of hydrogen.

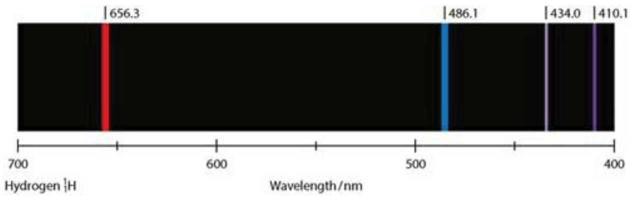


Figure 7.1

ABSORPTION SPECTRUM The set of wavelengths of light that are absorbed when white light is transmitted through a gas. These wavelengths correspond exactly to the wavelengths in the emission spectrum.

PHOTON The particle of light; a bundle of energy and momentum. The energy of a photon is given by $E = hf$, where f is the frequency of the light and h is the Planck constant, $h = 6.63 \times 10^{-34}$ Js.

ENERGY LEVEL The energy of an atom is **discrete** – that is, it can only have certain specific values. The possible energies that an atom can have are called its energy levels.

ATOMIC TRANSITIONS A photon is emitted when an atom makes a transition from a higher energy level to a lower energy level. The photon carries away an energy equal to the *difference* ΔE in energy of the levels involved in the transition.

Energy levels in an atom

The existence of discrete energy spectra is evidence of energy levels.

For every transition from a high to a lower energy level, one photon is emitted (Figure 7.2).

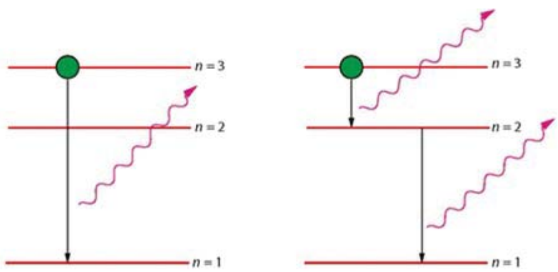


Figure 7.2

TEST YOURSELF 7.1

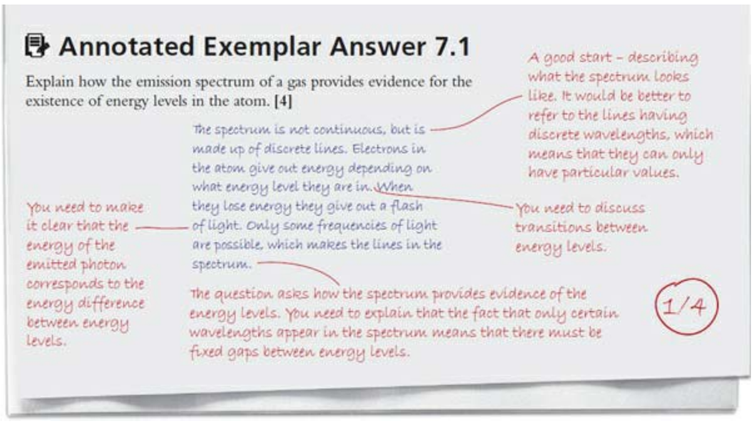
Of the three transitions shown in Figure 7.2, which one corresponds to the photon with the longest wavelength?

hint

Keep track of the energy units you are using.

TEST YOURSELF 7.2

Calculate the wavelength of the photon emitted in a transition between two energy states that differ in energy by 1.9 eV.


Annotated Exemplar Answer 7.1

Explain how the emission spectrum of a gas provides evidence for the existence of energy levels in the atom. **[4]**

The spectrum is not continuous, but is made up of discrete lines. Electrons in the atom give out energy depending on what energy level they are in. When they lose energy they give out a flash of light. Only some frequencies of light are possible, which makes the lines in the spectrum.

A good start – describing what the spectrum looks like. It would be better to refer to the lines having discrete wavelengths, which means that they can only have particular values.

You need to discuss transitions between energy levels.

You need to make it clear that the energy of the emitted photon corresponds to the energy difference between energy levels.

The question asks how the spectrum provides evidence of the energy levels. You need to explain that the fact that only certain wavelengths appear in the spectrum means that there must be fixed gaps between energy levels.

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Forces operating in the nucleus

The dominant forces acting within a nucleus are:

- the electrical repulsion force between the **protons**
- the strong nuclear force acting between nucleons (the **protons and neutrons**), which is (mostly) attractive.
- the weak nuclear force, which is responsible for beta decay (see section 7.2, Nuclear stability and radioactivity).

The strong force has a *short range*, which means that a nucleon only attracts its immediate neighbours (by contrast, the electrical force, with its infinite range, implies that any one proton repels all other protons in the nucleus).

The presence of the strong nuclear force prevents the electrical force from ripping the nucleus apart.

(The gravitational force is negligible because the masses involved are so small.)

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Annotated Exemplar Answer 7.1

Explain how the emission spectrum of a gas provides evidence for the existence of energy levels in the atom. **[4]**

You need to make it clear that the energy of the emitted photon corresponds to the energy difference between energy levels.

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You need to discuss transitions between energy levels.

The question asks how the spectrum provides evidence of the energy levels. You need to explain that the fact that only certain wavelengths appear in the spectrum means that there must be fixed gaps between energy levels.

1/4

7.2 Nuclear stability and radioactivity

DEFINITIONS

NUCLEON NUMBER The number of protons and neutrons in a nucleus.

PROTON OR ATOMIC NUMBER The number of protons in a nucleus.

NUCLIDE A nucleus with a specific number of protons and neutrons.

ISOTOPE Nuclei with the same proton number but different nucleon number are called *isotopes* of one another. Thus $^3_2\text{He}, ^4_2\text{He}, \ldots, ^{10}_2\text{He}$ are the eight isotopes of helium. Isotopes have the same *chemical* properties (same number of electrons) but different *physical* properties (for example, different mass, nuclear radius, specific heat capacity).

ATOMIC MASS UNIT, *u* One atomic mass unit is $\frac{1}{12}$ of the mass of the neutral atom of the carbon isotope $^{12}_6\text{C}$. $1u = 1.66 \times 10^{-27}$ kg..

NUCLEAR NOTATION A nucleus with *Z* protons and *N* neutrons is denoted ^A_ZX, where *A* is the *nucleon number*, *Z* is the *proton number* and X is the chemical symbol of the element. Thus ^4_2He represents the nucleus of helium, with 2 protons and 4 − 2 = 2 neutrons. The *approximate* mass of a nucleus of mass number *A* is *A* atomic mass units (*u*).

ACTIVITY The rate of radioactive decay, measured in becquerel (Bq); 1 Bq = 1 decay per second.

Radioactive decay

Of some 2500 known nuclides, less than about 300 are stable. Figure 7.3 is a plot of neutron number versus proton number (atomic number) for the known nuclides.

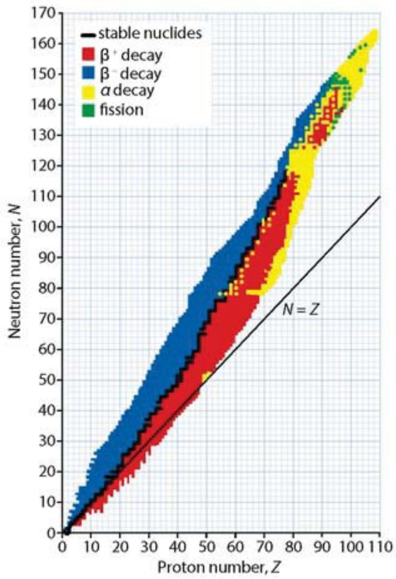


Figure 7.3

The squares in black are the stable nuclides. The figure shows that a small stable nucleus has about the same number of protons and neutrons, while a large stable nucleus has more (but not many more) neutrons than protons.

If a nucleus contains **too many protons** compared to neutrons – the squares to the right of the black ones in the figure – it is unstable because the electrical repulsion force between protons overwhelms the strong nuclear force. But it turns out that a nucleus can also be unstable if it contains **too many neutrons** compared to protons – the squares to the left of the black ones in the figure.

An unstable nucleus can reduce its energy (and so become more stable) by the emission of particles and energy, a phenomenon called *radioactive decay*. A decay is:

- a *random* process: it cannot be predicted *which* nucleus will decay and *when* it will decay.
- a *spontaneous* process: it cannot be induced to happen or prevented from happening.

There are three types of radioactive emission – *alpha*, *beta* and *gamma* – and a nuclear decay can involve one or more of these:

- Alpha: emission of helium-4 nuclei, ^4_2He or $^4_2\alpha$.
- Beta: emission of fast electrons, $^{0}_{-1}e$.
- Gamma: emission of energetic photons, $^0_0\gamma$.

The energies in alpha and gamma decay are *discrete* whereas those in beta decay are *continuous*.

These emissions are *ionising* – that is, they can knock electrons off the atoms they collide with, creating ions and radicals. For a given energy, alpha radiation is the least penetrating and the most ionising, while gamma radiation is the most penetrating and the least ionising.

Examples of alpha decay: In alpha (α) decay, an alpha particle is emitted; for example:

$$^{242}_{94}\text{Pu} \rightarrow {}^{4}_{2}\alpha + {}^{238}_{92}\text{U}$$

$$^{238}_{92}\text{U} \rightarrow {}^{4}_{2}\alpha + {}^{234}_{90}\text{Th}$$

$$^{218}_{84}\text{Po} \rightarrow {}^{4}_{2}\alpha + {}^{214}_{82}Pb$$

Examples of beta minus decay: In beta minus (β^-) decay, an electron and an anti-neutrino are emitted; for example:

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N} + {}^{0}_{0}\bar{\nu}_e$$

$$^{40}_{19}\text{K} \rightarrow {}^{0}_{-1}\text{e} + {}^{40}_{20}\text{Ca} + {}^{0}_{0}\bar{\nu}_e$$

$$^{60}_{27}\text{Co} \rightarrow {}^{0}_{-1}\text{e} + {}^{60}_{28}\text{Ni} + {}^{0}_{0}\bar{\nu}_e$$

Examples of beta plus decay: In beta plus (β^+) decay, a positron and a neutrino are emitted; for example:

$$^{22}_{11}\text{Na} \rightarrow {}^{0}_{+1}\text{e} + {}^{22}_{10}\text{Ne} + {}^{0}_{0}\nu_e$$

$$^{13}_{7}\text{N} \rightarrow {}^{0}_{+1}\text{e} + {}^{13}_{6}\text{C} + {}^{0}_{0}\nu_e$$

Examples of gamma decay: In gamma decay a photon is emitted; for example:

$$^{60}_{28}\text{Ni} \rightarrow {}^{0}_{0}\gamma + {}^{60}_{28}\text{Ni}$$

$$^{24}_{12}\text{Mg} \rightarrow {}^{0}_{0}\gamma + {}^{24}_{12}\text{Mg}$$

$$^{238}_{92}\text{U} \rightarrow {}^{0}_{0}\gamma + {}^{238}_{92}\text{U}$$

hint

Momentum is conserved; what is the momentum of the Po nucleus before the decay?

TEST YOURSELF 7.3

Calculate the ratio $\frac{K_\alpha}{K_{Pb}}$ of kinetic energies in the alpha decay $^{218}_{84}\text{Po} \rightarrow {}^{4}_{2}\alpha + {}^{214}_{82}\text{Pb}$ when the Po nucleus decays at rest.

The radioactive decay law

Despite the random nature of radioactive decay, we can still formulate a law of radioactive decay. This states that the *rate of decay is proportional to the number of nuclei present*: $\frac{\mathrm{d}N}{\mathrm{d}t} \propto N$. This implies an exponential decrease in activity as time passes.

The unit of activity is the becquerel (Bq): 1 Bq = 1 decay per second.

The time interval during which the activity of a sample is reduced by a factor of 2 is called its *half-life*, $T_{1/2}$; see Figure 7.4. The chance that, within an interval equal to the half-life, a randomly chosen nucleus will decay is 50%.

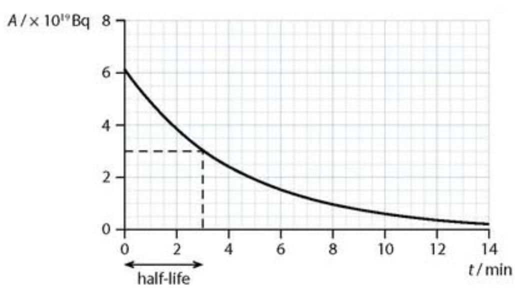


Figure 7.4

TEST YOURSELF 7.4

The half-life of an isotope is 4.0 days. Calculate the fraction of the original activity 12 days after it has been prepared.

TEST YOURSELF 7.5

An isotope X has a half-life of 2.0 minutes. It decays into isotope Y that is stable. Initially no quantity of isotope Y is present. After how much time will the ratio of Y atoms to X atoms be equal to 7?

TEST YOURSELF 7.6

A radioactive isotope has a half-life of 2.0 minutes. A particular nucleus has not decayed within a 2.0 min time interval. A correct statement about the next 2.0 min interval is that this nucleus:

a has a lower than 50% chance of decaying.

b will certainly decay.

c has a 50% chance of decaying.

d has a better than 50% chance of decaying.

hint

Remember the definition of half-life.

Mass defect and binding energy

The mass of a nucleus is smaller than the sum of the masses of its individual nucleons. The difference is known as the **mass defect**:

$$\delta = \underbrace{Zm_p + Nm_n}_{\text{total mass of individual nucleons}} - \underbrace{M}_{\text{mass of nucleus}}.$$

From Einstein's theory of relativity (see Option A), a mass m is equivalent to an energy E according to the equation $E = mc^2$. The energy corresponding to the mass defect is called the *binding energy* (BE) of the nucleus, and is the minimum energy needed to completely separate its nucleons. The larger the binding energy of a nucleus, the more stable it is.

It is often useful to know how much energy corresponds to a mass of 1 u. From $E = mc^2$:

$$\begin{aligned} 1\,u \times c^2 &= 1.660\,5402 \times 10^{-27} \times (2.9979 \times 10^8)^2\,\text{J} \\ &= 1.492\,394\,6316 \times 10^{-10}\,\text{J} \end{aligned}$$

Using 1 eV = 1.602 177 × 10^{-19} J, this is:

$$\frac{1.492\,394\,6316 \times 10^{-10}\,\text{J}}{1.602\,177 \times 10^{-19}\,\text{J eV}^{-1}} = 931.5 \times 10^6\,\text{eV} = 931.5\,\text{MeV}$$

So the energy equivalent of 1 u is 1 $u \times c^2$ = 931.5 MeV

A graph of BE/A, the binding energy per nucleon, versus nucleon number A (Figure 7.5) shows that most nuclei have approximately the same binding energy per nucleon, roughly 8.5 MeV

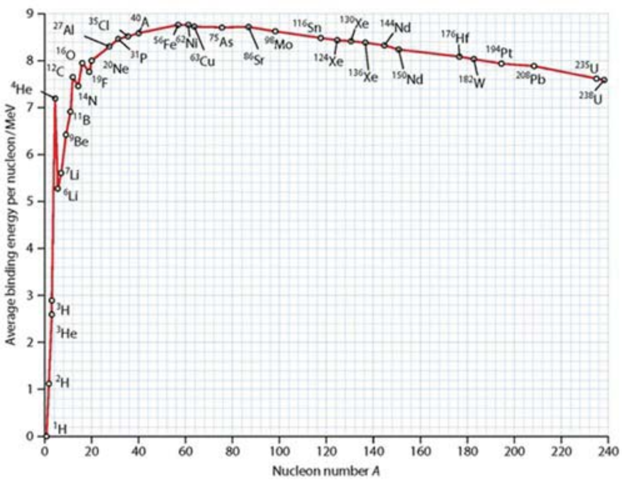


Figure 7.5

This is a consequence of the short range of the strong nuclear force: in a large nucleus, any one nucleon is surrounded by approximately the same number of nucleons as any other, so it requires the same amount of energy to remove it.

The graph also shows a maximum at nickel-62 (^{62}Ni), which makes this the most stable nucleus.

Nature of Science. The graphs of proton number against neutron number (Figure **7.3**) and binding energy per nucleon (Figure **7.5**) show very clear trends and patterns. Scientists can use these graphs to make predictions of the characteristics of the different isotopes, such as whether an isotope will decay by beta plus or beta minus decay.

TEST YOURSELF 7.7

a Show that the energy released in the alpha decay $^{218}_{84}\mathrm{Po} \rightarrow {}^{4}_{2}\alpha + {}^{214}_{82}\mathrm{Pb}$ is equal to the difference in binding energies of the products minus that of the reactants.

hint

The answer to this 'show' question requires some detail at each step of the argument.

b Using Figure 7.5, estimate the energy that is released in this decay.

TEST YOURSELF 7.8

The binding energy per nucleon of the nucleus $^{11}_{5}\mathrm{B}$ is approximately 7 MeV. The energy needed to completely separate the nucleons of this nucleus is approximately:

hint

How many nucleons are there in the $^{11}_{5}\mathrm{B}$ nucleus?

a 35 MeV
b 42 MeV
c 77 MeV
d 112 MeV.

7.3 Fission and fusion

Fission: The splitting (fission) of a large nucleus into two smaller nuclei plus neutrons plus energy (and usually photons). An example is:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{98}_{40}Zr + ^{135}_{52}Te + 3^{1}_{0}n$$

The nuclear masses in this example are 235.0439 *u* for U-235, 97.9128 *u* for Zr-98, 134.9165 *u* for Te-135 and 1.0087 *u* for the neutron. The mass difference is (235.0349 + 1.0087) − (97.9128 + 134.9165 + 3 × 1.0087) = 0.1972 *u*.

This is positive in this case and so it corresponds to a *release* of energy. This energy can be calculated from Einstein's equation, and is Q = 0.1972 × 931.5 = 184 MeV.

Fusion: The joining (fusion) of two light nuclei to form a heavier nucleus plus energy. An example is:

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$$

The nuclear masses in this example are: 2.0141 *u* for deuterium, 3.0160 *u* for helium 1.0087 *u* for the neutron. The mass difference is 2 × 2.0141 − (3.0160 + 1.0087) = 0.0035 *u*.

This is positive in this case and so it corresponds to a *release* of energy of 0.0035 × 931.5 = 3.26MeV.

A star produces its energy by nuclear fusion in its core, where the temperature and pressure are both high. The temperature must be high for the nuclei to overcome their electrical repulsion, and the pressure must be high to ensure a high probability of collisions and hence fusion.

Artificial transmutation: Ernest Rutherford was awarded the 1908 Nobel Prize in Chemistry for the discovery of *artificial transmutation*, in which one nuclear species is converted to another.

TEST YOURSELF 7.9

Rutherford observed the following artificial transmutation reaction, in which a very energetic alpha particle (helium-4 nucleus) collides with a nitrogen-14 nucleus:

$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H$

The masses are: 4.002 60 *u* for He, 14.003 07 *u* for N, 16.999 13 *u* for O and 1.007 83 *u* for H. Explain why the reaction cannot occur unless the alpha particle supplies sufficient kinetic energy.

7.4 Particle Physics

The accepted model of particle interactions at the present time is called the **standard model.** The model assumes that the so-called **elementary particles** – particles that are not made out of smaller constituents – belong to three different classes: quarks, leptons and exchange particles. Exchange particles are associated with the four fundamental interactions (forces) of nature. In addition there is also one special particle, the Higgs boson.

DEFINITIONS

QUARKS Elementary particles which combine to form a class of particles called **hadrons.**

HADRON A particle made of quarks. There are two kinds of hadrons: baryons and mesons.

BARYON A hadron which is made of three quarks. Protons and neutrons are examples of baryons: proton = uud, neutron = ddu.

MESON A hadron made of one quark and one antiquark.

COLOUR Quarks have a fundamental property which particle physicists call **colour** (though this has nothing to do with colour as we know it). Quarks have one of three colours: red, green or blue.

CONFINEMENT Quarks cannot be observed as isolated free particles and so colour cannot be observed directly. Hadrons (which are formed from quarks) always appear as combinations with no colour. For example, a proton consists of 2 u and 1 d quarks. The colour quantum numbers of the three quarks cannot be anything other than the colourless combination RGB (red-green-blue). We cannot, for example, have a proton whose combined quarks have colour. Similarly, the quarks in the meson $\pi^+ = u\bar{d}$ must have colours that are, for example, $\mathbf{R\overline{R}}$ but never $\mathbf{R\overline{G}}$ or $\mathbf{B\overline{R}}$.

QUANTUM NUMBER A number (or property) used to characterise a particle. Quantum numbers in the standard model include charge, baryon number, strangeness, lepton number and colour.

BARYON NUMBER, *B* All quarks are assigned a baryon number of $B=+\frac{1}{3}$ and all antiquarks a baryon number of $B = -\frac{1}{3}$.

STRANGENESS, *S* A quantum number that applies only to hadrons. For every strange quark in a hadron we assign a strangeness quantum number $S = -1$. For every anti-strange quark we assign a strangeness $S = +1$.

LEPTON NUMBER, *L* A quantum number that applies to leptons only. Each lepton is assigned a lepton number of +1. Thus the electron and the electron neutrino both have $L = +1$.

ANTI-PARTICLES To every particle there corresponds an anti-particle of the same mass but with *opposite values of electric charge and all other quantum numbers.* Some particles – such as the photon and the graviton – are their own anti-particles (among other things this requires that they be electrically neutral).

The Rutherford–Geiger–Marsden experiment

In this famous experiment, alpha particles were directed at a thin gold foil (Figure 7.6a). Most of the alpha particles went straight through with small deflections, but a very few were scattered back at large scattering angles (Figure 7.6b).

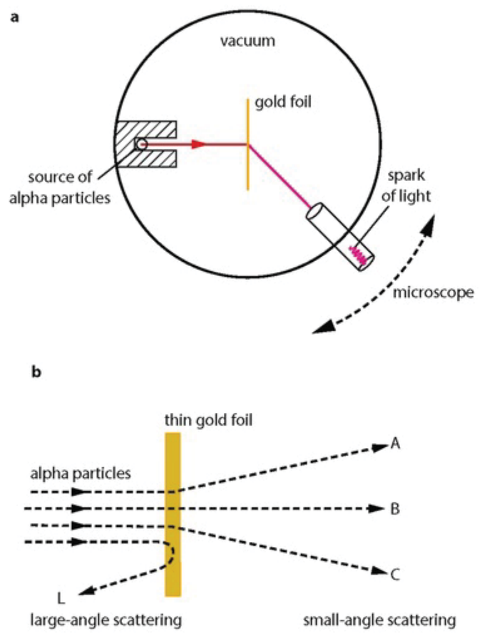


Figure 7.6

The large scattering angles implied a very large force of deflection. The force responsible for the deflection was the electric Coulomb force between the positive charge of the alpha particles and the positive charge of the atom. Since the electric force is given by $F = k\frac{Q_1 Q_2}{r^2}$, such a large force requires a very small separation r. In turn this requires that the positive charge of the atom must be concentrated in a very small volume: the nucleus. Rutherford thus deduced that the positive charge of the atom is concentrated in a spherical volume with a radius of the order of just 10^{-15} m.

TEST YOURSELF 7.10

Explain why, in the Rutherford experiment, **a** the alpha particle beam must be very narrow and **b** the foil must be very thin.

Model Answer 7.1

Describe the Rutherford experiment and state what conclusions may be reached from it about the distribution of matter in an atom.

Alpha particles were directed at a thin foil of gold. Most of the particles were deflected by small angles, as might be expected from the prevailing model of the atom that had the positive charge of the atom spread out over a volume of radius 10^{-10} m. Very rarely, large-angle deflections of the alpha particles took place. These large deflections require a very large electric force of repulsion. To provide such a force, the positive charge of the atom must be concentrated in a volume of radius around 10^{-15} m. Thus, most of the atomic volume is empty, except for a very small positively charged and massive object, the nucleus.

Particles galore!

Rutherford's experiment was followed by a rapid sequence of discoveries of new particles within the atom: protons and neutrons were discovered inside the nucleus. The proton was discovered in 1920 and the neutron in 1932. Also in 1932, a particle with the same mass as the electron but with positive charge was discovered; this was the positron – the anti-particle of the electron. By the late 1940s and all through the 1950s and 1960s, hundreds of other particles were discovered in experiments with cosmic rays and particle accelerators. It was urgent to make sense of these particles.

A breakthrough came in 1964 when Murray Gell-Mann proposed the existence of three new elementary particles, which he called **quarks.** Combinations of quarks could explain the plethora of observed particles and made possible an understanding of their properties. Since Gell-Mann's original proposal, three more quarks have been discovered, making a total of six.

The present standard model explains the properties of all the known particles on the basis of:

- **Six quarks** (called u, d, s, c, b and t)
- **Six leptons** (electron e^-, muon μ^-, tau τ^- and their corresponding neutrinos, ν_e, ν_μ, ν_τ.)
- **Exchange particles** (photon γ, $W^\pm$ and Z^0 bosons, and the gluons)

A special particle called the Higgs boson, H, completes the list of the elementary particles of the standard model.

Table 7.1 lists the properties of quarks. Except for the entry in the last column, this information is given in the IBO data booklet.

Table 7.1

Quark	Charge, Q / e	Baryon number, B	Strangeness, S
u	$\frac{2}{3}$	$\frac{1}{3}$	0
c	$\frac{2}{3}$	$\frac{1}{3}$	0
t	$\frac{2}{3}$	$\frac{1}{3}$	0
d	$-\frac{1}{3}$	$\frac{1}{3}$	0
s	$-\frac{1}{3}$	$\frac{1}{3}$	−1
b	$-\frac{1}{3}$	$\frac{1}{3}$	0

Particles such as the proton and neutron consist of three quarks (see Figure 7.7):

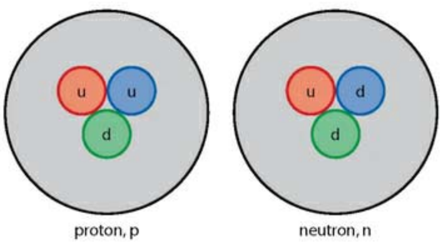


Figure 7.7

Thus a proton has quantum charge number $Q = \frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$ and the neutron has $Q = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$. (You should be able to describe protons and neutrons in terms of quarks.)

Particles made of three quarks are called **baryons.** All baryons have baryon number $B = +\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$. A particle such as the negative pion π^- consists of a quark and an anti-quark: $\pi^- = \bar{u}d$. Quark–antiquark combinations are called **mesons.** All mesons have zero baryon number: $B = +\frac{1}{3} - \frac{1}{3} = 0$.

Quarks and antiquarks do not combine in any other way.

Strangeness: For every strange quark in a hadron the hadron gets a strangeness quantum number of −1. Thus the positive pion $\pi^+ = \mathrm{u\bar{d}}$ has $S = 0$ (it does not have any s quarks), the baryon Σ^+ = uus has $S = -1$ and the baryon $\Xi^+ = \mathrm{\bar{d}\bar{d}\bar{s}}$ has $S = +1$.

In any nuclear reaction, electric charge Q, baryon number B and lepton number L are always conserved, which means that Q, B and L before and after the reaction must be the same. Strangeness S is conserved in electromagnetic and strong interactions but gets violated in weak interactions.

Confinement: Isolated quarks, and hence colour, cannot be observed. If enough energy is supplied to a hadron, the separation of the quarks will increase but eventually a quark–antiquark pair will be created out of the vacuum rather a single quark being ejected from the hadron itself (Figure 7.8).

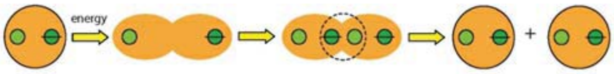


Figure 7.8

This is like breaking a bar magnet and hoping to isolate a magnetic pole. Instead you will only create two magnets!

Conservation laws: In interactions between particles, various quantum numbers are conserved – that is, they have the same value before and after the interaction. In addition to energy and momentum, the quantum numbers electric charge, baryon number, lepton number and colour are always conserved in all interactions. An important exception is strangeness, which is conserved in electromagnetic and strong interactions but not in the weak interaction.

TEST YOURSELF 7.11

State and explain whether the neutral K meson, $\mathrm{K}^0 = \mathrm{d\bar{s}}$, can be its own anti-particle.

TEST YOURSELF 7.12

The following reactions do not take place because they violate one or more conservation laws. In each case, name the conservation laws that are violated.

a $p \rightarrow e^- + \gamma$

b $p \rightarrow \pi^- + \pi^+$

c $n \rightarrow p + e^-$

d $e^- + e^+ \rightarrow \gamma$

e $e^+ \rightarrow \mu^+ + \nu_\mu$

f $p \rightarrow n + e^+ + \nu_e$.

Interactions

There are four fundamental interactions in nature, with differing interaction strengths:

- the electromagnetic interaction (acts on particles with electric charge)
- the strong (or colour) interaction (acts on particles with colour – that is, quarks)
- the weak interaction (acts on quarks and leptons)
- the gravitational interaction (acts on particles with mass).

In particle physics the gravitational interaction is neglected since the masses of the particles involved are so small.

Richard Feynman gave a new interpretation of a force or interaction between particles. For the electromagnetic interaction he considered that two particles, say two electrons, interact when one electron emits a particle and the other electron absorbs it. The particle exchanged between the two electrons is called an **exchange particle.** For the electromagnetic interaction, the particle exchanged is a photon. Other interactions have other exchange particles. Emitting a photon changes the momentum of the electron that emitted it. Absorbing a photon also changes the momentum of the electron that absorbed it. This change in momentum is what we normally expect a force to do.

Feynman diagrams

Feynman went one step further by representing interactions in terms of exchange particles in a pictorial way. He noticed that every conceivable interaction in electromagnetism can be constructed out of a single basic **interaction vertex.** In Figure 7.9 the interaction vertex has been used twice. In the top vertex the electron emits a photon. In the lower vertex the electron absorbs a photon.

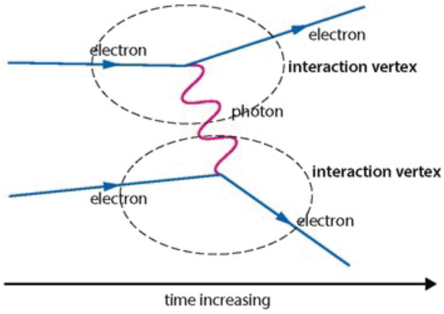


Figure 7.9

(The weak and strong interactions have many other interaction diagrams but in an exam these will be given where necessary.)

In Figure 7.10 the interaction vertex is again used twice to construct the process in which an electron annihilates with a positron, producing a photon which then rematerialises as a particle–anti-particle pair. Notice that we must specify the direction of time. Particles are shown as solid lines with arrows in the direction of time. Anti-particles are shown as solid lines with arrows opposite to the direction of time.

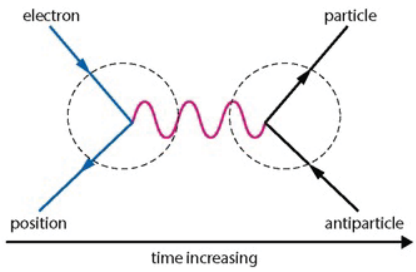


Figure 7.10

In Figure 7.11 the basic vertex has been used four times to create the process of a photon scattering off another photon: $\gamma + \gamma \rightarrow \gamma + \gamma$.

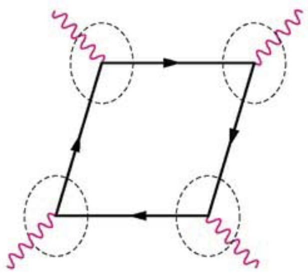


Figure 7.11

The weak interaction has many basic interaction vertices and this makes this interaction more complicated. A few vertices are shown in Figure 7.12.

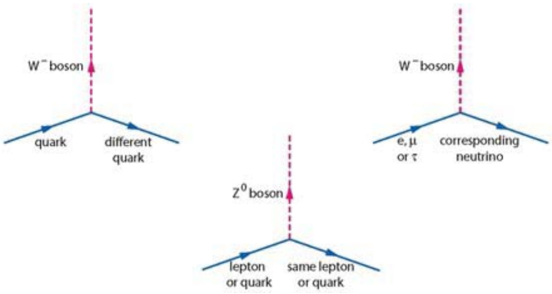


Figure 7.12

These can be used to describe beta minus decay, as Figure 7.13 shows.

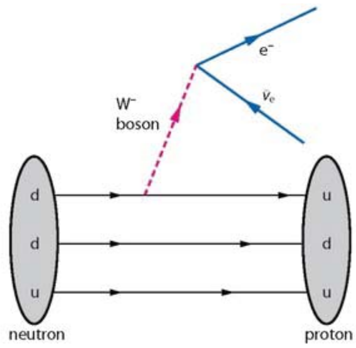


Figure 7.13

A d quark inside a neutron emits a W^- boson, which then turns into an electron and an electron anti-neutrino.

Similarly, there are also very many complicated interaction vertices for the strong interaction.

TEST YOURSELF 7.13

The Σ^+ = uus particle decays into a proton and a neutral pion ($\pi^0 = u\bar{u}$): $\Sigma^+ \rightarrow p + \pi^0$.

a State and explain whether the interaction involved is strong, weak or electromagnetic.

b Draw a Feynman diagram for this process.

TEST YOURSELF 7.14

A neutral meson consisting of a charmed quark and an anti-charmed quark decays into two photons. Construct a Feynman diagram for the process.

TEST YOURSELF 7.15

Construct a Feynman diagram for the process $e^- + e^+ \rightarrow e^- + e^+ + e^- + e^-$.

TEST YOURSELF 7.16

Use one of the basic interaction vertices for the weak interaction to draw a Feynman diagram for the decay of a neutron into a proton by beta decay.

The Higgs particle

In addition to the quarks, leptons and exchange particles we have discussed so far, the standard model requires the existence of just one more particle: the Higgs particle. This is the last particle of the standard model to be discovered. Its discovery was announced by the Large Hadron Collider team at CERN on July 4, 2012.

The standard model is a **mathematical theory based on symmetry.** The symmetry of the theory requires that the particles of the model (quarks, leptons and exchange particles) be massless. This is fine for the photon but not for the exchange particles of the weak interaction. They must be massive.

The Higgs particle achieves the trick of giving masses to some of the particles through their interactions with the Higgs but at the same time preserves the symmetry of the theory to a sufficient degree for the theory to work.

Checklist

After studying this chapter you should be able to:

- □ **calculate wavelengths of photons emitted in atomic transitions**
- □ **describe how discrete energy levels explain atomic spectra**
- □ **work with radioactive decay equations and the concepts of activity and half-life**
- □ **calculate mass defects, bonding energies and energy released in nuclear reactions**
- □ **apply conservation laws in particle physics**
- □ **describe quarks, leptons and exchange particles**
- □ **construct simple Feynman diagrams.**

8 ENERGY PRODUCTION

This chapter covers the following topics:

- □ Specific energy and energy density
- □ Sankey diagrams
- □ Primary energy sources
- □ Electricity as a secondary and versatile form of energy
- □ Renewable and non-renewable energy sources
- □ Conduction, convection and radiation
- □ Black body radiation, including the Stefan–Boltzmann law and Wien’s law
- □ Albedo and emissivity
- □ The solar constant
- □ The greenhouse effect and the energy balance of the Earth–atmosphere system

8.1 Energy sources

DEFINITIONS

PRIMARY ENERGY SOURCE A source of energy that has not been processed in any way.

SECONDARY ENERGY SOURCE A source of energy that has been processed.

NON-RENEWABLE ENERGY SOURCE A finite energy source which is being depleted much faster than it can be produced, and so will run out. Non-renewable sources include fossil fuels (e.g. oil, natural gas and coal) and nuclear fuels (e.g. uranium).

RENEWABLE ENERGY SOURCE This includes solar energy (and the other forms indirectly dependent on solar energy, such as wind energy, wave energy and bio-fuels) and tidal energy. At present, about 12% of our energy comes from these sources.

SPECIFIC ENERGY The amount of energy that can be obtained from a unit mass of a fuel. In the S.I. system it is measured in J kg^{-1}.

ENERGY DENSITY The amount of energy that can be obtained from a unit volume of a fuel. In the S.I. system it is measured in J m^{-3}.

DEGRADATION OF ENERGY Energy, while being always conserved, becomes less useful, meaning that it cannot be used to perform mechanical work. This is called *energy degradation*. For example, the thermal energy that comes out of the exhaust of a car is degraded energy. It cannot easily be further used.

SANKEY DIAGRAM A pictorial energy-flow block diagram showing energy flows using arrows whose widths are proportional to the respective energy transfers.

TEST YOURSELF 8.1

A power plant produces power by burning M kg of natural gas per second. The specific energy of natural gas is E. The plant operates at an overall efficiency of e. What is the net power produced by the plant?

a eME **b** $\frac{ME}{e}$ **c** $\frac{eE}{M}$ **d** $\frac{E}{eM}$

Efficiency

If a machine (for example, an electric motor, automobile engine or power plant) is used to do work, energy must be provided to the machine. It uses some of this energy to do useful work (for example, raising a load, moving an automobile or turning an electric turbine), but some of the input energy is lost to non-useful forms, such as thermal energy in overcoming friction. The ratio

$$\frac{\text{useful energy out}}{\text{total energy in}} \text{ or } \frac{\text{useful power out}}{\text{total power in}}$$

is therefore less than one. This ratio is called the **efficiency** of the machine.

An automobile engine typically has an efficiency of 30% to 40%, a coal power plant about 30%.

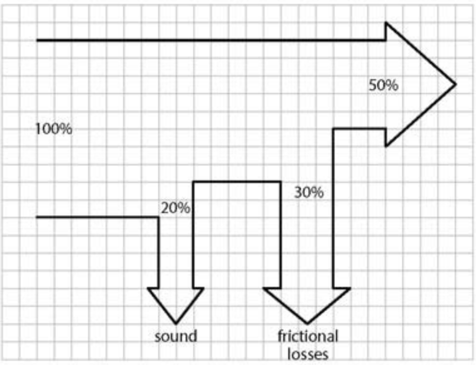


Figure 8.1

In the Sankey diagram for an electric motor shown in Figure 8.1, 100 units of input energy corresponds to an arrow width of 10 squares. Sound losses of 20% are represented by an arrow two squares wide, and frictional losses of 30% by an arrow three squares wide. The useful-energy arrow is five squares wide, thus 50% of the original input energy. The efficiency of the motor is useful $\frac{\text{energy}}{\text{input energy}} = 0.50$.

TEST YOURSELF 8.2

What is the efficiency of the engine whose Sankey diagram is shown in Figure 8.2?

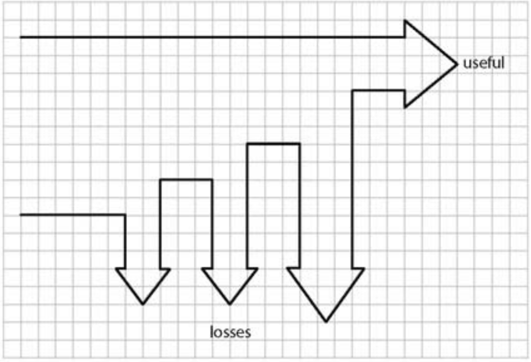


Figure 8.2

Fossil fuels

Fossil fuels (oil, coal and natural gas) have been created over millions of years. They are produced by the decomposition of buried animal and plant matter under the combined action of the high pressure of the material on top and bacteria.

Burning coal and oil have been the traditional ways of producing electricity. Worldwide, about 86% of our energy comes from fossil fuels. A typical fossil fuel power plant is shown in Figure 8.3.

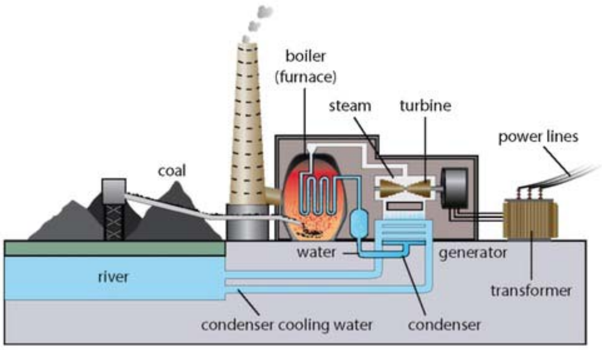


Figure 8.3

Advantages of fossil fuels:

- High energy density
- Used directly and easily by a variety of engines and devices
- Extensive distribution network in place.

Disadvantages of fossil fuels:

- Release of greenhouse gases into the atmosphere
- Especially with oil, serious environmental problems due to leakages along the production–distribution line
- Need for extensive storage facilities.

Annotated Exemplar Answer 8.1

a What is the energy density of a fuel? **[1]**

b A power plant produces electricity by burning gas with energy density $3.6 \times 10^7\,\text{J m}^{-3}$ at a rate of $19.2\,\text{m}^3\text{s}^{-1}$. The power output of the power plant is 400 mW. What is the efficiency of the power plant? **[2]**

a Energy density is the energy that can be extracted from the fuel.

This gains no marks because the key idea is missing – that it is the energy extracted per unit volume of fuel.

b Efficiency $= \dfrac{400\,\text{MW}}{3.6 \times 10^7 \times 19.2} = 0.578 \approx 58\%$

This is a straightforward calculation, which has given a sensible answer. If you had forgotten to change the 400 mW into 400×10^6 W, the efficiency would have worked out ridiculously small. Always check that your answer is reasonable.

2/3

Electricity production

Many of the energy sources that are available to us today are used to produce electricity (Figure 8.4).

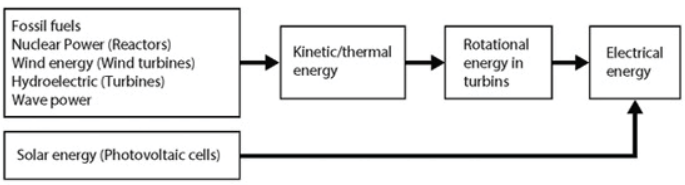


Figure 8.4

< Back to chapter

Annotated Exemplar Answer 8.1

a What is the energy density of a fuel? **[1]**

b A power plant produces electricity by burning gas with energy density 3.6×10^{7} Jm^{-3} at a rate of 19.2 m^{3} s^{-1}. The power output of the power plant is 400 mW. What is the efficiency of the power plant? **[2]**

a *Energy density is the energy that can be extracted from the fuel.*

This gains no marks because the key idea is missing - that it is the energy extracted per unit volume of fuel.

b $\text{Efficiency} = \frac{400\ MW}{3.6 \times 10^{7} \times 19.2} = 0.578 \approx 58\%.$

This is a straightforward calculation, which has given a sensible answer. If you had forgotten to change the 400 mW into 400 × 10^{6} W, the efficiency would have worked out ridiculously small. Always check that your answer is reasonable.

2/3

8.2 Nuclear Power

Worldwide, about 11% of our energy comes from nuclear fuels. Fission reactions were discussed in Section 7.3, Fission and fusion.

A schematic diagram of the core of one type of nuclear reactor, a pressurised water reactor, is shown in Figure 8.5.

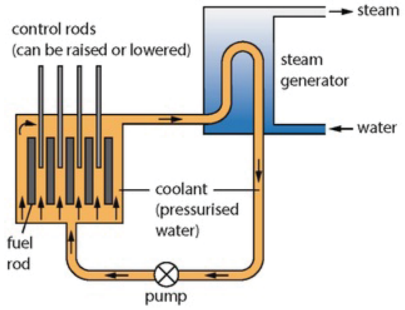


Figure 8.5

DEFINITIONS

NUCLEAR REACTOR A machine in which nuclear reactions take place, producing energy. The 'fuel' of a nuclear reactor is typically uranium-235. A typical energy-producing fission reaction is ${}^{1}_{0}n + {}^{235}_{92}\mathrm{U} \to {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2{}^{1}_{0}n$, in which a neutron splits a ${}^{235}\mathrm{U}$ nucleus into two smaller nuclei and two fast neutrons.

CHAIN REACTION The fast neutrons produced in the fission of ${}^{235}\mathrm{U}$ are then slowed down and collide with other ${}^{235}\mathrm{U}$ nuclei in the reactor, producing more fission, energy and neutrons. The reaction is thus self-sustaining.

CRITICAL MASS The minimum mass of uranium-235 for a chain reaction to sustain itself; with a lower mass, many neutrons escape without causing further reactions.

URANIUM ENRICHMENT The uranium commonly mined is ${}^{238}\mathrm{U}$, which contains small traces (0.7%) of ${}^{235}\mathrm{U}$. Enrichment is the process of increasing the concentration of the useful ${}^{235}\mathrm{U}$ in uranium samples. For nuclear power plants, the concentration is increased to 5%. Uranium used in nuclear weapons is enriched to much higher values, around 80%.

FUEL RODS Rods containing the nuclear fuel in pellet or powder form. In the case of uranium, the fuel rods contain a mixture of fissionable ${}^{235}\mathrm{U}$ and non-fissionable ${}^{238}\mathrm{U}$.

CONTROL RODS These rods can be inserted to absorb excess neutrons in order to control the rate of the fission reaction.

MODERATOR The neutrons produced in the fission reaction must be slowed down if they are to be used to cause further fissions. This is achieved through collisions of the neutrons with atoms of the moderator, the material surrounding the fuel rods.

HEAT EXCHANGER The kinetic energy of the reaction products is converted to thermal energy in the moderator. A coolant (water, for example) passing through the moderator can extract this thermal energy and use it to turn water to steam at high temperature and pressure. This can be used to turn the turbines of a power station, producing electricity.

Advantages of energy production by nuclear fission:

- High power output (high energy density)
- Large available reserves of nuclear fuel
- No greenhouse gas emissions.

Disadvantages of energy production by nuclear fission:

- The fast neutrons produced in a fission reaction may be used to bombard uranium-238 and produce plutonium-239:

$$n + {}^{238}_{92}\mathrm{U} \to {}^{238}_{92}\mathrm{U} \to {}^{239}_{93}\mathrm{Np} + {}^{0}_{-1}e + \overline{v} \quad {}^{239}_{93}\mathrm{Np} \to {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}e + \overline{v}.$$

  Thus, non-fissionable uranium-238 is being converted to fissionable plutonium-239, which can be used in the production of nuclear weapons or as nuclear fuel in *fast breeder* reactors.

- The spent fuel and the reaction products are highly radioactive, with long half-lives. Safe disposal of this material is a serious issue in commercial energy production.
- Even if the reactor is shut down, production of thermal energy continues due to beta decay of the product nuclei.
- A fission reactor can become a major public health hazard if control or back-up systems fail and an uncontrolled chain reaction leads to reactor meltdown.
- Uranium decays into radon gas, a known strong carcinogen. Inhalation of this gas and of radioactive dust particles is a major hazard in uranium mining.

TEST YOURSELF 8.3

In a nuclear fission reaction involving uranium-235, approximately 200 MeV of energy is produced. What mass of uranium is required per year in a nuclear reactor that produces 400 MW of electrical power with an overall efficiency of 40%?

hint

Recall the definition of efficiency.

TEST YOURSELF 8.4

A nuclear reactor operating without a moderator would:

a lead to meltdown.

b produce very little energy.

c produce much more energy.

d be more efficient.

8.3 Solar power

The nuclear fusion reactions in the Sun produce an incredible supply of free and essentially inexhaustible energy, which reaches the Earth's surface at a rate of about 1000 Wm^{-2} (1 kWm^{-2}). There are two ways to extract energy from this solar radiation.

Photovoltaic systems (solar to electric): These are cells made out of **semiconductors.** Sunlight incident on each cell produces a DC current and a DC voltage (by a process similar to the photoelectric effect). The currents and voltages are generally small, so photovoltaic cells are of limited use except in large arrays.

Solar active devices (solar to thermal): These are commonly used in many countries to extract thermal energy from sunlight and use it to heat water for domestic use. These simple, cheap collectors are usually put on the roof of a house. A disadvantage is that they tend to be bulky and cover a large area.

Advantages of solar energy production:

- Unlimited energy supply
- No extraction costs
- Clean.

Disadvantages of solar energy production:

- Works only during the day
- Affected by cloudy weather
- Low energy density
- Requires large area for collectors
- High initial costs (for photovoltaic systems).

8.4 Hydroelectric power

Consider a volume ΔV of water with a height h above some chosen horizontal level (Figure 8.6).

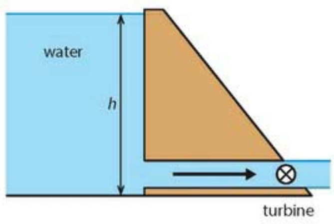


Figure 8.6

If this volume of water drops by h, its gravitational potential energy is converted to kinetic energy as it flows through the pipe at the bottom. Its mass is $m = \rho\Delta V$, where ρ is the density of water, and so its potential energy is $mgh = \rho\Delta Vgh$. The rate P (power) at which this potential energy is converted is therefore $P = \rho\frac{\Delta V}{\Delta t}gh$.

If the surface area of the dam is A, then its volume is $V = Ah$. Then the rate it which its volume changes is $\frac{\Delta V}{\Delta t} = A\frac{\Delta h}{\Delta t}$, so $P = \rho A\frac{\Delta h}{\Delta t}gh$.

In other words, the power depends on the height of the water and the rate at which the height changes.

Advantages of hydroelectric energy:

- Unlimited energy supply
- No extraction costs
- Clean.

Disadvantages of hydroelectric energy:

- Very location-dependent
- Dams and reservoirs require serious changes to the environment
- High initial costs.

Pumped storage is a way to store hydroelectric energy on a large scale. Water is pumped to a higher reservoir during low-demand periods, and then flows back down to generate electricity during high-demand periods. This may seem nonsensical, since pumping it up requires energy – more energy, in fact, than is regained when it is again allowed to flow down. But if the electricity to pump it up comes from other sources at off-peak times when energy prices are lower, then the net effect is an increase in energy available at peak times but at off-peak rates.

TEST YOURSELF 8.5

In an experiment, water (density 1000 kg m^{-3}) kept in a small basin of surface area 2.0 m^2 is allowed to descend from an initial height of 20 m until the basin empties (Figure 8.7).

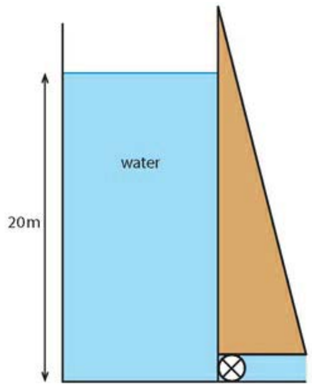


Figure 8.7

The height of the water in the basin varies with time as shown in Figure 8.8.

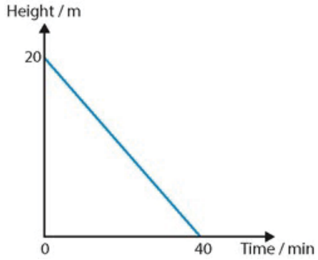


Figure 8.8

Estimate the *average* power that can be generated by the falling water.

hint

Because the generated power falls to zero as the water depth goes to zero (as the basin empties), the *average* power will be half the amount produced at maximum depth. Another way to look at this is that we will use the average depth of the water, 10m.

8.5 Wind power

This ancient method for exploiting the kinetic energy of wind is particularly useful for isolated areas and for agricultural use. Modern large wind turbines can produce up to a few megawatts of power.

To estimate the scale of energy transfer in a wind turbine, consider a mass of air, moving at wind velocity v, that in time Δt passes through the cross-sectional area A spanned by the blades of a wind turbine (Figure 8.9).

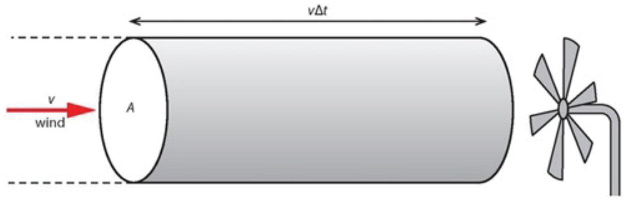


Figure 8.9

This is equal to the mass of a tubular volume of length $v\Delta$. If ρ is the density of air, this mass is $\rho A v \Delta t$. The kinetic energy of this mass of air is $\frac{1}{2}(\rho A v \Delta t)v^2 = \frac{1}{2}\rho A \Delta t v^3$. The rate at which this energy passes through the blades is $P = \frac{1}{2}\rho A v^3$.

Advantages of wind power:

- Essentially unlimited energy supply
- No extraction costs
- Clean
- Practical for remote locations away from other energy sources.

Disadvantages of wind power:

- Works only if there is wind!
- Visually intrusive in large numbers
- Low-frequency operating noise intrusive for some people
- High maintenance costs due to wind stresses.

TEST YOURSELF 8.6

a Estimate the number of wind turbines that are needed to supply a village whose power needs total 5.0 MW. wind turbine has blade diameter 12 m, the density of air is 1.2 kg m^{-3}; assume the average wind speed is 8.0 m s^{-1}.

b Comment on your answer.

Society demands action

Nature of Science. Society's demand for ever-increasing amounts of energy raises ethical debates. How can present and future energy needs be best met, without compromising the future of the planet? There are many aspects to the energy debate, and all energy sources have associated risks, benefits and costs. Although not all governments and people support the development of renewable energy sources, scientists across the globe continue to collaborate to develop new technologies that can reduce our dependence on non-renewable energy sources.

8.6 Thermal energy transfer

DEFINITIONS

INTENSITY Power received per unit area: $I = \frac{P}{A}$. If a source emits equally in all directions, then, at a distance d from the source, the intensity is $I = \frac{P}{4\pi d^2}$.

SOLAR CONSTANT The intensity of solar radiation at the Earth's distance from the Sun, d_{Earth}. Since the total power radiated by the Sun is $P = 3.8 \times 10^{26}$ W, the solar constant is $\frac{P}{4\pi d_{Earth}^2} \approx 1400\,\text{Wm}^{-2}$.

BLACK BODY A theoretical body that absorbs *all* the energy that is incident on it, reflecting none.

STEFAN–BOLTZMANN LAW The power emitted per unit area of a black body at an absolute (kelvin) temperature of T is $\frac{P}{A} = \sigma T^4$, where $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}. Thus $P = \sigma A T^4$.

ALBEDO $\alpha = \frac{\text{reflected power}}{\text{total incident power}}$ The Earth as a whole has an average albedo of about 0.30, indicating that it reflects about 30% of the radiation incident on it. The albedo is different for different parts of the Earth, depending on soil type, water depth, forestation and cloud cover.

Thermal energy can be transferred by conduction, by convection and by radiation.

Conduction: If you place a metal rod into hot coals, the other end will soon get warm. The heat of the coals causes atoms at the hot end to vibrate about their equilibrium positions more violently – that is, there is an increase in temperature. Collisions between neighbouring atoms result in an increase in the amplitude of vibration of neighbouring atoms – that is, their temperature increases too. The rate at which heat is transferred through a material is proportional to its cross-sectional area A and to the temperature difference $T_H - T_C$ across it, and inversely proportional to its thickness L (see Figure 8.10).

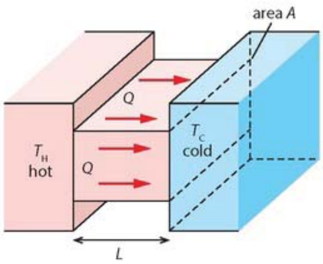


Figure 8.10

Convection: This applies mainly to fluids. Part of a fluid that is warmer than its surroundings will expand and become less dense. Buoyant forces will then make this part of the fluid rise while colder fluid will take its place. This upward motion of the warm fluid creates convection currents. Strong convection currents in the Sun's interior bring warm gases to the surface in this way. Weather patterns depend on convection currents within the oceans and the atmosphere.

Radiation: Unlike conduction and convection, radiation can travel through a vacuum. All bodies at a given absolute (Kelvin) temperature T radiate electromagnetic waves whose energy is distributed over an infinite range of wavelengths. Surfaces that are black and dull, as opposed to polished and shiny, are good approximations to a theoretical **black body,** which radiates according to the Stefan–Boltzmann law, $P = \sigma A T^4$. Everything else radiates according to $P = e\sigma A T^4$. The constant e, called the **emissivity** of the surface, may be defined as $\frac{\text{power per unit area radiated at absolute temperature } T}{\text{power per unit area radiated by a black body at absolute temperature } T}$. It measures how effectively a body radiates, and varies between 0 and 1.

Figure 8.11 shows the intensity of radiation from the surface of a black body as a function of the wavelength of the radiation. The area under each curve represents the total power radiated from the surface, and is proportional to T^4. (For the temperatures shown here, the peak wavelengths are in the infrared region of the electromagnetic spectrum.)

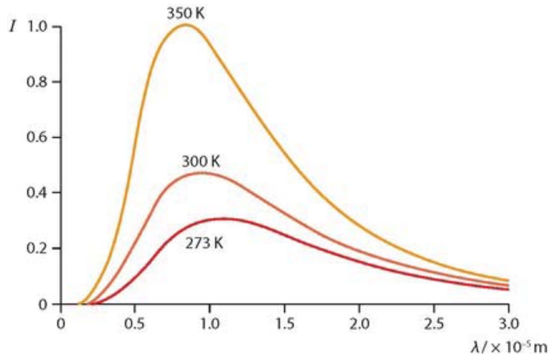


Figure 8.11

The graph shows that, as the temperature increases, the wavelength of the peak, at which most of the energy is emitted, becomes smaller. This is a consequence of Wien's law, which states that $\lambda T = 2.9 \times 10^{-3}$ Km.

A body of emissivity e kept at a temperature T_1 will radiate power at $P_{out} = e\sigma A T_1^4$. In surroundings at temperature T_2 it will also *absorb* power at a rate $P_{in} = e\sigma A T_2^4$. Thus the net power lost by the body is $P_{net} = P_{out} - P_{in} = e\sigma A(T_1^4 - T_2^4)$. At equilibrium, $P_{net} = 0$; that is, the body loses as much energy as it gains and so its temperature stays constant and equal to that of its surroundings: $T_1 = T_2$.

Figure 8.12 shows the intensity distribution of radiation from various surfaces kept at the same temperature (300 K). The difference in the curves is due entirely to their different emissivities (e = 1.0, 0.8 and 0.2). (The units on the vertical axis are arbitrary.)

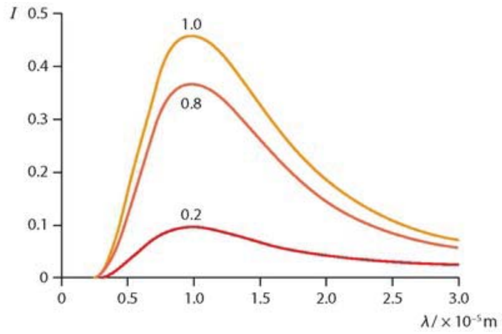
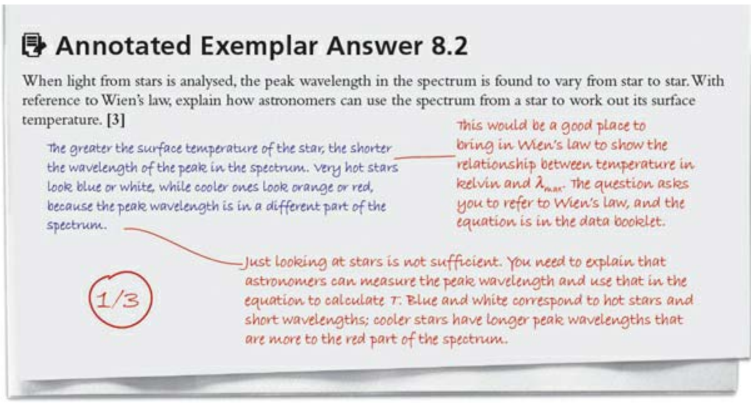


Figure 8.12

Table 8.1 shows typical emissivities of various materials.

Table 8.1

Surface	Emissivity
Black body	1.0
Ocean water	0.80–0.90
Ice	0.60–0.80
Snow	0.05–0.60
Dry land	0.70–0.80
Forest	0.85–0.95
Land with vegetation	0.75



Annotated Exemplar Answer 8.2

When light from stars is analysed, the peak wavelength in the spectrum is found to vary from star to star. With reference to Wien's law, explain how astronomers can use the spectrum from a star to work out its surface temperature. [3]

The greater the surface temperature of the star, the shorter the wavelength of the peak in the spectrum. Very hot stars look blue or white, while cooler ones look orange or red, because the peak wavelength is in a different part of the spectrum.

This would be a good place to bring in Wien's law to show the relationship between temperature in kelvin and λ_{max}. The question asks you to refer to Wien's law, and the equation is in the data booklet.

Just looking at stars is not sufficient. You need to explain that astronomers can measure the peak wavelength and use that in the equation to calculate T. Blue and white correspond to hot stars and short wavelengths; cooler stars have longer peak wavelengths that are more to the red part of the spectrum.

1/3

TEST YOURSELF 8.7

Four slabs of the same cross sectional area have various widths and temperature differences between the left and right side, as indicated in Figure 8.13.

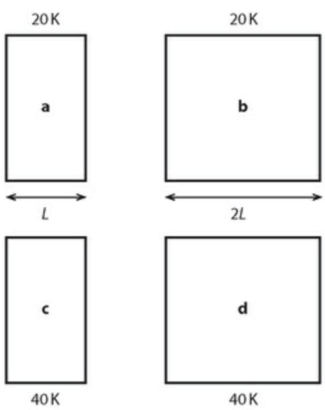


Figure 8.13

In which one is the rate of flow of heat the greatest?

TEST YOURSELF 8.8

The temperature of a body in kelvin is doubled. By what factor does the power radiated increase?

a 2

b 4

c 8

d 16

Model Answer 8.1

A container holds 220 kg of water. When the container is hidden from sunlight, the temperature of the water decreases at a rate of 3.5×10^{-3} K^{-1} s^{-1}. During the day, the solar intensity is 640W m^{-2}. The specific heat capacity of the water is 4.2 kJ kg^{-1} K^{-1}. Calculate the area of the water that must be exposed to sunlight so that the water temperature remains constant.

The rate at which thermal energy is lost is

$$\frac{\Delta Q}{\Delta t} = mc\frac{\Delta \theta}{\Delta t} = 220 \times 4200 \times 3.5 \times 10^{-3} = 3234\,W.$$

This must be provided by sunlight. If the exposed area is A, then the power supplied is $640 \times A$ and so $640\,A = 3234 \Rightarrow A = 5.1\ m^2$.

< Back to chapter

Annotated Exemplar Answer 8.2

When light from stars is analysed, the peak wavelength in the spectrum is found to vary from star to star. With reference to Wien's law, explain how astronomers can use the spectrum from a star to work out its surface temperature. **[3]**

> *The greater the surface temperature of the star, the shorter the wavelength of the peak in the spectrum. very hot stars look blue or white, while cooler ones look orange or red, because the peak wavelength is in a different part of the spectrum.*

This would be a good place to bring in Wien's law to show the relationship between temperature in kelvin and λ_{max}. The question asks you to refer to Wien's law, and the equation is in the data booklet.

Just looking at stars is not sufficient. You need to explain that astronomers can measure the peak wavelength and use that in the equation to calculate T. Blue and white correspond to hot stars and short wavelengths; cooler stars have longer peak wavelengths that are more to the red part of the spectrum.

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8.7 The energy balance of the Earth

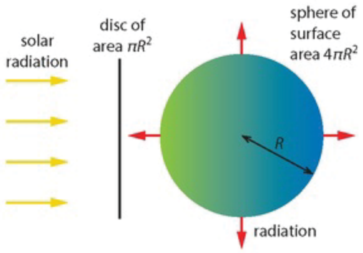

Figure 8.14

Average intensity: The solar constant is S = 1400 Wm^{-2}. The radiation that reaches the Earth can be thought of as passing through a disc of radius equal to the Earth's radius, R (see Figure 8.14). The power incident on this disc is therefore $P = S\pi R^2$. This power is thought to be distributed over the entire surface of the Earth and so the average intensity incident on Earth is $I_{ave} = \frac{S\pi R^2}{4\pi R^2} = \frac{S}{4} \approx 350\,\mathrm{W\,m^{-2}}$. About 30% is reflected, which means that the Earth's surface receives an *average* radiation intensity of 350 × 0.70 ≈ 250 Wm^{-2}.

TEST YOURSELF 8.9

Assume that the Earth's surface has a fixed temperature T and that it radiates as a black body. The intensity of the solar radiation arriving at the upper atmosphere is S = 1400 Wm^{-2}. Take the albedo of the atmosphere to be α= 0.30.

a Write down an equation expressing the fact that the power received by the Earth equals the power radiated by the Earth into space (an energy-balance equation). Refer to Figure 8.15.

b Solve the equation to calculate the constant Earth temperature.

c Comment on your answer.

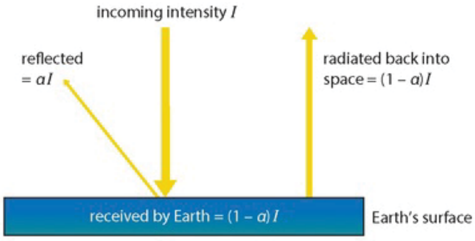

Figure 8.15

The greenhouse effect

The **greenhouse effect** is represented by the dashed box in Figure 8.16. The crucial difference between this figure and Figure 8.15 is that, with an atmosphere, some of the radiation emitted by the Earth's surface gets absorbed by gases in the atmosphere and then re-radiated in all directions, including back down to Earth, warming the Earth.

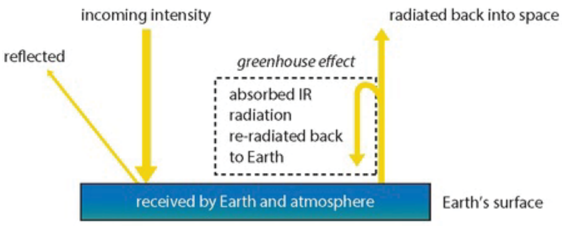

Figure 8.16

The mechanism of photon absorption by greenhouse gases: Like atoms, molecules exist in discrete energy states. The differences between molecular energy levels correspond to infrared photon energies. The radiation emitted by the Earth's surface is in the infrared region. Hence some of this radiation is absorbed by these gases, exciting their molecules to a higher energy state. This radiation is released when the molecule makes a transition back down to its original state.

Greenhouse gases have natural as well as human-related sources, as indicated in Table 8.2.

Table 8.2

Greenhouse gas	Natural sources	Human-related sources
H_2O (water vapour)	evaporation of water from oceans, rivers and lakes	
CO_2 (carbon dioxide)	forest fires, volcanic eruptions, evaporation of water from oceans	burning of fossil fuels in power plants and cars, burning of forests

CH_4 (methane)	wetlands, oceans, lakes and rivers	flooded rice fields, farm animals, termites, processing of coal, natural gas and oil, and burning of biomass
N_2O (nitrous oxide)	forests, oceans, soil, grasslands	burning of fossil fuels, manufacture of cement, fertilisers, deforestation (reduction of nitrogen fixation in plants)

Checklist

After studying this chapter you should be able to:

- ☐ **distinguish between specific energy and energy density and do calculations with these quantites**
- ☐ **use Sankey diagrams to estimate efficiency**
- ☐ **distinguish between primary and secondary energy sources and between renewable and non-renewable energy sources**
- ☐ **describe energy transformations**
- ☐ **understand the mechanisms of conduction, convection and radiation**
- ☐ **use the Stefan–Boltzmann law and Wien’s law**
- ☐ **work with albedo and emissivity**
- ☐ **calculate the solar constant**
- ☐ **construct simple energy-balance equations**
- ☐ **describe the mechanism of photon absorption by greenhouse gases.**

9 WAVE PHENOMENA (HL)

This chapter covers the following topics:

- □ Quantitative simple harmonic motion
- □ Interference
- □ Thin-film interference
- □ Diffraction
- □ Resolution
- □ Multiple-slit interference
- □ The diffraction grating
- □ The Doppler effect

9.1 Simple harmonic motion (SHM)

In simple harmonic motion (SHM), the acceleration and the net force are *proportional to and opposite to the displacement from equilibrium.* Mathematically this can be written as $a = -\omega^2 x$, where ω is a constant called the **angular frequency**. SHM consists of periodic oscillations with a period that is *independent of the amplitude.*

The period is given by $T = \frac{2\pi}{\omega}$.

DEFINITIONS

x, v AND a $x = x_0 \cos(\omega t + \phi)$, $v = -\omega x_0 \sin(\omega t + \phi)$, $x = -\omega^2 x_0 \cos(\omega t + \phi)$, where x_0 is the maximum displacement.

PHASE, ϕ An angle added to ωt in the formulas for x, v and a which indicates their values at t = 0.

POTENTIAL ENERGY $E_p = \frac{1}{2} m\omega^2 x^2$

KINETIC ENERGY $E_K = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

Two special cases of SHM are:

- a mass m at the end of a spring with spring constant k: the period is $T = 2\pi\sqrt{\frac{m}{k}}$
- a simple pendulum of length l: the period is $T = 2\pi\sqrt{\frac{l}{g}}$.

The expression for the pendulum is true provided the amplitude is small. Note also that the period of a pendulum does not depend on its mass.

A common choice of phase is $\phi = 0$, in which case $x = x_0 \cos \omega t$, $v = -\omega x_0 \sin \omega t$, $a = -\omega^2 x_0 \cos \omega t$. These three quantities are plotted together in Figure 9.1. The maximum speed is $v_{max} = \omega x_0$ and the maximum acceleration is $a_{max} = \omega^2 x_0$.

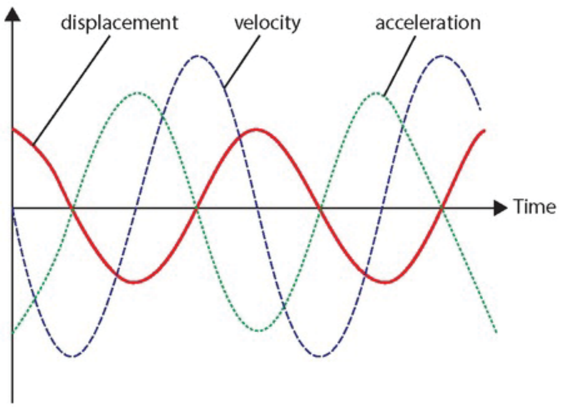

Figure 9.1

Another common choice of phase is $\phi = -\frac{\pi}{2}$, in which case $x = x_0 \sin \omega t$, $v = \omega x_0 \cos \omega t$, $a = -\omega^2 x_0 \sin \omega t$.

You should be able to draw graphs of displacement, speed and acceleration as a function of time for SHM, showing the relative positions of these three graphs, and to state the phase difference between them.

Energy in simple harmonic motion

In SHM, kinetic energy and potential energy are constantly transformed into one another. The maximum potential energy $\frac{1}{2} m\omega^2 x_0^2$ occurs at maximum displacement, $x = \pm x_0$, and here the speed $v = 0$.

Therefore the kinetic energy is also zero at this point, so $\frac{1}{2} m\omega^2 x_0^2$ is a measure of the total energy: $E_T \frac{1}{2} m\omega^2 x_0^2. Since\ E_T = E_K + E_P$ at any point, the kinetic energy at any displacement x is given by $\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x^2$ so $v = \pm\omega\sqrt{x_0^2 - x^2}$.

As the oscillating body goes past the equilibrium position, $x = 0$ and so $v = \pm\omega x_0$.

TEST YOURSELF 9.1

Figure 9.2 shows the variation with displacement of the acceleration of a particle.

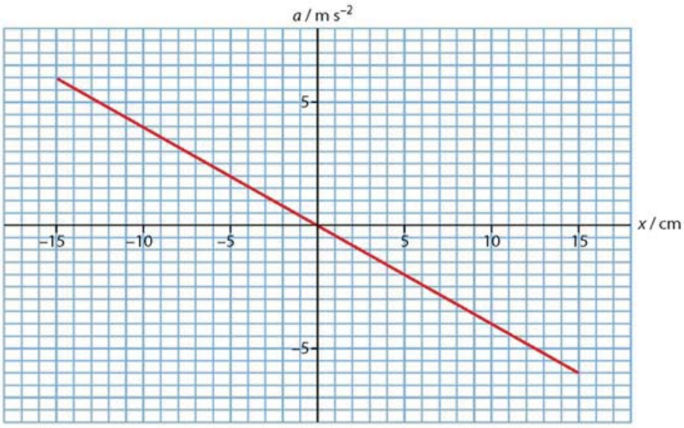

Figure 9.2

a Use the graph to explain why the oscillations are SHM. Calculate
b the period of the motion,
c the maximum speed and
d the speed at a displacement of 5.0cm.

TEST YOURSELF 9.2

Figure 9.3 shows the kinetic energy of a mass of 0.25 kg undergoing SHM as a function of the displacement.

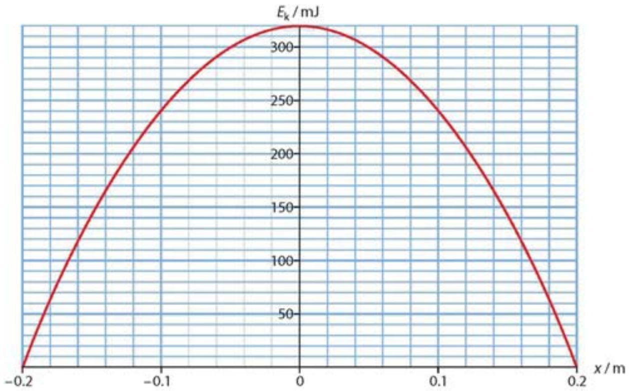

Figure 9.3

Calculate **a** the period of the motion and **b** the maximum speed and the maximum acceleration.

c On a copy of this figure, draw a sketch graph to show the variation with displacement of the potential energy.

d Estimate the displacement when the potential energy is equal to the kinetic energy.

Worked Example 9.1

Figure 9.4 shows the variation with time of the speed of a particle of mass 0.20 kg that is executing SHM.

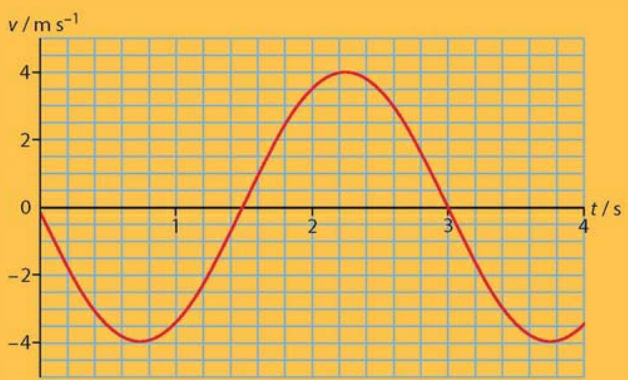

Figure 9.4

Use the graph to find:

a the amplitude of the motion
b the displacement at 2.0 s
c the total energy of the particle.

The equation for the speed graph is $v = -\omega x_0 \sin(\omega t)$ and so $x = x_0 \cos(\omega t)$.

The period is 3.0 s and so

$\omega = \frac{2\pi}{T} = 2.0944 \mathrm{s}^{-1}$.

a The maximum speed is 4.0 m s^{-1} and so from $v_{max} = \omega x_0$ we find
$x_0 = \frac{V_{max}}{\omega} = \frac{4.0}{2.0944} = 1.9099 \approx 1.9 \mathrm{m}$

b From $x = x_0 \cos(\omega t)$ we find $x = 1.9099 \times \cos(2.0944 \times 2.0) \approx -0.95$m. You would make a mistake here if your calculator is not set to radian mode.

c The total energy is $E_T = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} \times 0.20 \times 2.0944^2 \times 1.9099^2 \approx 1.6 \mathrm{J}$

9.2 Diffraction

Recall from Chapter 4 that diffraction is the spreading of a wave as it goes through an opening (aperture) or past an obstacle.

Single-slit diffraction

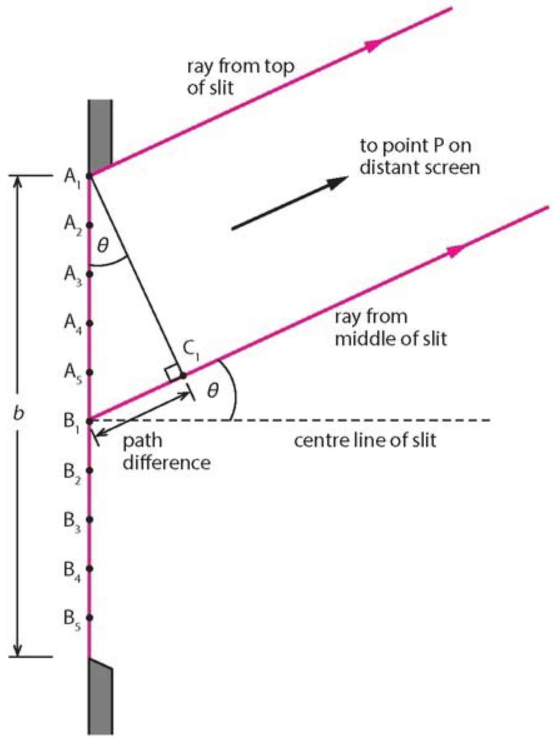

Figure 9.5

Figure 9.5 shows two light rays of identical wavelength (from points A_1 and B_1) which both make an angle of θ with the centre line. The path difference between these two rays is $\frac{b}{2}\sin\theta$, where b is the width of the slit.

If the angle is very small, this is approximately equal to $\frac{b}{2}\theta$ if the angle is expressed in radians.

If this path difference happens to be half a wavelength $\left(\frac{b}{2}\theta \approx \frac{\lambda}{2}\right)$, we will have destructive interference when these rays meet at a screen (assumed to be at a distance very large compared with the slit width). The same condition holds for a pair of rays from *just below* the top and *just below* the middle of the opening, or indeed for any other such pair of rays at an angle θ. The result will be a dark band at the screen at the **diffraction angle** $\theta_D \approx \frac{\lambda}{b}$.

On the other hand, all rays leaving the slit in a direction along the centre line ($\theta = 0$) will arrive in phase, so we will have constructive interference, and a bright band, at the centre of the diffraction pattern.

Similar calculations for other angles results in the complex intensity pattern at the screen shown in Figure 9.6. This is the basic pattern for single-slit diffraction, and you should be familiar with its features.

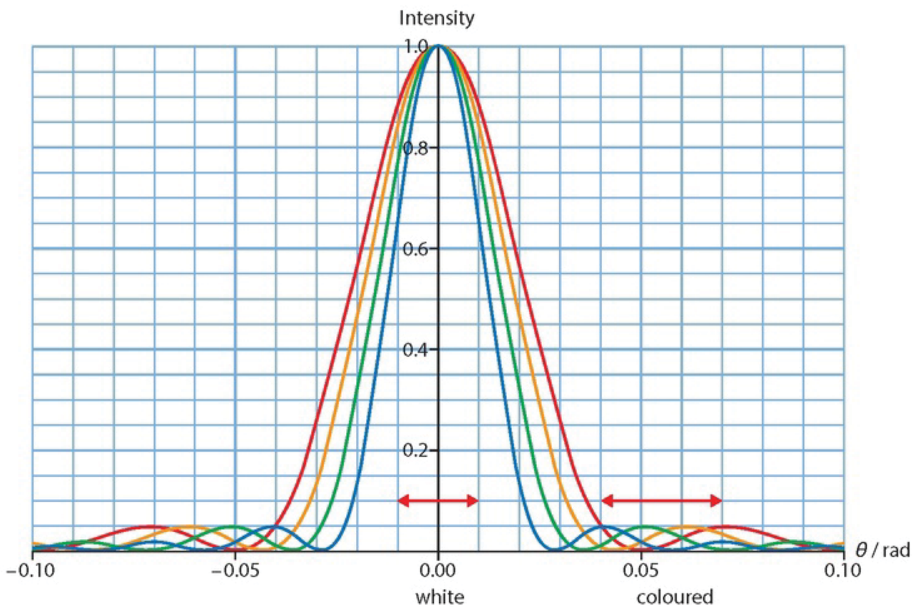

Figure 9.6

The intensity of the first secondary maximum is about 4.5% of that of the central maximum.

Notice that as the wavelength increases the width of the diffraction pattern increases. Thus if *white* light is incident on the slit, each constituent colour will have its own characteristic pattern, and their combination will be a pattern which is white at the centre but coloured to the sides.

Circular apertures: A detailed analysis of the diffraction pattern for a circular aperture of diameter *b* shows that the first diffraction minimum is observed at a diffraction angle of $\theta_{\mathrm{D}} \approx 1.22\frac{\lambda}{b}$.

TEST YOURSELF 9.3

The wavelength of blue light in the graph of Figure 9.6 is 4.80×10^{-7}m. Estimate the width of the slit, assuming it is rectangular.

Model Answer 9.1

Describe the changes to the single-slit diffraction pattern for a circular aperture when its diameter is reduced.

Reducing the diameter makes the diffraction pattern wider, since $\theta_{\mathrm{D}} \approx 1.22\frac{\lambda}{b}$. At the same time, less light gets through the aperture and so the intensity of the pattern will be reduced.

Resolution

DEFINITION

RESOLUTION A measure of the ability of a detection device such as a telescope, microscope or your eye to distinguish two objects – that is, to *see* them as separate objects.

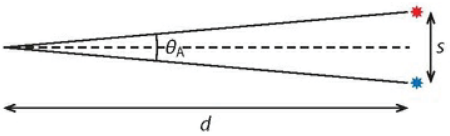

Figure 9.7

Light from two point sources will diffract when it passes through an aperture. If the two sources are separated by a distance *s* which is small compared with the distance *d* to the aperture, their angular separation is $\theta_A \approx \frac{s}{d}$ (Figure 9.7).

If this separation is small enough, the diffraction patterns of the two sources will overlap and the two sources may appear as one (Figure 9.8).

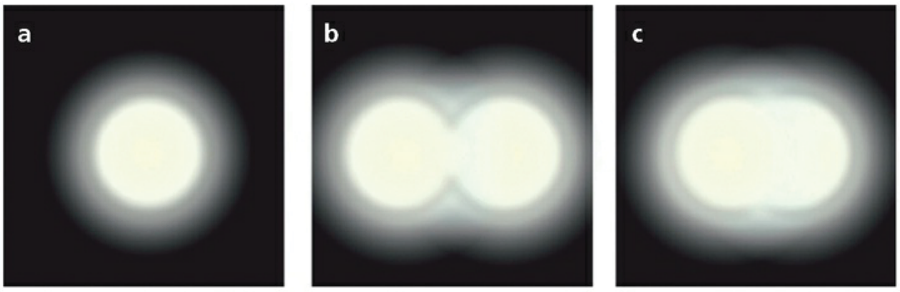

Figure 9.8 Two images that are **a** not resolved, **b** clearly resolved and **c** barely resolved.

DEFINITION

RAYLEIGH CRITERION Two sources are said to be *just resolved* if the central maximum of the diffraction pattern of one source falls on the first minimum of the other.

This means that the angular separation θ_{A} of the sources must satisfy $\theta_{\mathrm{A}} \approx \theta_{\mathrm{D}} = 1.22\frac{\lambda}{b}$, where *b* is the diameter of the circular aperture used to collect the light from the sources. In other words, to know whether two sources are resolved we compare the angular separation $\theta_{\mathrm{A}}$and the diffraction angle $\theta_{\mathrm{D}} = 1.22\frac{\lambda}{b}$.

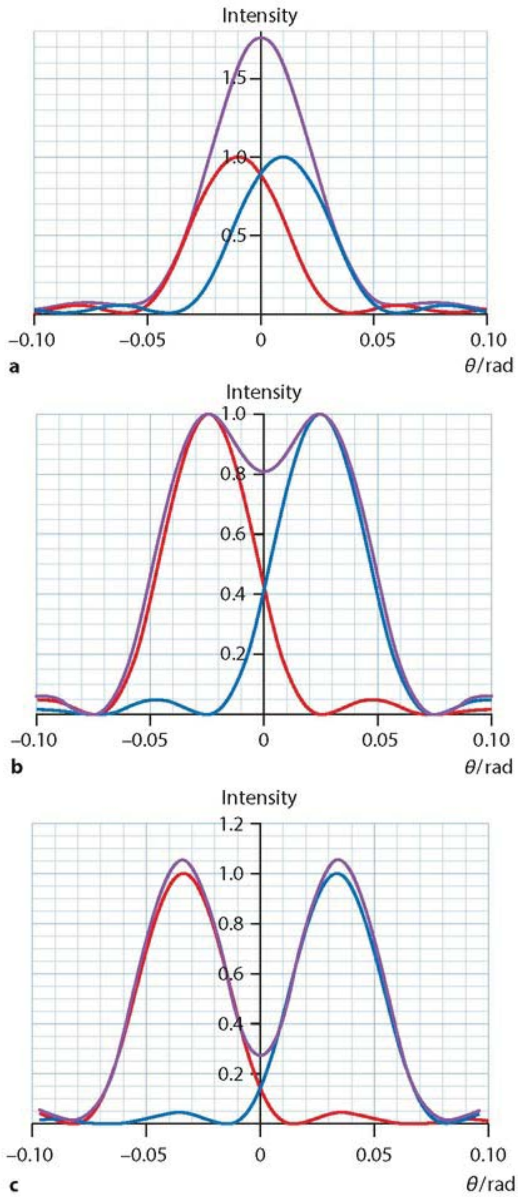

Figure 9.9 The combined diffraction patterns of two sources that are **a** not resolved, **b** barely resolved and **c** clearly resolved.

Sources not resolved: $\theta_A < \theta_D \approx 1.22\frac{\lambda}{b}$ (Figure 9.9a).

Sources just resolved: the central maximum of one coincides with the first minimum of the other; $\theta_A \approx \theta_D \approx 1.22\frac{\lambda}{b}$ (Figure 9.9b)

Sources well resolved: $\theta_A > \theta_D \approx 1.22\frac{\lambda}{b}$ (Figure 9.9c)

TEST YOURSELF 9.4

Mars has a diameter of 7×10^6 m and is 7×10^{10} m from Earth. It reflects light of wavelength 650 nm. Does the naked human eye with pupil diameter 3 mm see the planet as a point source of light or as an extended disc?

hint

To see the planet as a disc you should be able to resolve two points a diameter apart.

Annotated Exemplar Answer 9.1

A passenger on a ship some distance from the shore uses a telescope to observe coloured lights along the seafront. The passenger finds she can resolve blue light sources that are closer together than red sources. Explain this observation. **[3]**

Diffraction angle θ is given by $\theta = \frac{1.22\lambda}{b}$, so the blue light gives a smaller diffraction angle.

Why does the blue light give a smaller diffraction angle? You need to show you know the relative sizes of the wavelengths of the two colours of light.

The explanation is unfinished. What is the significance of the smaller diffraction angle? According to the Rayleigh criterion, it is just possible to resolve two sources if the angular separation is equal to the diffraction angle. A smaller wavelength gives a smaller diffraction angle, so with blue light a smaller angular separation is resolvable. This means the blue lights can be closer together and still be resolved.

1/3

< Back to chapter

Annotated Exemplar Answer 9.1

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1/3

9.3 Interference

Figure 9.10 shows plane wavefronts approaching two rectangular slits.

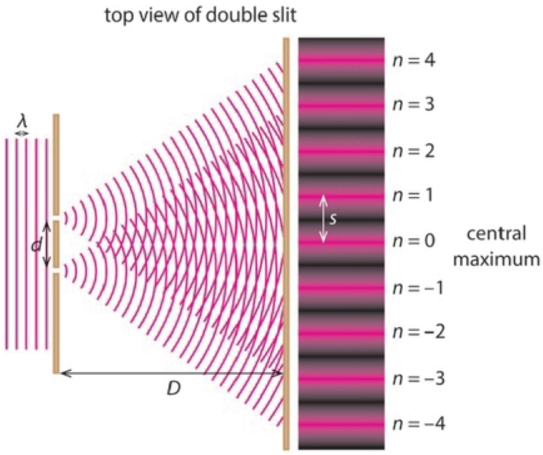


Figure 9.10

Diffraction takes place at each slit, so waves spread out from each slit. Far from the slits, rays from one slit meet rays from the other and interfere. In this way regions of maximum and minimum intensity of light are created. We will get maximum intensity (constructive interference) at those points where the path difference is an integral multiple of the wavelength. We will get points of minimum intensity (destructive interference) when the path difference is a half-integral multiple of the wavelength. Figure 9.11 shows that the path difference is $d \sin \theta$, where d is the separation of the slits.

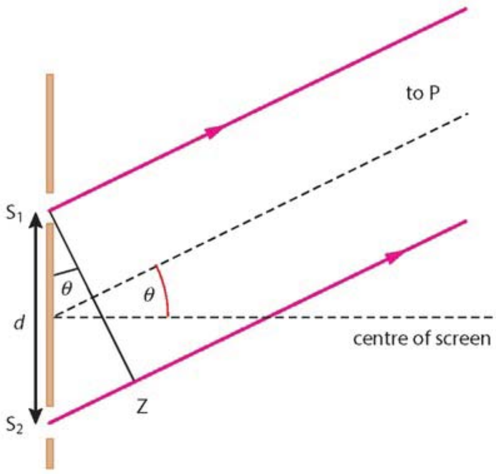


Figure 9.11

So we have the conditions:

$$d \sin \theta = \begin{cases} n\lambda & \text{constructive} \\ \left(n + \frac{1}{2}\right)\lambda & \text{destructive} \end{cases}$$

Figure 9.12 shows how the intensity of the light on the screen varies with the angle θ if the slit width is negligible. Here I_0 is the intensity of light at the centre of the screen due to one slit alone.

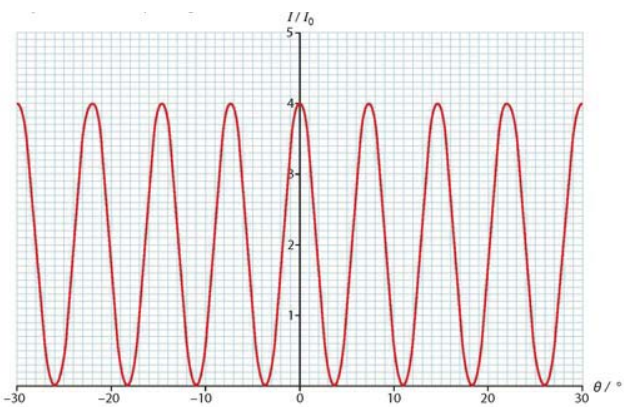


Figure 9.12

Why is the maximum intensity $4I_0$? The amplitude of the wave from one slit is A_0, so its intensity is $I_0 = kA_0^2$ since intensity is proportional to the square of the amplitude. For constructive interference the combined amplitude is $A_0 + A_0 = 2A_0$. The combined intensity is therefore $I = k(2A_0)^2 =$

$4kA_0^2 = 4I_0$. In general the maximum intensity increases by a factor of n^2, where n is the number of slits.

Effect of non-zero slit width

In addition to interference between rays from each slit, we also have interference between rays *within* each slit. The effect of this is to modify the intensity pattern to the one shown in red in Figure 9.13.

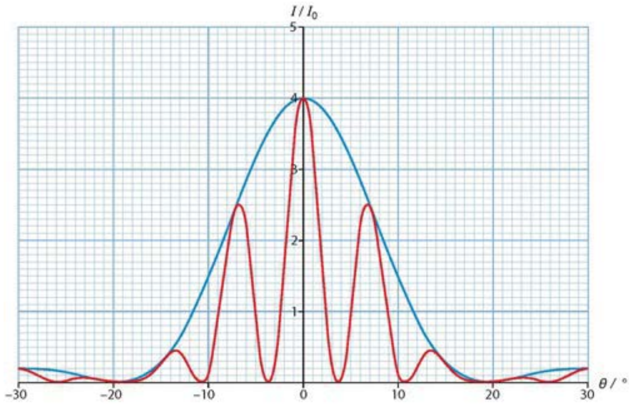


Figure 9.13

The blue curve is the diffraction pattern for a single slit (compare this with Figure 9.6), which has effectively been imposed on top of the two-slit pattern of Figure 9.12. The maxima are no longer equal in intensity but they remain equally separated and at the same positions.

Increasing the number of slits

Figure 9.14 shows what happens when the number of slits increases to four.

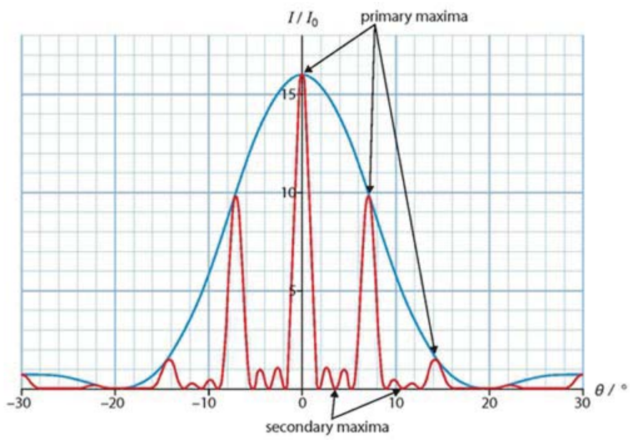


Figure 9.14

There are two secondary maxima in between two primary maxima. The primary maxima have narrowed and the central maximum has intensity 16 I_0.

You should be able to describe what happens to the intensity pattern as the number of slits increases further.

Model Answer 9.2

Figure 9.15 shows the intensity pattern when light is incident on a number of parallel rectangular slits. I_0 is the intensity of light at the centre of the screen due to a single slit.

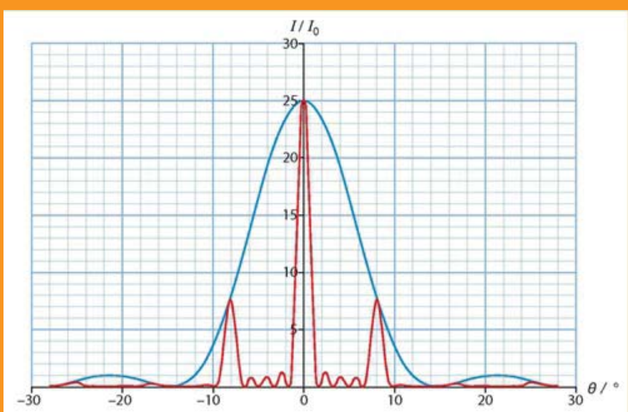


Figure 9.15

Use the graph to determine a the number of slits, and b **the slit width *d* and slit separation *b***

in terms of the wavelength.

hint

Be careful with degrees and radians.

a The intensity of the central maximum is $25I_0$ implying five slits.

b The $n = 1$ primary maximum is observed at $\theta = 8.0°$ and so from $d \sin\theta = n\lambda$ we find $d \sin 8.0° = 1 \times \lambda$, giving $d = 7.2\lambda$.

The first minimum of the single-slit diffraction pattern is observed at $\theta = 14.5°$. To use $\theta = \frac{\lambda}{b}$ we must express the angle in radians. Hence $b = \frac{\lambda}{\theta} = \frac{\lambda}{14.5 \times \frac{\pi}{180}} \approx 4\lambda$.

The diffraction grating

The pattern illustrated in Figure 9.15 continues as the number of slits increases further. As the number of slits increases, the secondary maxima shrink in size and the primary maxima grow very intense and very narrow.

If we further assume that the width of each slit grows negligibly small, the arrangement becomes that of a **diffraction grating**, with very many narrow slits. This produces well-separated, very narrow, very bright maxima of essentially equal intensity (Figure 9.16).

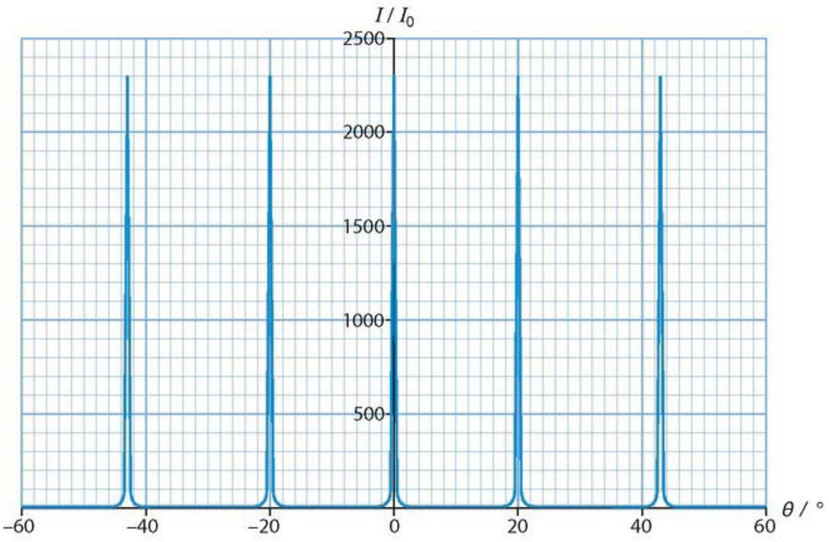


Figure 9.16

The position of the maxima is found from $d \sin\theta = n\lambda$. This formula gives the primary maxima for any number of slits.

TEST YOURSELF 9.5

The graph in Figure 9.16 shows the intensity pattern for a diffraction grating with 700 lines per mm. Use the graph to calculate the wavelength of the light used here.

hint

How do you find the slit separation from the number of lines per mm?

Resolvance of a diffraction grating

One use of a diffraction grating is to measure the wavelength of light from its diffraction pattern, using $d \sin\theta = n\lambda$.

It is often the case that a spectrum under investigation (the spectrum of a star, for example) contains lines whose wavelengths are very similar. How good is a particular diffraction grating at distinguishing lines whose wavelengths are very close to one another?

The **resolvance** or **resolving power** of a diffraction grating – its ability to distinguish between a particular pair of wavelengths – is defined as $R = \frac{\bar{\lambda}}{\Delta\lambda}$, where $\bar{\lambda}$ is the average of the two wavelengths and $\Delta\lambda$ is their difference. It can be shown that $R = mN$, where m is the order of the intensity maximum at which the observation is made and N is the total number of slits in the diffraction grating. So the smallest difference in wavelength that can be resolved with a particular diffraction grating is $\Delta\lambda = \frac{\bar{\lambda}}{mN}$.

Model Answer 9.3

The spectrum of mercury has two lines with wavelengths 579.0 nm and 577.0 nm. Calculate the smallest number of slits needed to resolve these lines in the second order.

From $\Delta\lambda = \frac{\bar{\lambda}}{mN}$ we find $N = \frac{\bar{\lambda}}{m\Delta\lambda} = \frac{578.0}{2\times 2.00} = 145$.

TEST YOURSELF 9.6

The spectrum of sodium contains two lines with wavelengths 588.9950 nm and 589.5924 nm. Determine if these lines can be resolved in the second order, using a diffraction grating with 1000 lines.

Thin-film interference

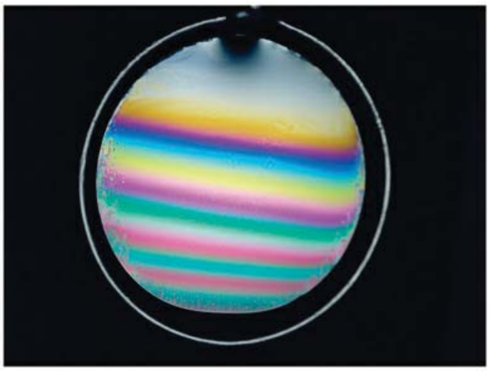


Figure 9.17

A thin, transparent film viewed from above will appear coloured because of interference of rays reflected from its upper and lower boundaries. Figure 9.17 shows the interference pattern in a film of soapy water.

Figure 9.18 illustrates this film as seen from the side.

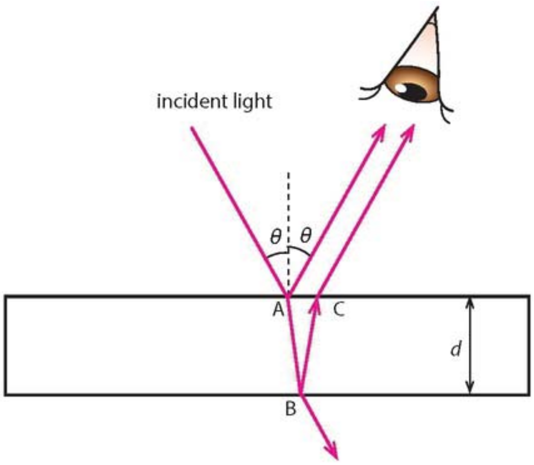


Figure 9.18

The ray reflected at point A will interfere with the ray reflected at point B. The path difference is approximately 2*d*, where *d* is the thickness of the film (assuming we are looking directly from above). We now apply the conditions of constructive and destructive interference.

There is a complication, however, because the rays may undergo a phase change upon reflection. In fact, there is no phase change when a ray is reflected at a boundary into a medium of *lower* refractive index, but a phase change of π (half a cycle) when it is reflected at a boundary into a medium of *higher* refractive index. (Compare this with the reflection of pulses off fixed and free ends in Section 4.3.)

Hence the conditions for interference are as follows (λ is the wavelength of the light in air, $\frac{\lambda}{n}$ is the wavelength of the light in soapy water, n is the refractive index of the soapy water film and m is an integer):

No phase change, or phase change at both boundaries:

$$2d = \begin{cases} m\frac{\lambda}{n} & \text{constructive} \\ \left(m + \frac{1}{2}\right)\frac{\lambda}{n} & \text{destructive} \end{cases} \text{ which can be rewritten as } 2dn = \begin{cases} m\lambda & \text{constructive} \\ \left(m + \frac{1}{2}\right)\lambda & \text{destructive} \end{cases}$$

Phase change at only one boundary:

$$2d = \begin{cases} m\frac{\lambda}{n} & \text{destructive} \\ \left(m+\frac{1}{2}\right)\frac{\lambda}{n} & \text{constructive} \end{cases} \text{ which can be rewritten as } 2dn = \begin{cases} m\lambda & \text{destructive} \\ \left(m+\frac{1}{2}\right)\lambda & \text{constructive} \end{cases}$$

The refractive index of air is about $n = 1$, while that of soapy water is greater than 1. In Figure 9.18, with air above and below the film of soapy water, the wave reflected at A will therefore change phase, while the one reflected at B will not.

If the film is illuminated with white light, at each point some wavelengths (that is, colours) interfere constructively and others destructively. Because the thickness d varies across the film, it will display the swirl of colours shown in Figure 9.17.

Worked Example 9.3

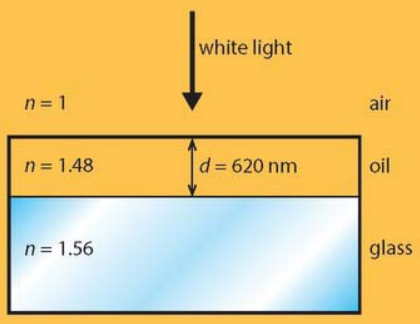


Figure 9.19

A sheet of glass of refractive index 1.56 is coated with a layer of oil of refractive index 1.48 (Figure 9.19). The thickness of the oil is 620 nm.

Determine the visible wavelengths at which a reflected ray of white light interferes constructively and those at which it interferes destructively. Hence determine the colour of the reflected light as seen from above. Neglect any reflection from the glass–air interface at the bottom.

Here we have a phase shift at both reflecting boundaries. Therefore the condition for constructive interference of waves from each boundary is $2dn = m\lambda$. Solving for λ and putting in numbers, we find $\lambda = \frac{2dn}{m} = \frac{2\times 620\text{nm}\times 1.48}{m} = \frac{1840\text{nm}}{m}(3\text{s.f.})$. Visible wavelengths vary from 400 nm to 700 nm, and $m = 3$ gives a wavelength in this range, $\lambda = 612$ nm, a yellow wavelength. The condition for destructive interference in this situation is $2dn = \left(m+\frac{1}{2}\right)\lambda$, or $\lambda = \frac{2dn}{m+\frac{1}{2}} = \frac{1840\text{nm}}{m+\frac{1}{2}}$. This gives visible wavelengths for both $m = 3$ and $m = 4$: $\lambda = 526$nm (green) and $\lambda = 409$nm (blue), and these will therefore be absent. The reflected light will be yellowish orange since yellow is dominant and blue and green are absent.

TEST YOURSELF 9.7

Calculate the minimum thickness of a soap film ($n = 1.33$), suspended in air, that will show constructive interference of light of wavelength 520 nm.

9.4 The Doppler effect

DEFINITION

DOPPLER EFFECT The change in the observed frequency of a wave whenever there is *relative* motion between the emitter and the receiver (Figure 9.20).

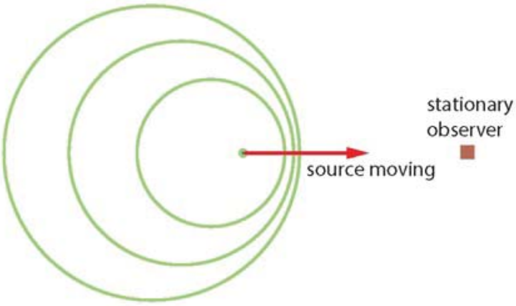


Figure 9.20

Note that the effect is described here in terms of *observed frequency*. For example:

- The horn of an approaching car is heard at a higher frequency than what is actually emitted.
- Light from a galaxy moving away from Earth is observed to have a redder colour (a lower frequency) than what is actually emitted.

The Doppler effect has many applications. Two common applications are: determining the speed of a car on a highway and of blood cells in an artery.

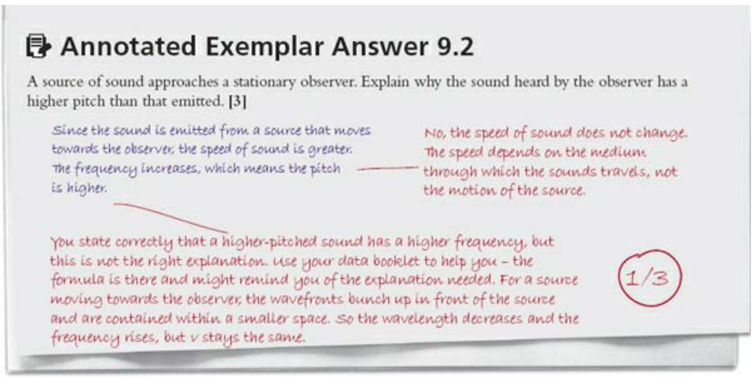
General equations for the frequency shift are as follows (f_o = observed frequency, f_s = source frequency, v_o = speed of observer, v_s = speed of source, v = wave speed):

Source moving towards observer:
$$f_o = f_s \frac{v}{v - v_s}$$

Observer moving towards source:
$$f_o = f_s \frac{v}{v + v_s}$$

Source moving away from observer:
$$f_o = f_s \frac{v + v_o}{v}$$

Observer moving away from source:
$$f_o = f_s \frac{v - v_o}{v}$$



Annotated Exemplar Answer 9.2

A source of sound approaches a stationary observer. Explain why the sound heard by the observer has a higher pitch than that emitted. **[3]**

Since the sound is emitted from a source that moves towards the observer, the speed of sound is greater. The frequency increases, which means the pitch is higher.

No, the speed of sound does not change. The speed depends on the medium through which the sounds travels, not the motion of the source.

You state correctly that a higher-pitched sound has a higher frequency, but this is not the right explanation. Use your data booklet to help you – the formula is there and might remind you of the explanation needed. For a source moving towards the observer, the wavefronts bunch up in front of the source and are contained within a smaller space. So the wavelength decreases and the frequency rises, but v stays the same.

1/3

TEST YOURSELF 9.8

A sound wave of frequency 300 Hz is emitted towards an approaching car. The wave is reflected from the car and is then received back at the emitter at a frequency of 315 Hz. What is the speed of the car? (Take the speed of sound to be 340 m s^{-1}.)

hint

This is a rather complex, double-Doppler-effect problem.

TEST YOURSELF 9.9

A child on a carousel is sounding a whistle. The frequency measured by a stationary observer far away from the carousel varies between 510 Hz and 504 Hz. Take the speed of sound to be 340 m s^{-1}. Use this information to calculate the speed of rotation of the carousel.

hint

Why do you think the frequency varies?

The Doppler effect for light

DEFINITIONS

RED-SHIFT A shift of received light to a longer wavelength (lower frequency) than that emitted.

BLUE-SHIFT A shift of received light to shorter wavelength (higher frequency) than that emitted.

The analysis of Doppler shift for light is more complicated than for sound, because Einstein's theory of special relativity applies. However, if the speed of the source is small compared to the speed of light, we have the approximate formula $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$, where $\Delta\lambda = \lambda' - \lambda$ is the shift in wavelength, λ is the wavelength *emitted* at the source, λ' is the observed wavelength, v is the speed of the source and c is the speed of light.

Since $\lambda = \frac{c}{f}$, we have $\frac{\Delta\lambda}{\lambda} = \frac{\lambda'-\lambda}{\lambda} = \frac{\frac{c}{f'} - \frac{c}{f}}{\frac{c}{f}} = \frac{f-f'}{f'} = \frac{\Delta f}{f'}$, where f and f' are the emitted and observed frequencies, respectively. The minus sign means that when $\lambda' > \lambda$ then $f' < f$.

Your IBO data booklet has f in the denominator rather than f'. Because of the extremely small size of Δf, it makes little difference whether f or f' is used in the denominator. Using f, and considering only magnitudes, we then have $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$.

(In fact this formula can also be used for sound if the speed of the source is much smaller than the speed of sound.)

TEST YOURSELF 9.10

Hydrogen atoms in a distant galaxy emit light of wavelength 658 nm. The light received on Earth is measured to have a wavelength of 720 nm. Calculate the speed of the galaxy and state whether the galaxy is approaching the Earth or moving away.

hint

Do we have a blue-shift or a red-shift?

Nature of Science. The Doppler effect was first proposed to explain changes in the wavelength of light from binary stars moving in relation to each other. The effect also explains the change in pitch that occurs when a fast moving source of sound passes by. Applying the theory to different types of wave in different areas of physics has led to Doppler imaging in medicine using ultrasound, hand-held radar guns to check for speeding vehicles, and improvements in weather forecasting using the Doppler shift in radio waves reflected from moving cloud systems.

The expansion of the Universe was discovered by Edwin Hubble (1889–1953) through applying the red-shift formula that is based on the Doppler effect to light from distant galaxies. Observers on Earth who measure the light emitted by galaxies find that the wavelength is longer than that emitted. The galaxies must be moving away. Yet the modern view is that space in-between galaxies is itself being stretched.

This stretching makes all distances, including wavelengths, get larger. So the reason we observe red-shift is not the Doppler effect, but a much more complicated phenomenon predicted by Einstein's general theory of relativity. The Doppler effect remains as a simple, intuitive yet wrong description of what is actually going on, which gives the right answer for galaxies that are not too far away.

Checklist

After studying this chapter you should be able to:

- □ **describe simple harmonic motion using the equations as well as graphs**
- □ **understand the phenomenon of diffraction and draw the intensity pattern for single-slit diffraction**
- □ **solve problems with two slit interference including drawing the intensity graphs for zero and non-zero slit width**
- □ **solve problems with resolution and the Rayleigh criterion**
- □ **describe the changes in the intensity pattern when the number of slits increases**
- □ **solve problems with diffraction grating resolvance**
- □ **solve problems with thin-film interference paying attention to phase changes**
- □ **describe the Doppler effect with wavefront diagrams and equations**

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Annotated Exemplar Answer 9.2

A source of sound approaches a stationary observer. Explain why the sound heard by the observer has a higher pitch than that emitted. **[3]**

since the sound is emitted from a source that moves towards the observer the speed of sound is greater. The frequency increases, which means the pitch is higher.

No, the speed of sound does not change. The speed depends on the medium through which the sounds travels, not the motion of the source.

You state correctly that a higher-pitched sound has a higher frequency, but this is not the right explanation. use your data booklet to help you – the formula is there and might remind you of the explanation needed. For a source moving towards the observer, the wavefronts bunch up in front of the source and are contained within a smaller space. so the wavelength decreases and the frequency rises, but v stays the same.

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10 FIELDS (HL)

This chapter covers the following topics:

- □ Gravitational and electrical potential
- □ Gravitational and electrical potential energy
- □ Equipotental surfaces
- □ The relation between potential and field
- □ Orbital and escape speeds

10.1 Gravitational and electrical potential

DEFINITIONS

GRAVITATIONAL POTENTIAL AT A POINT The work done (by you) per unit mass to move a point mass m from infinity to that point at a small constant speed: $V = \frac{W}{m}$. The unit of gravitational potential is J kg^{-1}.

ELECTRICAL POTENTIAL AT A POINT The work done (by you) per unit charge to move a positive point charge q from infinity to that point at a small constant speed: $V = \frac{W}{q}$. The unit of electrical potential is the volt (V) and equals JC^{-1}.

EQUIPOTENTIAL SURFACE A surface on which the potential is the same everywhere.

GRAVITATIONAL POTENTIAL ENERGY AT A POINT The work done (by you) to move a point mass m from infinity to that point in a gravitational field.

ELECTRICAL POTENTIAL ENERGY AT A POINT The work done (by you) to move a positive point charge q from infinity to that point in an electric field.

Gravitational potential is a scalar quantity. The work done in moving a mass in a gravitational field is independent of the path followed. The gravitational potential at a distance r from a particle of mass M is $V = -\frac{GM}{r}$.

Electrical potential is a scalar quantity. The work done in moving a charge in an electric field is independent of the path followed. The electrical potential at a distance r from a particle of charge Q is $V = \frac{kQ}{r}$.

Unlike gravitation, where the sign is negative, the sign here is positive, but the charge Q must include its sign. Thus the electrical potential at a distance of 2.0×10^{-10} m from an *electron* is $V = \frac{9\times10^{9}\times(-1.6\times10^{-19})}{2.0\times10^{-10}} = -7.2\,\text{V}$.

Because potential is a scalar quantity, to find the gravitational (or electrical) potential from more than one mass (or charge), we find the potential from each mass (or charge) separately and add them.

Gravitational work: To move a mass m from one point to another in a gravitational field requires that work be done. The work done on the mass by you is $W = m\Delta V$, where $\Delta V = V_{\text{final}} - V_{\text{initial}}$ is the change in gravitational potential from the initial to the final position. (The work done *by the gravitational force* is $W = -m\Delta V$.)

Electrical work: To move a charge q from one point to another in an electric field requires that work be done. The work done on the charge by you is $W = q\Delta V$, where $\Delta V = V_{\text{final}} - V_{\text{initial}}$ is the change in electrical potential from the initial to the final position. (The work done *by the electric force* is $W = -q\Delta V$.)

Worked Example 10.1

Figure 10.1 shows the variation of the electrical potential with distance from the centre of a charged sphere of radius 2.0 m.

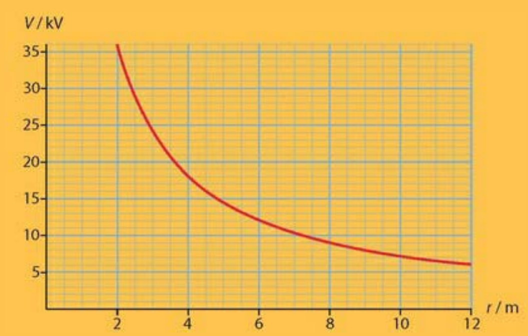


Figure 10.1

a Determine the charge on the sphere.

b Calculate the work that must be done on a point negative charge of 2.5 nC in order to move it from a distance of 4.0 m to a distance of 12 m from the centre of the sphere.

a The potential at the surface of the sphere is 36 kV and so from $V = \frac{kQ}{r}$ we find $3.6 \times 10^{4} = \frac{8.99\times10^{9}\times Q}{2.0} \Rightarrow Q = 8.0\text{mC}$. The potential is positive so the charge is positive.

b The work done to move the negative point charge is $W = q\Delta V = (-2.5 \times 10^{-9}) \times (6.0 \times 10^{3} - 18 \times 10^{3}) = +3.0 \times 10^{-5}$ J.

TEST YOURSELF 10.1

Figure 10.2 shows two unequal spherical masses. At which point is the gravitational potential largest?

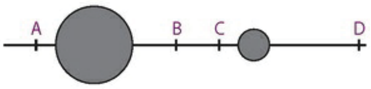


Figure 10.2

TEST YOURSELF 10.2

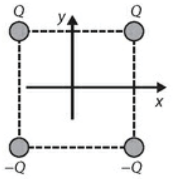


Figure 10.3

Four charges of equal magnitude are placed at the vertices of a square, as shown in Figure 10.3. The origin of the axes is at the centre of the square. The electrical potential is zero:

a at the origin only.

b along the *x*-axis only.

c along the *y*-axis only.

d along both the *x*- and *y*-axes.

TEST YOURSELF 10.3

The centres of two equal spherical masses *M* are a horizontal distance 2*d* apart. A small mass *m* is placed at the midpoint of the line joining the two masses.

a What was the work done to bring *m* from infinity to this position?

b The mass *m* is now moved a distance $\frac{d}{2}$ closer to the right-hand mass. How much work must be done to do this?

The relation between field strength and potential

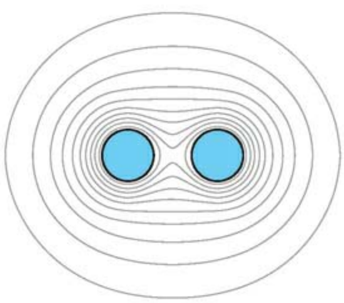
The field strength (gravitational or electrical) is the negative gradient of the potential:

$$g = -\frac{dV}{dr},\ E = -\frac{dV}{dr}$$

This means that in a graph of potential versus distance the gradient is equal to the negative of the field strength. For example, for a positive point charge *Q* the potential is $V = \frac{kQ}{r}$ and so $E = -\frac{d}{dr}\left(\frac{kQ}{r}\right) = \frac{kQ}{r^2}$, as expected.

This also means that field lines are always at right angles to equipotential surfaces. Figure 10.4 shows equipotential surfaces (panel a) and field lines (panel b) for two equal masses or equal negative charges (for positive charges the field lines would have the arrows reversed).

a



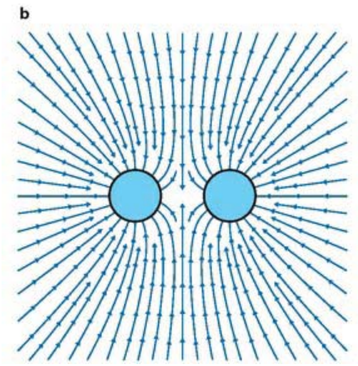


Figure 10.4

Model Answer 10.1

Figure 10.5 shows equipotential lines for two electric charges. The charge on the left is negative.

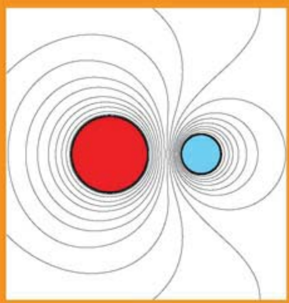


Figure 10.5

a State and explain the sign of the other charge.
b Draw the field lines for this arrangement of charges.
c Suggest whether this diagram could represent equipotential surfaces for a pair of masses.

a The right charge is positive. In between the charges the field is nowhere zero, which it would be if the charges had the same sign.
b We must draw lines normal to the equipotential lines, with arrows leaving the positive charge on the right and ending at the negative charge on the left; see Figure 10.6.

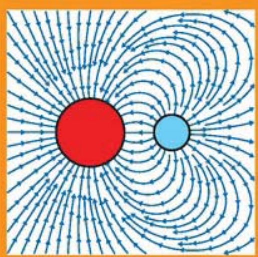


Figure 10.6

c This could never represent masses because there is no equipotential surrounding both sources as in Figure 10.4a.

TEST YOURSELF 10.4

Consider the electric field illustrated in Figure 10.7.

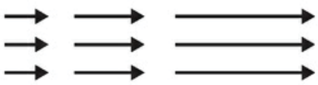
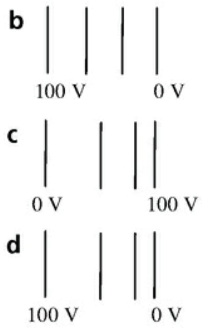


Figure 10.7

Which set of equipotential surfaces gives rise to this electric field pattern?



Model Answer 10.2

Figure 10.8 shows the equipotential lines around a pair of objects.

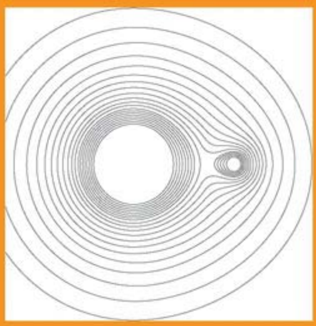


Figure 10.8

Consider the following possible sources for these lines.

I Unequal charges of equal sign
II Unequal charges of opposite sign
III Unequal masses

Which is correct?

a I or II
b I or III
c II or III
d I, II or III

Since there are equipotential lines surrounding both sources the sources can be either masses or charges of equal sign, hence **b**.

10.2 Gravitational and electrical potential energy Gravitational potential energy

The gravitational potential energy of a mass m placed at point where the gravitational potential is V is $E_p = mV$. If the field is caused by a mass M then, since $V = -\frac{GM}{r}$, we have that $E_p = -\frac{GMm}{r}$ (Figure 10.9).

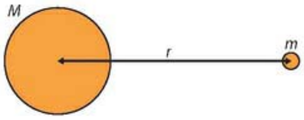


Figure 10.9

Electrical potential energy

The electrical potential energy of a charge q placed at point where the electrical potential is V is $E_p = qV$.

If the field is caused by a charge Q then since $V = \frac{kQ}{r}$ we have that $E_p = \frac{kQq}{r}$ (Figure 10.10).

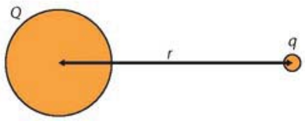


Figure 10.10

Unlike for gravitation, where the sign is negative, the sign here is positive, but the charges Q and q must include their signs.

TEST YOURSELF 10.5

Calculate the electrical potential energy of an electron in orbit around a proton when the orbital radius is 2.0×10^{-10} m.

TEST YOURSELF 10.6

A proton is kept on the surface of a positively charged metallic sphere. The radius of the sphere is 0.50 m and the potential at its surface is 180 V. The proton is now released.

Calculate the speed of the proton **a** as it passes a point 1.0 m from the centre of the sphere, **b** after it has moved an infinite distance away from the sphere.

c Sketch a graph to show the speed of the proton as a function of distance from the centre of the sphere.

TEST YOURSELF 10.7

Calculate the gravitational potential energy of a satellite of mass 3500 kg in orbit around the Earth at a height of 520 km above the surface of the Earth. (The Earth's radius is $R = 6.4 \times 10^6$ m and its mass is $M = 6.0 \times 10^{24}$ kg.)

Worked Example 10.2

Figure 10.11 shows the gravitational potential due to two stars whose centres are a distance $d = 1.5 \times 10^{11}$ m apart.

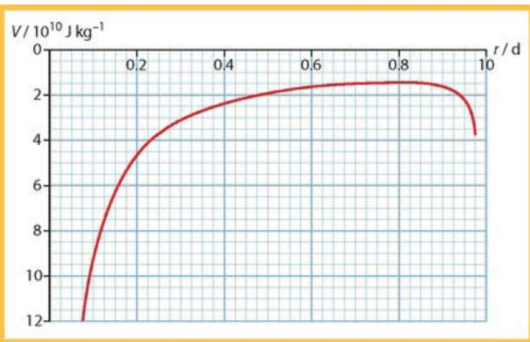


Figure 10.11

a State the significance of the point at $r = 0.80\ d$.

b Calculate the ratio of the masses of the two stars.

c Estimate the combined gravitational field strength of the two stars at the point with $r = 0.20\ d$.

a The gradient of the curve at $r = 0.80\ d$ is zero, indicating that the gravitational field strength is zero there.

b Since $g = 0$ at $r = 0.80\ d$ we have that

$$\frac{GM_1}{(0.80d)^2} = \frac{GM_2}{(d - 0.80d)^2}$$

$$\frac{M_1}{M_2} = \frac{0.80^2}{0.20^2} = 16$$

c We need to draw the tangent to the curve at $r = 0.20\ d$ (Figure 10.12):

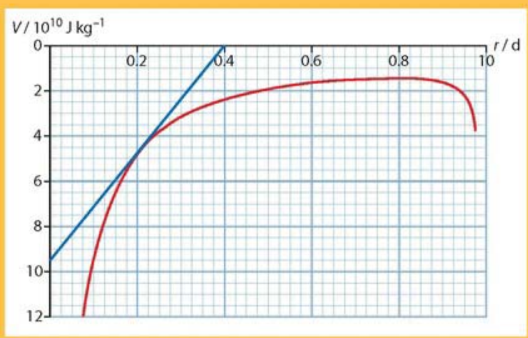


Figure 10.12

The tangent line has a gradient equal to:

$$g = \frac{9.5 \times 10^{10} - 0}{0.40 \times 1.5 \times 10^{11} - 0} \approx 1.6\text{Nkg}^{-1}.$$

10.3 Motion in a gravitational field

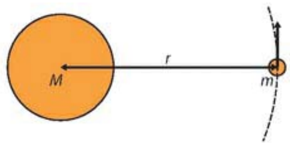


Figure 10.13

A point mass *m* orbits a spherical mass *M* (Figure 10.13).

The orbital radius is *r* and the orbital speed is *v*. The gravitational force provides the centripetal force on *m* and so $\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. The formula says that the closer *m* is to *M* the faster it moves.

(You should be able to quickly derive this formula in an exam.)

For circular motion in general, we have that $v = \frac{2\pi r}{T}$, where *T* is the period, the time for one full revolution.

Kepler's third law relates the period to the orbital radius. Taking *M* to be the mass of the Sun and *m* the mass of a planet we deduce, by combining the last two formulas, that $\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \Rightarrow T^2 = \frac{4\pi^2}{Gm}r^3$. (You should be able to quickly derive this formula in an exam.)

TEST YOURSELF 10.8

The average distance between the Sun and the Earth is 1.5×10^{11} m. Estimate the mass of the Sun.

hint

How long does it take the Earth to go around the Sun?

Orbital motion: a satellite orbits a spherical mass M. The kinetic energy of the orbiting satellite is $E_K = \frac{1}{2}mv^2$, and since $v^2 = \frac{GM}{r}$, we have that $E_k = \frac{1}{2}\frac{GMm}{r}$. The total energy is therefore

$$E_T = E_k + E_p = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}.$$

The total energy is negative (Figure 10.14). The satellite is in a **potential well**, so energy must be supplied if it is to move away.

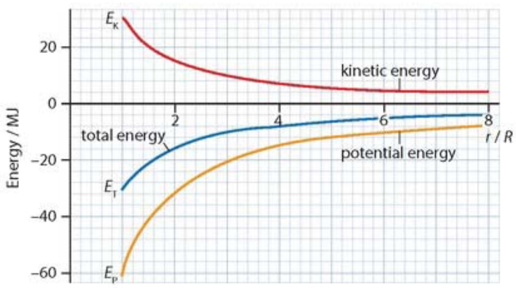
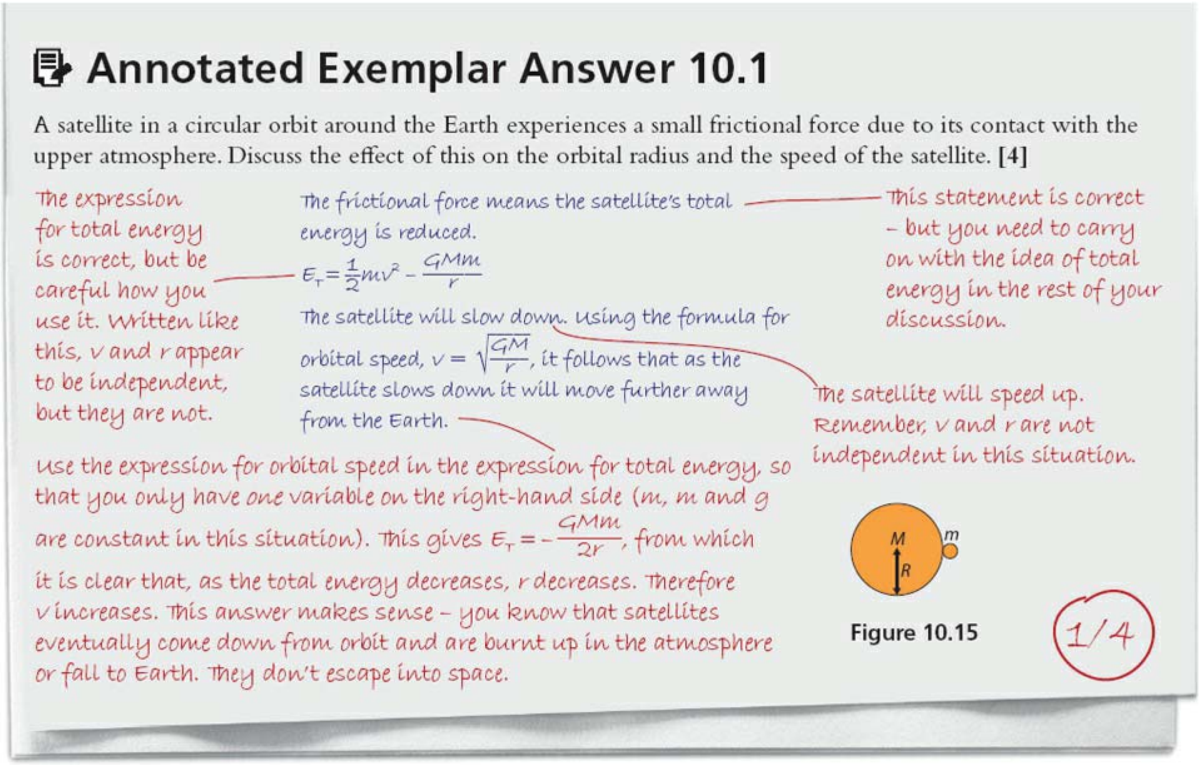


Figure 10.14

Nature of Science. The Sun influences the motion of the Earth a distance of 1.5×10^{n}m away. How is the 'influence' of the Sun 'transmitted' to the position of the Earth? If the Sun were to suddenly disappear, how long would it take the Earth to leave its orbit and plunge into darkness? To answer such questions required a huge shift in scientific thinking, known as a paradigm shift. Fields were introduced, which replaced the idea of direct, observable action with a mechanism for 'action at a distance'. Fields were further refined with the concept of waves. If a charge were to suddenly appear, an electric field would be established in space at the speed of light. The 'information' about the existence of the charge is carried by electromagnetic waves travelling through vacuum at the speed of light. It is similarly believed that gravitational waves carry the 'information' about the existence of mass. But unlike electromagnetic waves, gravity waves have not yet been observed, even though no one doubts their existence.



Annotated Exemplar Answer 10.1

A satellite in a circular orbit around the Earth experiences a small frictional force due to its contact with the upper atmosphere. Discuss the effect of this on the orbital radius and the speed of the satellite. **[4]**

The frictional force means the satellite's total energy is reduced.

$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r}$

The satellite will slow down. Using the formula for orbital speed, $v = \sqrt{\frac{GM}{r}}$, it follows that as the satellite slows down it will move further away from the Earth.

This statement is correct – but you need to carry on with the idea of total energy in the rest of your discussion.

The expression for total energy is correct, but be careful how you use it. Written like this, v and r appear to be independent, but they are not.

The satellite will speed up. Remember, v and r are not independent in this situation.

Use the expression for orbital speed in the expression for total energy, so that you only have one variable on the right-hand side (m, M and G are constant in this situation). This gives $E_T = -\frac{GMm}{2r}$, from which it is clear that, as the total energy decreases, r decreases. Therefore v increases. This answer makes sense – you know that satellites eventually come down from orbit and are burnt up in the atmosphere or fall to Earth. They don't escape into space.

Figure 10.15

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DEFINITION

ESCAPE SPEED The minimum launch speed of a projectile at the surface of a planet (Figure 10.15) so that the projectile can move far away (to infinity).

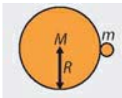

Figure 10.15

The total energy at launch at the surface of the planet is $E_T = E_K + E_P = \frac{1}{2}mv^2 - \frac{GMm}{R}$, where R is the radius of the Earth. To just escape, the total energy at infinity will be *zero*, so, by conservation of energy, $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 \Rightarrow v^2 = \frac{2GM}{R}$.

TEST YOURSELF 10.9

A probe is launched from the surface of the Earth with a speed that is half the escape speed. How far away from the Earth does the probe get? (Give your answer in terms of R, the radius of the Earth.)

hint

Use conservation of energy.

Model Answer 10.3

A projectile is allowed to drop from rest from a height of 1500 km above the Earth's surface.

Calculate the speed just before impact, assuming no atmosphere. (Mass of Earth $M = 6.0 \times 10^{24}$ kg; radius of Earth $R = 6.4 \times 10^6$ m.)

At the point of release of the projectile, its total energy is $E = -\frac{GMm}{R+h}$. At impact it is $E = \frac{1}{2}mv^2 - \frac{GMm}{R}$.

By energy conservation $\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$. The mass of the projectile cancels out and so

$$V = \sqrt{\frac{2GM}{R} - \frac{2GM}{R+h}}$$

$$V = \sqrt{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \left(\frac{1}{6.4 \times 10^6} - \frac{1}{6.4 \times 10^6 + 1.5 \times 10^6}\right)}$$

$$V = 4.87 \times 10^3 \approx 4.9 \times 10^3 \text{ms}^{-1}$$

Checklist

After studying this chapter you should be able to:

- □ define and solve problems with gravitational and electrical potential
- □ define and solve problems with gravitational and electrical potential energy
- □ draw gravitational and electrical equipotental surfaces
- □ apply the relation between potential and field
- □ derive expressions for orbital and escape speeds
- □ solve problems on motion in gravitational and electric fields.

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Annotated Exemplar Answer 10.1

A satellite in a circular orbit around the Earth experiences a small frictional force due to its contact with the upper atmosphere. Discuss the effect of this on the orbital radius and the speed of the satellite. **[4]**

The expression for total energy is correct, but be careful how you use it. Written like this, v and r appear to be independent, but they are not.

The frictional force means the satellite's total energy is reduced.

$E_{\mathrm{T}} = \frac{1}{2}mv^2 - \frac{GMm}{r}$

The satellite will slow down. Using the formula for orbital speed, $V = \sqrt{\frac{GM}{r}}$*, it follows that as the satellite slows down it will move further away from the Earth.*

This statement is correct – but you need to carry on with the idea of total energy in the rest of your discussion.

use the expression for orbital speed in the expression for total energy, so that you only have one variable on the right-hand side (m, m and g are constant in this situation). This gives $E_{\mathrm{T}} = -\frac{GMm}{2r}$*, from which it is clear that, as the total energy decreases, r decreases. Therefore v increases. This answer makes sense – you know that satellites eventually come down from orbit and are burnt up in the atmosphere or fall to Earth. They don't escape into space.*

The satellite will speed up. Remember, v and r are not independent in this situation.

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11 ELECTROMAGNETIC INDUCTION (HL)

This chapter covers the following topics:

- □ Faraday's law
- □ Lenz's law
- □ AC current
- □ Transformers
- □ AC power transmission
- □ Capacitors
- □ Rectification

11.1 Magnetic flux and induced EMF

DEFINITIONS

MAGNETIC FLUX The product of the magnetic field through a loop, the area of the loop and the cosine of the angle between the field and the normal to the loop (Figure 11.1). $\phi = BA \cos\theta$.

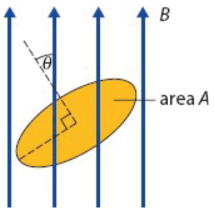


Figure 11.1

MAGNETIC FLUX LINKAGE The product of the flux trough a loop times the number of turns of the wire around the loop. If the loop consists of N turns of wire, the magnetic flux linkage through the loop is $\Phi = N\phi = NBA \cos\theta$.

FARADAY'S LAW The EMF induced in a loop is equal to the negative of the rate of change of magnetic flux linkage with time: $\text{emf} = -\frac{\Delta\Phi}{\Delta t}$.

LENZ'S LAW The direction of the induced EMF is such as to oppose the change in flux that created it. This is a restatement of the law of conservation of energy.

In Figure 11.2a the field is parallel to the surface. There is zero flux through the loop. In Figure 11.2b the field is normal to the loop and so the flux is a maximum.

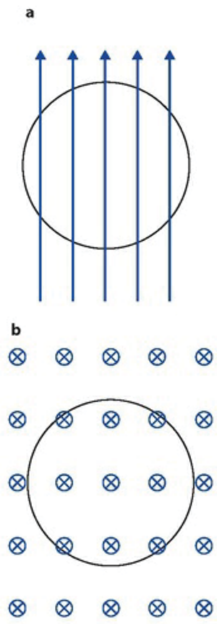


Figure 11.2 The flux through the loop is **a** zero if θ = 90°; **b** maximum if θ = 0.

Think of magnetic field lines as arrows. The flux is a measure of how many arrows pierce the loop's area. If the loop is made of conducting wire then the EMF will produce a current.

Examples of the application of Faraday's law

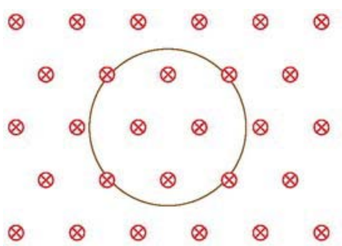


Figure 11.3

1 The magnetic field strength in Figure 11.3 is *increasing* at a rate of 5.0×10^{-3} Ts^{-1}. The area of the loop is 6.0×10^{-2} m^2 and there are 50 turns of wire. The rate of change of flux linkage is $\frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B}{\Delta t}A = 50 \times 5.0 \times 10^{-3} \times 6.0 \times 10^{-2} = 15\text{mV}$ and this is the induced EMF.

2 The rod in Figure 11.4 is pushed steadily to the right with speed v, so the area of the loop decreases, thus decreasing the flux linkage in the loop. The rate of change of flux linkage is $\frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BLx)}{\Delta t} = BL\frac{\Delta x}{\Delta t} = BLv$

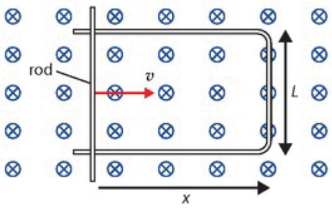


Figure 11.4

3 A loop is rotating at a constant angular speed about a vertical axis in a constant magnetic field directed horizontally to the right (Figure 11.5). The area is 2.4×10^{-2} m^2, there are 1500 turns of wire around the loop and the magnetic field has strength 0.20 T. The loop rotates at 50 revolutions per second.

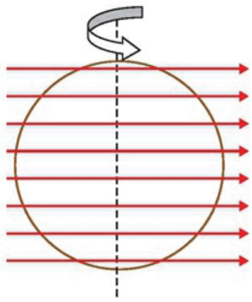


Figure 11.5

The loop rotates about the axis with constant angular speed $\frac{d\theta}{dt} = \omega = 2\pi f$. Hence $\theta = 2\pi ft$. The flux through the loop changes because the angle between the loop and the field changes. Using calculus,

$$\begin{aligned}\frac{d\Phi}{dt} &= \frac{d(NBA\ \cos(2\pi ft))}{dt} = -NBA\ \sin(2\pi ft)\frac{d(2\pi ft)}{dt} \\ &= -NBA(2\pi f)\sin(2\pi ft).\ \text{So the induced EMF is} \\ V &= \frac{d\Phi}{dt} = NBA(2\pi f)\sin(2\pi ft) \\ &= 1500 \times 0.20 \times 2.4 \times 10^{-2} \times (2\pi \times 50) \\ &\quad \sin(100\pi t) = 2.3 \times 10^{-3}\sin(100\pi t)\end{aligned}$$

In each of these examples, Lenz's law can be used to find the *direction* of the induced current. This law states that:

- if the flux is increasing, the induced current must produce a magnetic field opposite to the external field.
- if the flux is decreasing, the induced current must produce a magnetic field parallel to the external field.

Example 1: The flux is *increasing*. The external magnetic field is directed into the page. Lenz's law says that the induced current produces its own magnetic field out of the page – that is, opposite to the external field. The current is therefore counter-clockwise.

Example 2: The flux is *decreasing*. The external magnetic field is directed into the page. Lenz's law says that the induced current produces its own magnetic field into the page – that is, parallel to the external field. The current is therefore clockwise.

Example 3: The flux linkage is varying with time according to the graph in Figure 11.6.

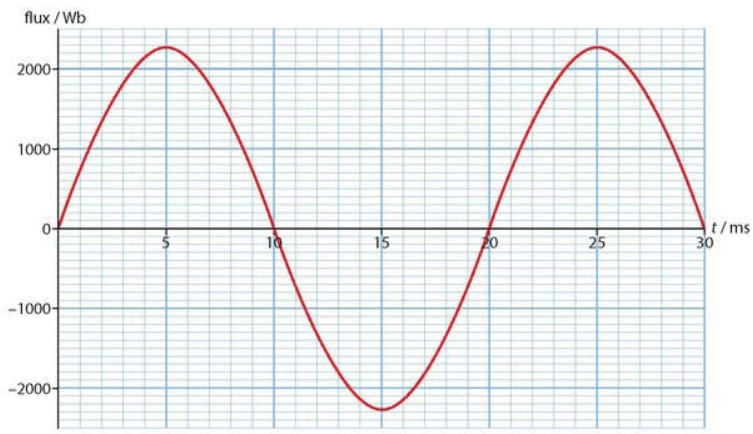


Figure 11.6

The flux increases for 10 ms and then decreases in the next 10 ms. This means that the induced current is changing direction every 10 ms.

Worked Example 11.1

A horizontal metallic ring is dropped in a region where there is a uniform upward magnetic field (see Figure 11.7).

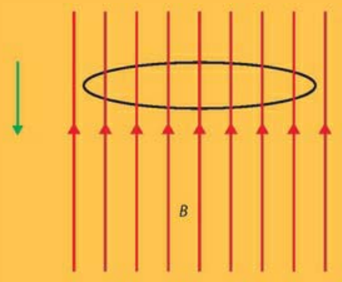


Figure 11.7

Neglecting air friction, what is the acceleration of the ring while it is moving in the region of the field?

a zero

b less than *g* but not zero

c *g*

d greater than *g*.

The field is uniform and so the change in flux is zero. Hence there is no induced EMF, no induced current and no force on the ring other than its weight. Hence the answer is *c*.

Model Answer 11.1

Figure 11.8 shows a loop of conducting wire of resistance 0.15Ω that is moved away from a straight wire carrying a constant current.

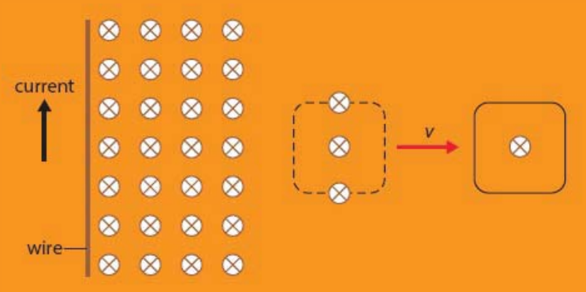


Figure 11.8

The constant current generates a magnetic field around the straight wire which decreases with distance from the wire. This field is into the page at the position of the loop. The area of the loop is 5.4×10^{-4} m^2 and the average magnetic field inside it decreases from 0.20 T to 0.12 T in 0.48 s.

a Calculate the EMF induced in the loop as it is being moved.

b State and explain the direction of the current induced in the loop.

a The loop is moved to the right, into a region of weaker magnetic field, so the flux linkage in the loop decreases. The average induced EMF is $\frac{\Delta\Phi}{\Delta} = \frac{(0.20-0.12)\times5.4\times10^{-4}}{0.48} = 9.0\times10^{-5}\text{V}$. The average current is then $I = \frac{9.0\times10^{-5}}{0.15} = 6.0\times10^{-4}\text{A}$

b The flux is decreasing, so the current induced in the loop must produce a field that is also into the page. Hence the current in the loop must be clockwise.

Motional EMF

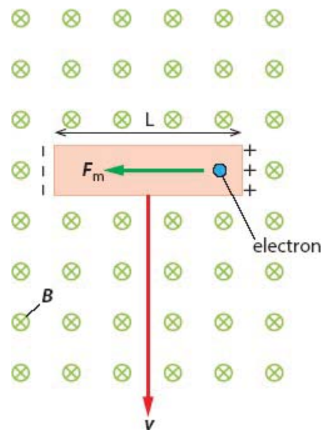


Figure 11.9

Consider a conducting rod of length L that moves at right angles to a uniform magnetic field $\boldsymbol{B}$ with speed $\boldsymbol{v}$, as in Figure 11.9. The field is into the plane of the page.

Each electron in the rod moves with speed v relative to the magnetic field and so experiences a magnetic force of magnitude $F_m = evB$ and directed to the left. Electrons will therefore move towards the left end of the rod, making it negatively charged and the right end positively charged. An electric field E will therefore be established in the rod (from right to left). Eventually the electric and magnetic forces will be equal and thus $evB = eE \Rightarrow E = vB$. This means there will be an EMF induced at the ends of the rod of magnitude V such that $E = \frac{V}{L}$, so $V = BLv$

TEST YOURSELF 11.1

In Example 2 (Figure 11.4), suppose the rod is given an initial velocity to the right and then released.

> hint
>
> Looking for a source for the energy means looking for work being done.

a Explain why the speed of the rod will decrease.

b A lamp placed in the loop will light up. State and explain where the energy to light up the lamp comes from.

TEST YOURSELF 11.2

A battery is connected in series to a device D, as in Figure 11.10a. The switch is closed at $t = 0$.

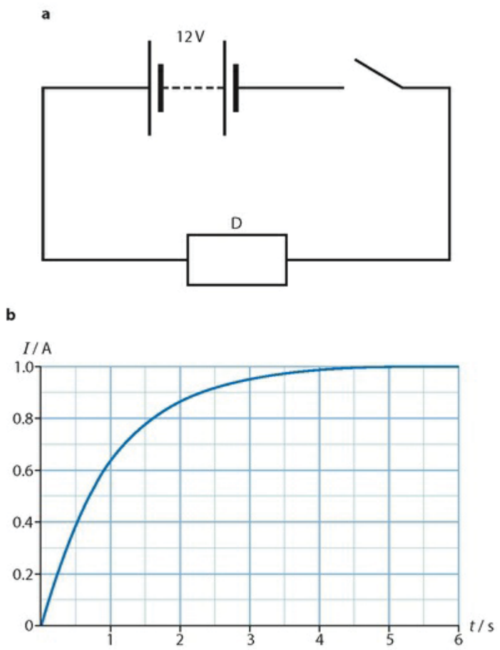


Figure 11.10

The current in the device varies with time as shown in Figure 11.10b. Suggest what the device is, and calculate its resistance.

11.2 Alternating current

DEFINITIONS

RMS CURRENT The square root of the mean (average) value of the square of the current in an AC circuit. This current would give the same power dissipation in a resistor in a DC circuit as the average power in the same resistor in an AC circuit.

TRANSFORMER A device that takes as input an alternating voltage and delivers as output an alternating voltage of the same frequency and different peak value.

Technology follows science

Nature of Science. Alternating current is the current universally produced. There are many reasons for this. It can be produced quite easily in generators. It can be turned on and off much more safely than dc currents: switching off large dc currents can create dangerous induction currents, but with ac the switching can be timed to when the currents are small. Finally, the use of ac current makes the use of transformers possible and this leads to a more economical transmission of power. For these reasons the use of ac is widespread. Solid scientific reasons dictate its use.

Electricity for commercial and home use comes from *generators* in which a coil rotates in a magnetic field (Figure 11.11).

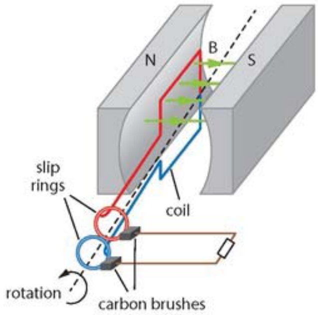


Figure 11.11

Suppose the coil in this figure rotates with a period of 20 ms – that is, at 50 revolutions per second. Figure 11.12 shows how the flux linkage in the rotating coil varies with time: $\Phi = NBA\cos(2\pi ft)$.

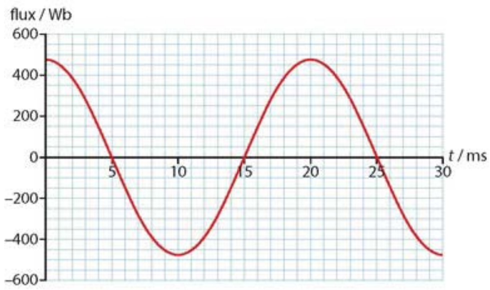


Figure 11.12

The negative gradient of this graph is the induced EMF in the coil (Figure 11.13):

$$V = \frac{d\Phi}{dt} = 2\pi fNBA\sin(2\pi ft\ ).$$

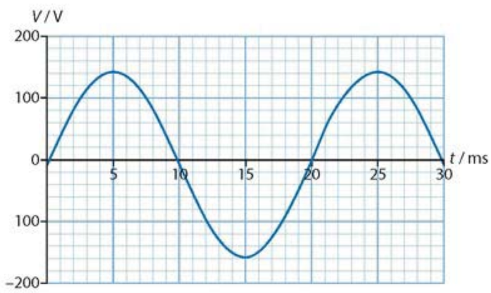


Figure 11.13

Figure 11.14 shows how the current in the loop varies with time (remember that the resistance of the coil is

$R = 15\Omega$ and recall that $I = \frac{V}{R}$) : $I = \frac{2\pi f NBA \sin(2\pi f t)}{R}$.

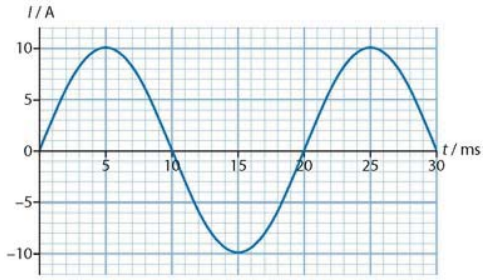


Figure 11.14

From these graphs we deduce that the peak values for voltage and current are V_0 = 150 V and I_0 = 10.0 A. This means that the rms values are $V_{rms} = \frac{150}{\sqrt{2}} = 106\text{V}$ and $I_{rms} = \frac{10.0}{\sqrt{2}} = 7.07\text{A}$.

Notice that when the flux is zero the EMF (and the current) have large values, and vice-versa. The current has the same dependence on time as the voltage (they are **in phase**).

Because the voltage and the current constantly vary in an AC circuit, the power dissipated in a resistor also varies. The *average* power dissipated is half of the peak power (Figure 11.15).

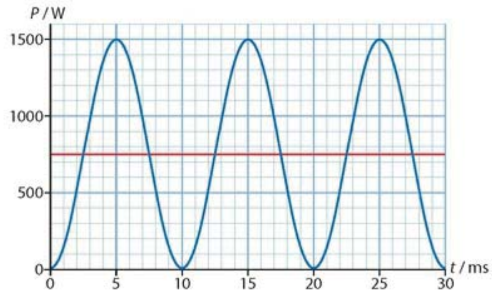


Figure 11.15

$$P = VI = RI^2 = \frac{V^2}{R} = \frac{(2\pi f NBA \sin(2\pi f t))^2}{R}$$

The peak power is $P_0 = \frac{(2\pi f NBA)^2}{R}$. The average power $\overline{P}$ is half of the peak power and equals $\overline{P} = \frac{1}{2}P_0 = \frac{1}{2}V_0 I_0 = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} = RI_{\text{rms}}^2$.

Worked Example 11.2

Figure 11.16 shows how the power in a generator varies with time.

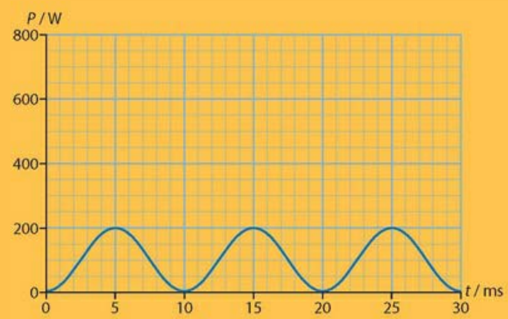


Figure 11.16

If the frequency of rotation is now doubled, sketch a graph to show how the power now varies with time.

Since power is proportional to the square of the frequency, the peak power will increase by a factor of $2^2 = 4$. The period will halve and so we have the red line in Figure 11.17.

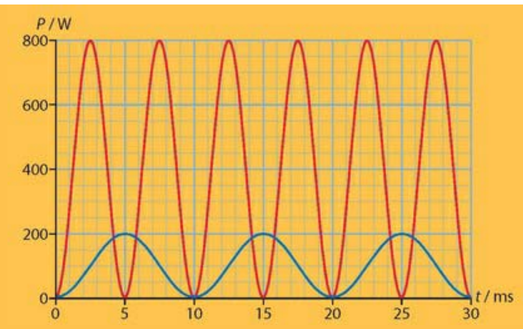


Figure 11.17

11.3 Transformers

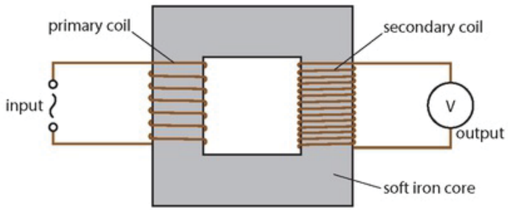


Figure 11.18

Consider two coils, a primary and a secondary, with N_p and N_s turns of wire, respectively (Figure 11.18).

An alternating voltage V_p is applied to the primary coil. Because the magnetic field it produces is varying, the flux linkage in the secondary coil is also changing and so an EMF V_s is induced such that $\frac{V_P}{V_s} = \frac{N_P}{N_s}$.

Assuming a perfectly efficient transformer, the power from the secondary coil equals the power into the primary coil, so $V_pI_p = V_sI_s$. Therefore $\frac{I_P}{I_s} = \frac{N_s}{N_P}$.

Step-up transformers increase the output voltage and step-down transformers decrease it.

Real transformers suffer from power losses: the output power is less than the input power. Reasons for this loss include heating of the coil wires, eddy currents, hysteresis and flux loss.

- Eddy currents: these are currents created when free electrons inside the iron core are forced to move due to the magnetic field they are exposed to. This creates heating in the core and power loss. These are reduced by laminating the core.
- Hysteresis: as the current increases so does the magnetic field and the stored magnetic energy. As the current decreases the magnetic energy decreases as well but not all of it is returned: some energy remains stored.
- Flux loss: some of the flux created in the primary coil does not link the secondary coil. The iron core reduces flux loss to a very large degree.

Power loss in transmission

Transmission cables with resistance R and current I have a power loss of $P_{lost} = RI^2$ as heat in the cables. This may be reduced by reducing the current. This means transmitting power at very high voltage, and this requires transformers.

Suppose a power station produces 1 GW of power at an rms voltage of 50 kV The rms current sent down the wires to consumers would then be $I = \frac{1\text{GW}}{50\text{kV}} = 2 \times 10^4\,\text{A}$. Assuming transmission cables of total resistance 1Ω, we would then have a power loss in the heating of the cables of $P_{lost} = RI^2 = 1 \times 4 \times 10^8 = 0.4$ GW. This means that 40% of the produced power would be wasted.

If a step-up transformer is used to raise the produced voltage from 50 kV to 500 kV, the current sent down the cables will be reduced to $I = \frac{1\text{GW}}{500\text{kV}} = 2 \times 10^3\,\text{A}$, and the power lost will be $P_{lost} = RI^2 = 1 \times 4 \times 10^6 = 0.004$ GW, just 0.4% of the produced power – so much less power is wasted.

A step-down transformer must then be used to reduce this high voltage to a safe low value to be supplied to consumers.

TEST YOURSELF 11.3

Figure 11.19 shows how the EMF in the primary coil of an ideal transformer varies with time.

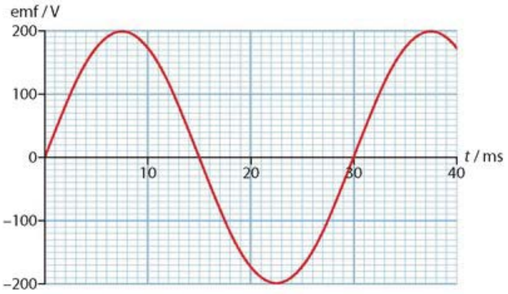


Figure 11.19

a There are 900 turns of wire in the primary coil. The EMF in the secondary coil has a peak value of 16 V. Calculate:

i the rms value of the voltage in the secondary coil

ii the number of turns in the secondary coil.

b The current in the secondary coil has an rms value of 2.4 A. Calculate:

i the resistance of the secondary coil

ii the average power and

iii the peak power in the secondary coil.

c Draw a sketch graph to show how the power in the secondary coil varies with time.

11.4 Capacitors

DEFINITIONS

CAPACITANCE The charge per unit voltage that can be stored on a capacitor: $C = \frac{Q}{V}$, where Q is the charge on one of the plates and V is the potential difference between the plates. The unit of capacitance is the farad (F); $1\,\mathrm{F} = 1\,\mathrm{C}\mathrm{V}^{-1}$

ENERGY STORED IN A CAPACITOR This is given by the equivalent expressions $E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$.

TOTAL CAPACITANCE OF CAPACITORS CONNECTED IN SERIES
$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

TOTAL CAPACITANCE OF CAPACITORS CONNECTED IN PARALLEL
$C_T = C_1 + C_2 + \cdots$

TIME CONSTANT The time after which the stored charge has decreased to about 37% of its initial value.

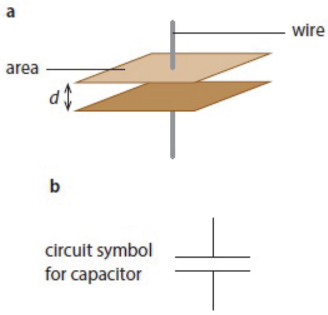


Figure 11.20

For a parallel-plate capacitor (see Figure 11.20a), $C = \varepsilon\frac{A}{d}$, where A is the area of one of the plates, d is the separation of the plates and ε is the **permittivity** of the medium between the plates. The standard symbol for a capacitor is shown in Figure 11.20b.

If the plates are in vacuum, $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm}^{-1}$. If an insulator (also called a dielectric) of permittivity ε is inserted between the plates of a capacitor the capacitance will increase because $\varepsilon > \varepsilon_0$

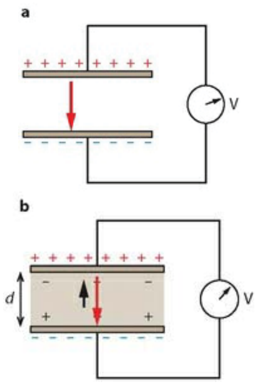


Figure 11.21

Why does this happen? The capacitor in Figure 11.21 does not discharge because of the infinite resistance voltmeter that does not allow the flow of any charge: the charge on the plates cannot change. There is an electric field in between the plates directed from positive to negative (red arrow). This electric field acts on the electrons of the dielectric, pulling them somewhat against the field – that is, upwards. So there is separation of charge in the dielectric. This creates a small electric field within the dielectric that is directed upward (black arrow).

This means that the net electric field in between the parallel plates is *reduced* compared to that in vacuum. Now, the work done to move charge q from one plate to the other is given by $W = Fd = qEd$ and since E is reduced, so is the work done. But the work done is also equal to $W = qV$. This implies that the potential difference across the plates has been reduced. From the definition $C = \frac{Q}{V}$ it follows that the capacitance increases.

Model Answer 11.2

A capacitor in vacuum is connected to a battery that establishes a *constant* potential difference between the plates. A dielectric is then inserted in between the plates of the capacitor. Predict the effect of this on the capacitance of the capacitor.

Following the discussion above the work done to move a charge across the plates must stay the same and so the net electric field in between the plates also stays the same. But the electric field inside the dielectric is opposite to the electric field of the plates. For the net field to stay the same, the field due to the charges on the plates must increase. Hence the charge on the plates must increase and from $C = \frac{Q}{V}$ the capacitance increases.

Capacitors in parallel and in series

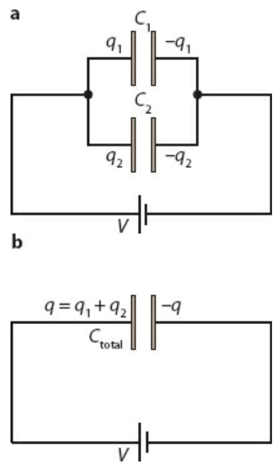


Figure 11.22

Figure 11.22a shows two capacitors of capacitance C_1 and C_2 connected in parallel.

Both are connected to a source of potential difference V, and this is the common potential difference across both of them.

The charge on the first capacitor is q_1 and that on the other is q_2. We have that $q_1 = C_1V$ and $q_2 = C_2V$.

The total charge on the two capacitors is $q = q_1 + q_2 = (C_1 + C_2)\, V$.

We may define the total capacitance C_{total} of the combination using $q = C_{total}\, V$, so $C_{total} = C_1 + C_2$.

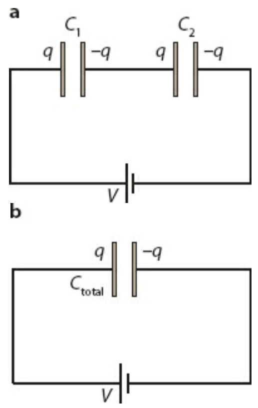


Figure 11.23

Figure 11.23a shows two capacitors of capacitance C_1 and C_2 connected in series.

In this case the charge on each capacitor is the same (if it were not, the segment between the two capacitors would have a net charge).

We know that $V_1 = \frac{q}{C_1}$ and $V_2 = \frac{q}{C_2}$, and that the source of potential difference V is equal to $V_1 + V_2$. The total capacitance C_{total} is then $V = \frac{q}{C_{total}} = \frac{q}{C_1} + \frac{q}{C_2}$, giving $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$.

Annotated Exemplar Answer 11.1

a State what is meant by capacitance. **[1]**

b Figure 11.24 shows an arrangement of three capacitors connected to a source of EMF 12 V.

Calculate the potential difference across each capacitor. **[4]**

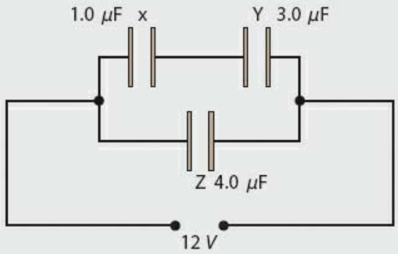


Figure 11.24

a Capacitance is the charge per unit voltage that can be stored on a capacitor:

$C = \frac{q}{V}$

Good – marks are easy to get if you have learned the definitions of key terms.

b The pd across Z is 12 V

Another easy mark.

X and Y together have a capacitance of $3.0 + 1.0 = 4.0\ \mu F$

Take care not to confuse the formulas for capacitors in series and for resistors in series. For capacitors in series, the capacitance of the combination is given by $\frac{1}{C_{total}} = \frac{1}{C1} + \frac{1}{C_2} + \ldots$. Here, this gives a combined capacitance of 0.75 μF and a charge of $q = 12 \times 0.75 \times 10^{-6} = 9.0\ \mu C$.

The pd across the combination is 12 V, so this is divided in the ratio of the capacitances of X and Y.

The pd across Y is $\frac{3.0}{4.0} \times 12 = 9.0$ V.

The pd across X is $\frac{1.0}{4.0} \times 12 = 3.0$ V.

Think about how the charge builds up on X and Y – the total charge q on each capacitor must be the same. Since $C = \frac{q}{V}$, and since for capacitors X and Y the value of q is the same, the pd is inversely proportional to the capacitance. The larger capacitor has the smaller pd across it. You can use the value for the charge stored to calculate the individual pds from $V = \frac{q}{C}$. The values of pd are then 9.0 V for X and 3.0 V for Y.

You can check your answers using a simple calculation. What charge q is stored on each capacitor? Using your values, X stores 3.0 μC and Y stores 27 μC. These are not equal, so the answer must be wrong.

2/5

Charging and discharging a capacitor

The circuit in Figure 11.25 may be used to investigate both the charging and the discharging of a capacitor.

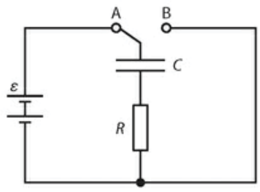
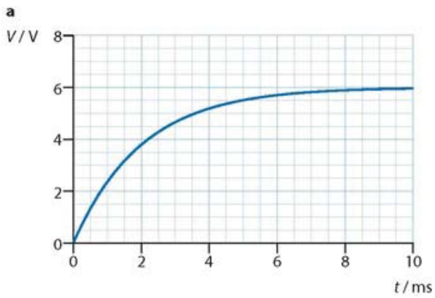


Figure 11.25

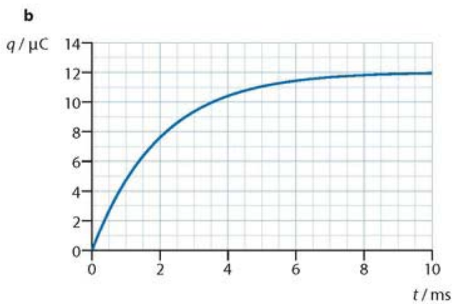
If the switch is moved to position A, the capacitor will charge. After the capacitor has charged, if the swirch is moved to B, the capacitor will discharge.

Charging: Initially the capacitor is uncharged. As soon as the switch is moved to A:

- the charge on the capacitor plates will increase to a final constant value (Figure 11.26a)



- the potential difference across the capacitor plates will increase to the EMF of the battery (Figure 11.26b)



- the current starts out large and then decreases exponentially (Figure 11.26c).

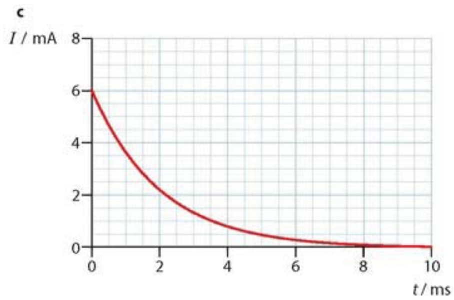


Figure 11.26 The variation of **a** potential difference, **b** charge and **c** current with time for a charging capacitor.

The capacitor is now fully charged, and the current becomes zero.

The final charge on the plates is $q_0 = C\varepsilon$. The initial current is equal to $I_0 = \frac{\varepsilon}{R}$.

Worked Example 11.3

The graphs in Figure 11.26 were obtained while charging a capacitor in the circuit in Figure 11.25. Determine:

a the EMF of the battery
b the capacitance of the circuit
c the resistance of the circuit.

a The EMF is the final value of the potential difference across the capacitor, and that is 6.0 V.
b The final charge on the capacitor plate is 12 μC, and from $q_0 = C\varepsilon$ we find

$$C = \frac{q_0}{\varepsilon} = \frac{12\times10^{-6}}{6.0} = 2.0\mu\text{F}$$

c The initial current is $I_0 = \frac{\varepsilon}{R}$ and so $R = \frac{\varepsilon}{I_0} = \frac{6.0}{6.0\times10^{-3}} = 1.0 \times 10^3 \Omega$

Model Answer 11.3

Figure 11.27 **shows how the charge on a capacitor plate increases as the capacitor is being charged**.

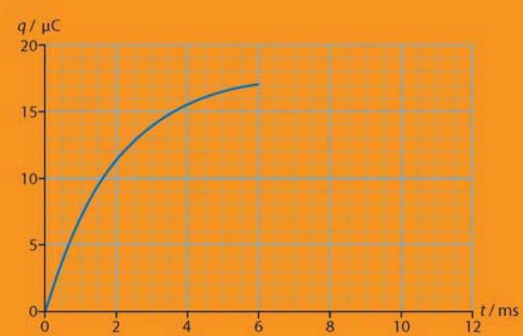


Figure 11.27

Estimate a **the initial current in the circuit, and** b **the charge on the plate after the capacitor is fully charged**.

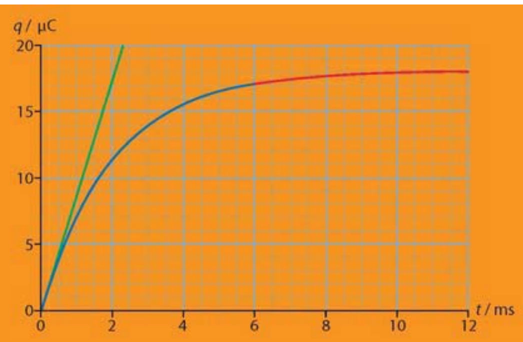


Figure 11.28

a Draw the tangent to the graph at t = 0 (green line). Its gradient is about 9 μA.
b Extend the graph (red line) to show the saturation value of the charge (Figure 11.28). This shows that the final charge is about 18 μC.

Discharging: Initially the capacitor is fully charged. As soon as the switch is moved to position B, Figure 11.25, the current, charge and voltage all decrease exponentially according to $q = q_0 e^{-\frac{t}{RC}}, V = V_0 e^{-\frac{t}{RC}}, I = \frac{V_0}{R} e^{-\frac{\tau}{RC}}$.

Quantities with a subscript of 0 indicate values at t = 0. The quantity RC is called the **time constant** and is denoted by τ. After a time $t = \tau$, the charge will be $q = q_0 e^{-\frac{\tau}{\pi}} = \frac{q_0}{e} \approx 0.37 q_0$, or 37% of its initial value.

By analogy with radioactive decay, we define the **half-life** $T_{1/2}$ of the capacitor as the time it takes for the charge to decrease to half its initial value. We substitute $q = \frac{1}{2} q_0$ to get $\frac{1}{2} q_0 = q_0 e^{-\frac{T_{1/2}}{\tau}}$, and taking logarithms gives $\ln 2 = \frac{T_{1/2}}{\tau}$ or $T_{1/2} = \tau \ln 2$.

Model Answer 11.4

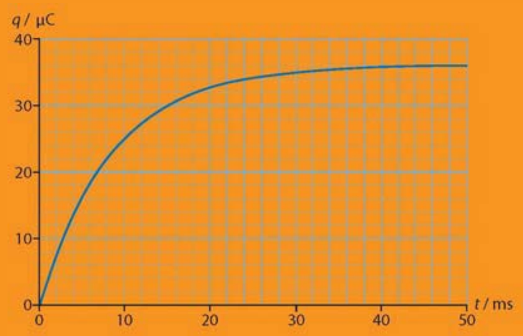


Figure 11.29

The graph in Figure 11.29 shows the variation with time t of the charge q on a capacitor plate as the capacitor is being charged.

The capacitance is 4.0 μF and the resistance of the resistor R is 2.0 kΩ.

a Use the graph to estimate the EMF of the charging battery.
b Sketch a graph to show the variation with time of the current in the circuit while the capacitor is being charged, putting numbers on the vertical axis.
c After how much time is the energy stored in the capacitor reduced to half its initial value?

a From the graph, the final charge is 36 μC. This equals $q_0 = C\varepsilon \text{ and so } \varepsilon = \frac{36\times10^{-6}}{4.0\times10^{-6}} = 9.0\text{V}$

b The initial current is $I_0 = \frac{\varepsilon}{R} = \frac{9.0}{2.0\times10^3} = 4.5\text{mA}$

The current drops exponentially and so we have a graph like the one in Figure 11.30.

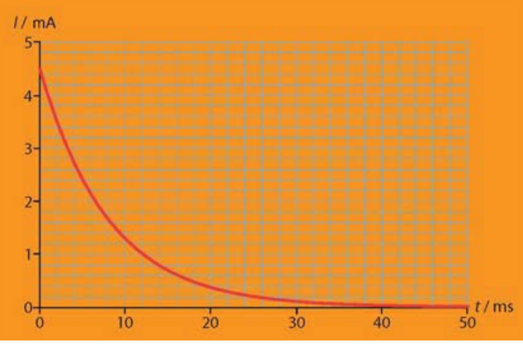


Figure 11.30

c The stored energy is given by $E = \frac{q_0^2 e^{-\frac{2t}{RC}}}{2C}$. We need $\frac{q_0^2 e^{-\frac{2t}{RC}}}{2C} = \frac{1}{2}\frac{q_0^2}{2C}$, thus $\frac{1}{2} = e^{-\frac{2t}{RC}}$, so, by taking logarithms, $\ln 2 = \frac{2t}{RC} \Rightarrow t = \frac{RC \ln 2}{2} = \frac{2.0 \times 10^3 \times 4.0 \times 10^{-6} \times \ln 2}{2} = 2.8\text{ms}$

TEST YOURSELF 11.4

The time constant of a resistor-capacitor system is τ. The capacitor is initially fully charged. After how much time will the energy stored in the capacitor be reduced to half its initial value?

a τ

b $\tau \ln 2$

c $\frac{\tau \ln 2}{2}$

d $2\tau \ln 2$

< Back to chapter

Annotated Exemplar Answer 11.1

a State what is meant by capacitance. **[1]**

b Figure 11.24 shows an arrangement of three capacitors connected to a source of EMF 12 V.

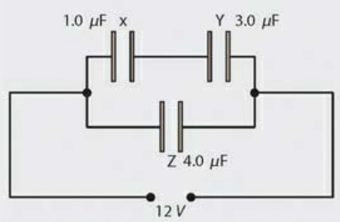


Figure 11.24

Calculate the potential difference across each capacitor. **[4]**

a *Capacitance is the charge per unit voltage that can be stored on a capacitor:* $c = \frac{q}{v}$

b *The pd across Z is 12 V*

X and Y together have a capacitance of 3.0 + 1.0 = 4.0 *μF*

The pd across the combination is 12 V, so this is divided in the ratio of the capacitances of X and Y.

The pd across $Y\ is\ \frac{3.0}{4.0} \times 12 = 9.0\,v$.

The pd across $X\ is\ \frac{1.0}{4.0} \times 12 = 3.0\,v$.

Good – marks are easy to get if you have learned the definitions of key terms.

Another easy mark.

Take care not to confuse the formulas for capacitors in series and for resistors in series. For capacitors in series, the capacitance of the combination is given by $\frac{1}{c_{\text{total}}} = \frac{1}{c1} + \frac{1}{c_2} + \ldots$.

Here, this gives a combined capacitance of 0.75 μF and a charge of q = 12 × 0.75 × 10^{-6} =9.0 *μc*.

You can check your answers using a simple calculation. What charge q is stored on each capacitor? using your values, X stores 3.0 μc and Y stores 27 μc. These are not equal, so the answer must be wrong.

Think about how the charge builds up on X and Y – the total charge q on each capacitor must be the same. Since $c = \frac{q}{v}$*, and since for capacitors X and Y the value of q is the same, the pd is inversely proportional to the capacitance. The larger capacitor has the smaller pd across it. You can use the value for the charge stored to calculate the individual pds from* $v = \frac{q}{c}$*. The values of pd are then 9.0 V for X and 3.0 V for Y*

2/5

11.5 Rectification

For many applications it is necessary to convert an AC current to a DC current, in which the electrons all flow in the same direction.

Half-wave rectification

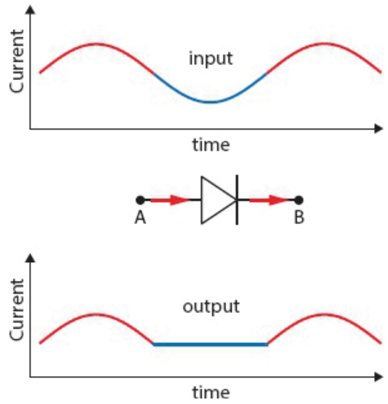


Figure 11.31

This conversion can be partially achieved with a single diode. A **diode** allows current to pass through it in only one direction. When the current is positive (red) it is allowed to pass through the diode. When it is negative (blue) it does not (Figure 11.31).

The output is direct current, although it is not constant. In half-wave rectification, therefore, half the power is lost.

Full-wave rectification

A better way to rectify uses four diodes in a **diode-bridge rectifier**, shown in Figure 11.32. Notice that all diodes point the same way, in this case to the 'left'.

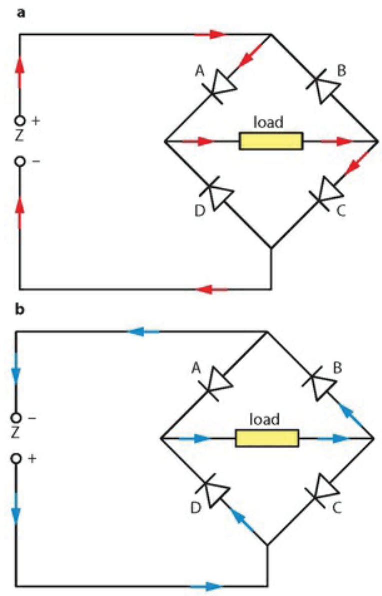
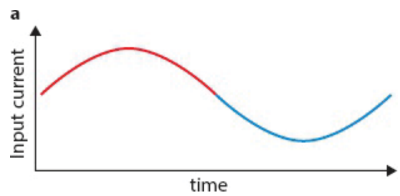


Figure 11.32

- During the first half-cycle (Figure 11.32a), the current moves clockwise, entering the bridge through diode A, passing through the load resistor from left to right and exiting though diode C. During this half-cycle diodes B and D do not conduct any current.
- In the next half-cycle (Figure 11.32b), the current moves counterclockwise, entering the bridge through diode D, moving through the load from *left to right again* – that is, in the same direction as the first half-cycle – and exiting through diode B. During this half-cycle, diodes A and C do not conduct any current.

The input and output currents are shown in Figure 11.33.



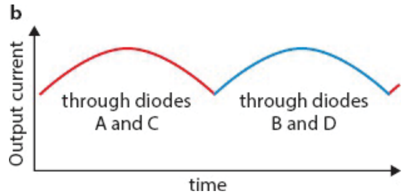


Figure 11.33 a Input current; **b** output current.

Smoothing the output

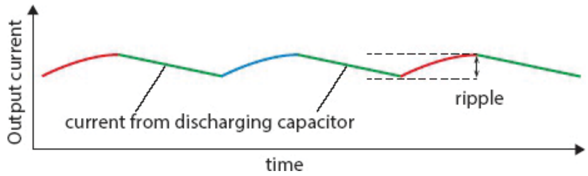


Figure 11.34

The output current of the diode-bridge rectifier may be made smoother by adding a capacitor in parallel with the load resistor. The output (Figure 11.34) is now smoother, though it still has a 'ripple'.

The greater the capacitance the less the 'ripple'.

Checklist

After studying this chapter you should be able to:

- □ **solve problems with Faraday's and Lenz's laws**
- □ **work with AC circuits**
- □ **describe why a dielectric increases the capacitance of a capacitor**
- □ **solve simple problems with capacitors in series and in parallel**
- □ **solve problems with charging and discharging capacitors**
- □ **understand simple rectifier circuits.**

12 QUANTUM PHYSICS AND NUCLEAR PHYSICS (HL)

This chapter covers the following topics:

- □ The quantum nature of radiation
- □ The wave nature of matter
- □ Pair annihilation and creation
- □ The Bohr model of the hydrogen atom
- □ The Schrödinger theory
- □ The Heisenberg uncertainty principle and tunnelling
- □ Nuclear radii and nuclear energy levels
- □ Deviations from Rutherford scattering
- □ The radioactive decay law

12.1 The quantum nature of radiation

DEFINITIONS

PHOTOELECTRIC EFFECT The emission of electrons from a metal surface when electromagnetic radiation is incident on the surface. This is evidence for the behaviour of light as photons – that is, as particles or quanta of energy.

WORK FUNCTION, ϕ The minimum energy required to eject an electron from a metal.

STOPPING POTENTIAL The potential at which the photocurrent in a photoelectric experiment becomes zero.

The photoelectric effect can be investigated with the apparatus shown in Figure 12.1.

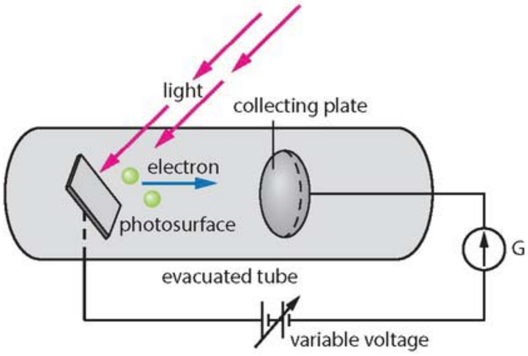


Figure 12.1

A sensitive galvanometer G measures the current flowing in the circuit, and Figure 12.1 shows the results for a range of positive and negative voltages.

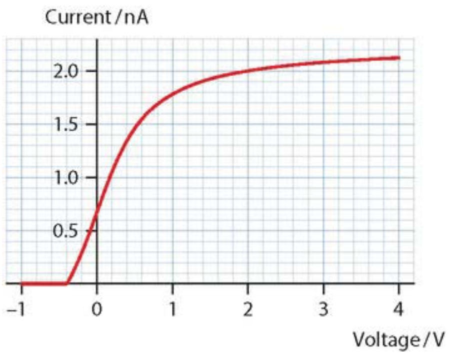


Figure 12.2

With the battery connected as shown, we can obtain data for the current for negative voltages (to obtain data for positive voltages, the polarity of the battery must be reversed).

As the applied voltage is made more and more positive, the number of electrons attracted to the collecting plate increases and so the current increases.

As the voltage is made more and more negative, the current becomes zero at what is called the **stopping voltage**, V_s. At more negative values of the voltage, no more electrons are emitted from the photo-surface, so the stopping voltage is a measure of the maximum kinetic energy of the emitted electrons: $E_{max} = eV_S$.

The key experimental facts are the following:

- The kinetic energy of the emitted electrons increases with increasing light frequency.
- The kinetic energy of the emitted electrons does not depend on the intensity of the light. The light intensity only affects the *number* of electrons emitted (i.e. the photocurrent).
- Below a particular value of the light frequency, called the **threshold frequency**, no electrons are emitted at all, no matter how high the light intensity.
- Electrons are emitted within 1 ns of the incidence of light on the surface, thus with essentially no time delay.

These facts contradict the classical theory of light as waves.

Einstein's explanation of the photoelectric effect

Einstein suggested that light is made up of *particles*, called photons – bundles or quanta of energy. In electron emission, one electron absorbs one photon of energy $E = hf$, where f is the frequency of the light. If the minimum energy needed for the electron to escape from the metal is called ϕ (known as the **work function**), then the electron will be emitted with kinetic energy $E_k = hf - \phi$. This means that, if the *maximum* kinetic energy of the emitted electrons is plotted against light frequency (Figure 12.3):

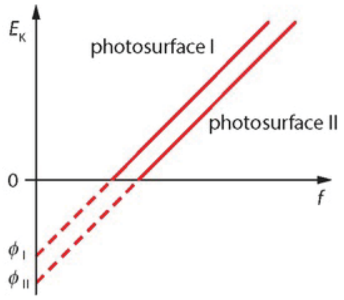


Figure 12.3

- the graph will be a straight line
- the gradient of the line will be h, the Planck constant
- the vertical intercept will be the work function, ϕ
- a different photosurface will give a different but parallel straight line.

Worked Example 12.1

Red light of intensity 100 Wm^{-2} is incident on a photosurface. The stopping potential is 4 V. What is the stopping potential when red light of intensity 200 Wm^{-2} is incident on the same photosurface?

a 4V
b 2V
c 1V
d 0.5V

The stopping potential is determined by the energy of the incoming photons and the work function of the photosurface. It does not depend on intensity, so the answer is **a**.

TEST YOURSELF 12.1

The work function of a metal surface is 1.8 eV. Monochromatic light of wavelength 4.8×10^{-7} m is incident on the metal. Calculate:

a the threshold frequency
b the speed of the emitted electrons.

hint

To find speed you must first find the kinetic energy of the electrons. How are kinetic energy and frequency related? How are frequency and wavelength related?

TEST YOURSELF 12.2

Figure 12.4 shows the variation with frequency of the stopping voltage in a photoelectric experiment.

hint

What is the equation relating stopping voltage and frequency?

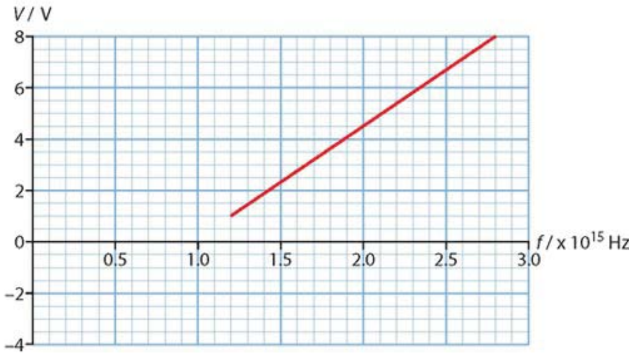


Figure 12.4

Determine the work function of the metal and a value for the Planck constant.

Model Answer 12.1

When light is incident on a metallic surface it is observed that electrons are emitted without any measurable time delay.

a Why can this observation not be understood in terms of the wave theory of light?

b Suggest how the photon theory of light explains this observation.

a According to the wave theory of light, the energy carried by a wave is spread over the wavefront. An electron would then have to accumulate energy from the wavefront and that would take time.

b In the photon theory, the electron is emitted when it absorbs a photon. Hence the transfer of energy to the electron is almost instantaneous.

12.2 The wave nature of matter

De Broglie's hypothesis states that all moving particles (electrically charged or neutral) show wave behaviour, with a characteristic wavelength given by $\lambda = \frac{h}{p}$, where h is the Planck constant and p is the momentum of the particle.

Figure 12.5 shows the apparatus used by Davisson and Germer to verify de Broglie's hypothesis.

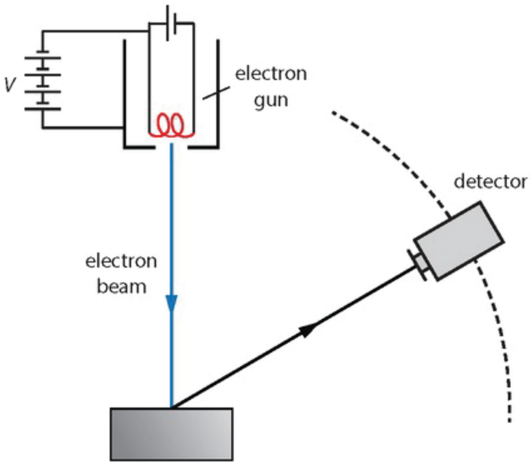
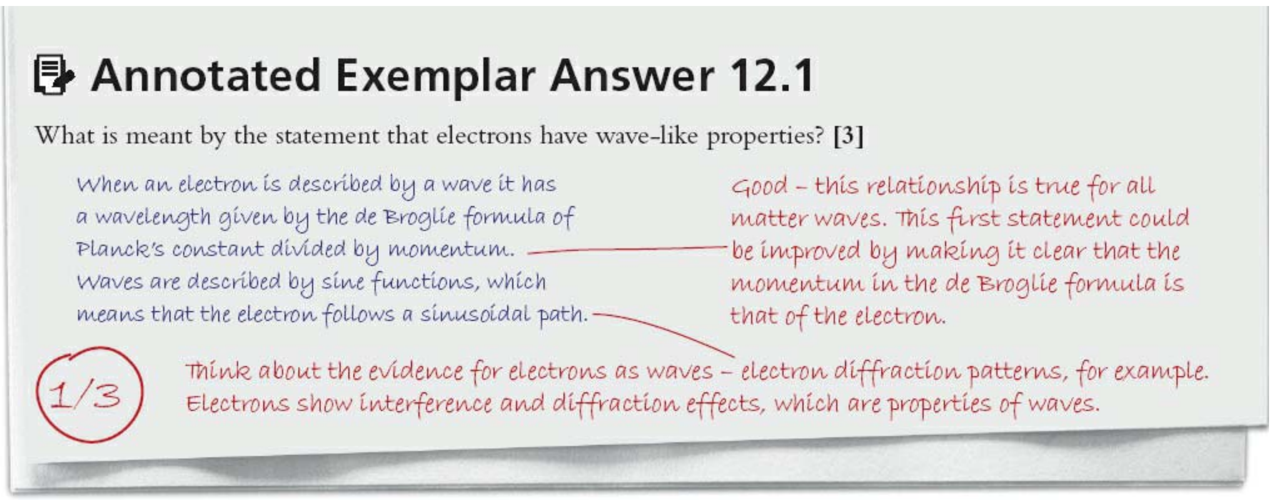


Figure 12.5

A beam of electrons is accelerated through a known potential difference and directed at a crystal, from which it scatters. The detector, which can be rotated as shown, shows peaks in the number of scattered electrons at specific angles. The results are consistent with the diffraction of waves, for a wavelength equal to the de Broglie wavelength of the electrons.



Annotated Exemplar Answer 12.1

What is meant by the statement that electrons have wave-like properties? **[3]**

When an electron is described by a wave it has a wavelength given by the de Broglie formula of Planck's constant divided by momentum.
Waves are described by sine functions, which means that the electron follows a sinusoidal path.

Good – this relationship is true for all matter waves. This first statement could be improved by making it clear that the momentum in the de Broglie formula is that of the electron.

1/3

Think about the evidence for electrons as waves – electron diffraction patterns, for example. Electrons show interference and diffraction effects, which are properties of waves.

TEST YOURSELF 12.3

Calculate the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 250 V.

TEST YOURSELF 12.4

Explain why an electron microscope can resolve objects that cannot be resolved using visible light.

hint

What is the work done when a charge is accelerated through a pd – and what becomes of this work? Think of a convenient formula for kinetic energy in terms of momentum.

< Back to chapter

Annotated Exemplar Answer 12.1

What is meant by the statement that electrons have wave-like properties? **[3]**

When an electron is described by a wave it has a wavelength given by the de Broglie formula of Planck's constant divided by momentum.

Good – this relationship is true for all matter waves. This first statement could be improved by making it clear that the momentum in the de Broglie formula is that of the electron.

Waves are described by sine functions, which means that the electron follows a sinusoidal path.

Think about the evidence for electrons as waves – electron diffraction patterns, for example. Electrons show interference and diffraction effects, which are properties of waves.

1/3

12.3 Pair annihilation and pair creation

One of the unusual predictions of quantum theory concerns the collision of a particle with its anti-particle. The mass and energy of the particle and its anti-particle are completely converted into the energy of at least two photons. The particle and anti-particle disappear – they *annihilate* each other.

So, for example, a proton with kinetic energy 2.00 GeV colliding with an anti-proton of the same kinetic energy moving in the opposite direction will produce two photons (see the Feynman diagram in Figure 12.6).

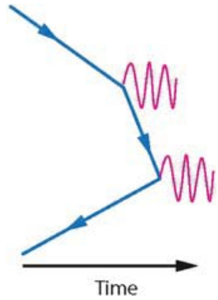


Figure 12.6

Because the total initial momentum is zero in this case, the final momentum must also be zero, so the photons move in opposite directions *and* have the same wavelength. The energy of the photons comes from the rest energy of each proton (0.938 GeV) and their kinetic energy, making a total available energy of (2 × 0.938 + 2 × 2.00) ≈ 5.88 GeV. So each photon will have an energy of about 2.9 GeV

The reverse phenomenon, pair *creation*, may also take place. This refers to a single photon producing a particle–anti-particle pair. In order for momentum to be conserved, there must be a nucleus nearby, and it participates in the reaction (Figure 12.7).

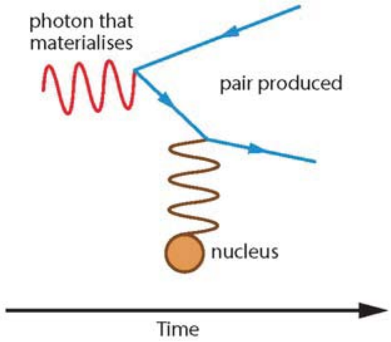


Figure 12.7

Leaving aside the nucleus for a moment, in order to produce a pair of particles each of mass m, the energy of the photon must be equal to at least $2mc^2$.

Annotated Exemplar Answer 12.2

a Estimate the wavelength of a photon needed to produce an electron–positron pair. **[2]**

b Explain whether the actual wavelength of the required photon will be smaller or larger than the answer to **a**. **[2]**

a The energy needed is $2m_ec^2 = 2 \times 0.511 = 1.02$ MeV.

Hence $\frac{hc}{\lambda} = 1.02 \times 10^6$ eV, leading to $\lambda = \frac{1.24 \times 10^{-6}}{1.02 \times 10^6} = 1.2 \times 10^{-12}$ m.

b Because a nucleus must be involved in the reaction, it will receive some energy as well, so the photon energy must be larger, hence its wavelength must be larger.

3/4

A good answer to both parts, though you have overlooked the fact that a larger photon energy requires a smaller wavelength.

< Back to chapter

Annotated Exemplar Answer 12.2

a Estimate the wavelength of a photon needed to produce an electron–positron pair. **[2]**

b Explain whether the actual wavelength of the required photon will be smaller or larger than the answer to **a.** **[2]**

a The energy needed is $2m_ec^2 = 2 \times 0.511 = 1.02$ MeV.

Hence $\frac{hc}{\lambda} = 1.02 \times 10^6\ ev$, *Leading to* $\lambda = \frac{1.24 \times 10^{-6}}{1.02 \times 10^6} = 1.2 \times 10^{-12} m$

b Because a nucleus must be involved in the reaction, it will receive some energy as well, so the photon energy must be larger: hence its wavelength must be larger.

3/4

A good answer to both parts, though you have overlooked the fact that a larger photon energy requires a smaller wavelength.

12.4 The Bohr model

Niels Bohr objected to the idea of an electron orbiting a nucleus in a circular orbit. He argued that such an electron experiences centripetal acceleration, and an accelerated charge radiates away energy. So such an orbit would be unstable: the electron would spiral into the nucleus. However, Bohr argued that, if the following condition is satisfied, the orbits *would be stable*:

hint

The mathematics here should be familiar from Chapter 10.

Bohr's first postulate: $mvr = n\frac{h}{2\pi}$, where *n* is a positive integer.

The quantity *mvr* is the angular momentum of the orbiting electron (see Option B).

Bohr could not completely justify his postulate; this was done in later work by Schrödinger.

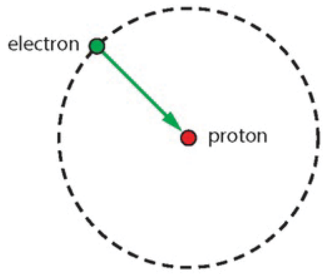


Figure 12.8

So an electron orbiting a nucleus in one of the allowed stable orbits has a total energy of $E = \frac{1}{2}mv^2 - \frac{ke^2}{r}$, where *r* is the orbital radius. The force on the electron is the Coulomb force between the electron and the proton in the hydrogen nucleus (Figure 12.8): $F = \frac{ke^2}{r^2}$.

Hence $\frac{ke^2}{r^2} = m\frac{v^2}{r}$ and so $\frac{ke^2}{r} = mv^2$, which simplifies the expression for total energy to $E = -\frac{1}{2}\frac{ke^2}{r}$.

A consequence of the Bohr postulate is that $v = n\frac{h}{2\pi rm}$. Substituting this into $\frac{ke^2}{r} = mv^2$ gives $\frac{ke^2}{r} = m\left(n\frac{h}{2\pi rm}\right)^2 \Rightarrow r = n^2\frac{h^2}{4\pi^2 ke^2 m}$. Substituting this radius into the formula for the total energy gives $E = -\frac{2\pi^2 k^2 e^4 m}{n^2 h^2}$. Putting in numerical values of the constants, we find that the energy, in units of eV, is given by $E = -\frac{13.6}{n^2}$.

This gives the discrete energy level structure we saw in Chapter 7.

Bohr's second postulate: Every time an electron makes a transition from a high energy state to a lower one, it emits one photon. The photon energy is equal to the difference in energy between the levels of the transition.

Since the energy of a photon is given by $E = hf = \frac{hc}{\lambda}$, it follows that in a transition from a state with $n = n_1$ to a state with $n = n_2$ we must have $\frac{1}{\lambda} = \frac{2\pi^2 k^2 e^4 m}{h^3 c}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$.

Thus the two Bohr postulates explained the atomic spectrum of hydrogen, a huge unsolved problem at the time.

Quantum physics

Nature of Science. The study of emission spectra from flames revealed characteristic lines that could be used to identify different elements. Observations of the spectrum of sunlight showed dark absorption lines. In the 1800s improved instruments allowed for more accurate measurements of the wavelengths of light corresponding to the bright and dark lines in spectra. But what was the reason for these lines? As we saw in Chapter **7**, a quantum model was needed to explain these patterns. The first of these quantum models was the Bohr model of the hydrogen atom, which could predict the wavelengths of the lines in the spectrum of hydrogen.

To explain the puzzling observations seen in the photoelectric effect, Einstein suggested that light existed as packets of energy called photons – a quantum model for light. Accepting the idea of a wave-particle duality revolutionised scientific thinking. Any moving particle could have wavelike characteristics! New ideas opened up new avenues for research, and led to the idea of the wave function for electrons in atoms, the uncertainty principle and probability functions. A whole new branch of physics was born – quantum physics.

12.5 The Schrödinger theory

Schrödinger's theory is the currently accepted basis for atomic and molecular structure. Unlike Bohr's theory, it can be applied to any atom, not just hydrogen.

An electron is described by a **wavefunction** ψ, a function of time and position much like an ordinary wave. The wavefunction here is a complex number, with the property that the value of $|\Psi|^2$ at any point gives the *probability* of finding an electron in a small volume ΔV in the neighbourhood of that point:

$$\text{Probability} \propto |\Psi|^2 \Delta V$$

In the Schrödinger theory, then, we can no longer think of the electron as localised in space. Its position is given by a probability function, a kind of 'electron cloud' around the nucleus.

The Schrödinger theory predicts discrete energy levels for the electrons in atoms. It also predicts the intensities of spectral lines, something the Bohr model does not do.

TEST YOURSELF 12.5

Figure 12.9 shows the variation with position of the wavefunction of an electron confined within a region of linear size 1.0×10^{-10} m. (The units on the vertical axis are arbitrary.)

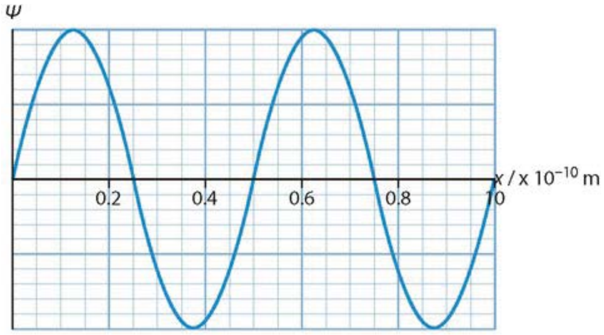


Figure 12.9

a State what is meant by a wavefunction.

b Use the graph to estimate **i** the momentum of the electron and **ii** the kinetic energy of the electron.

c State one position where it is highly *unlikely* for this electron to be found.

12.6 The Heisenberg uncertainty principle

The Heisenberg uncertainty principle states that it is not possible, in principle, to determine both the position and the momentum of an electron with infinite precision at the same time. The more precisely the position is known, the more imprecisely the momentum is known, and vice-versa:

$$\Delta x\,\Delta p \geq \frac{h}{4\pi}$$

Δx, Δp are the uncertainties in the measurements of position and momentum, and h is the Planck constant.

This principle also applies to measurements of energy and time: in measuring the energy of a state, a measurement that takes time Δt to complete results in an uncertainty ΔE in the measured value of the energy, such that:

$$\Delta E\,\Delta t \geq \frac{h}{4\pi}$$

The graph in Figure 12.10a is a one-dimensional illustration of the wavefunction of an electron with a well-defined wavelength.

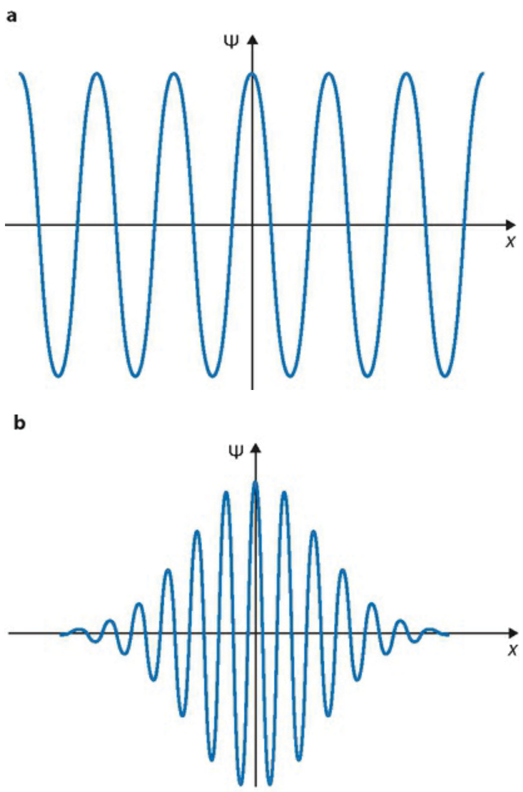


Figure 12.10

Since wavelength is related to momentum through the de Broglie formula $\lambda = \frac{h}{p}$, a well-defined wavelength implies a well-defined momentum. In turn, this implies a poorly-defined position – that is, we don't really know where this electron is.

For the electron whose wavefunction is illustrated in Figure 12.10b, it is possible to determine its position with relatively little uncertainty. In turn this implies a large uncertainty in its momentum.

The uncertainty principle may be used to make estimates of various quantities, as the next examples show.

Model Answer 12.2

An electron is confined within a hydrogen atom of linear size 10^{-10} m. Using the uncertainty principle, demonstrate that the kinetic energy of the electron is at least one eV, explaining your working.

hint

The size of the hydrogen atom gives an estimate of the uncertainty in the position of the electron.

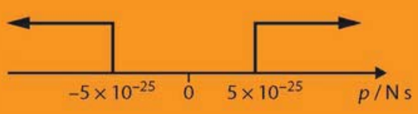


Figure 12.11

The uncertainty in the position of the electron is of order $\Delta x \approx 10^{-10}$m and so the uncertainty in the momentum is $\Delta p \approx \frac{h}{4\pi \Delta x} = \frac{6.6\times10^{-34}}{4\pi\times10^{-10}} \approx 5\times10^{-25}$Ns. That is, the momentum can be measured to be $p_0 \pm \Delta p$. The smallest size p_0 can have is zero, so the smallest possible magnitude of the electron's momentum is 5×10^{-25} Ns (Figure 12.11).

The energy of the electron is then at least:

$$E_k \approx \frac{(5\times10^{-25})^2}{2\times9\times10^{-31}} \approx 1.4\times10^{-19}\text{J} \approx \frac{1.4\times10^{-19}}{1.6\times10^{-19}} \approx 1\text{eV}.$$

TEST YOURSELF 12.6

In nuclei, nucleons exist in nuclear energy levels, and in atoms, electrons exist in atomic energy levels. The order of magnitude of the size of a nuclear energy level is 1 MeV, whereas an atomic energy level is of the order 1 eV. Use this information and the uncertainty principle to make an order-of-magnitude estimate of the ratio $\frac{\text{size of atom}}{\text{size of nucleus}}$.

TEST YOURSELF 12.7

The energy of an excited energy level of an electron is to be measured. The electron will stay in that excited level for less than 0.1 ns.

a What is the uncertainty in the measured value of the energy of the energy level?

b State and explain what your answer in a implies about the wavelengths of spectral lines.

hint

How is the wavelength of a photon in a transition calculated?

Tunnelling

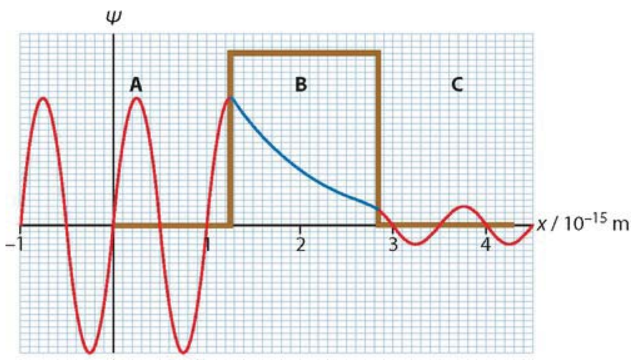


Figure 12.12

Consider the wavefunction of a proton approaching an energy barrier from region A, as illustrated in Figure 12.12. The barrier consists of a region of positive potential, which is superimposed on the wavefunction graph, in region B.

In classical theory, to make it over the barrier and appear in region C, the proton's total energy must be larger than eV, where V is the size of the potential barrier. If it isn't, the proton will be reflected back into region A. Region C, in other words, is classically forbidden to this proton.

But one of the most impressive phenomena of quantum mechanics, **tunnelling**, actually makes it possible for the proton to appear in region C. This is a consequence of the fact that particles have wave properties and are described by wavefunctions.

The Schrödinger theory requires that the wavefunctions of the proton in regions A, B and C *join smoothly together* (no jumps and no corners). As a consequence, the wavefunction cannot just drop to zero in region C. It is as if the wavefunction 'leaks' into region C.

Notice that the de Broglie wavelength in region C is the same as that in region A. This means that the proton has the same energy after going through the barrier!

The relative probability of finding a particle in region C is the ratio of the square of the wavefunction in region C to that in region A. Three factors affect this probability:

- the mass m of the particle
- the width w of the barrier
- the difference ΔE between the energy of the barrier and that of the particle.

The larger each of these quantities is, the smaller the transmission probability. So, everything else being equal, the transmission probability for an electron, for example, is greater than that for a proton.

Strange as it seems, the tunnelling phenomenon has very many practical applications, including in the so-called scanning tunnelling microscope (a microscope that can ‘see’ atoms) and the tunnel diode (a diode in which the current can be very quickly switched between on and off).

12.7 Nuclear radii and distance of closest approach

An alpha particle of electric charge 2*e* and kinetic energy E_k is directed head-on at a nucleus of atomic number *Z*. The alpha particle will come to rest for an instant a distance *d* from the centre of the nucleus, where $E_k = k\frac{(2e)(Ze)}{d} \Rightarrow d = k\frac{(2e)(Ze)}{E_k}$, and will then be repelled (Figure 12.13).

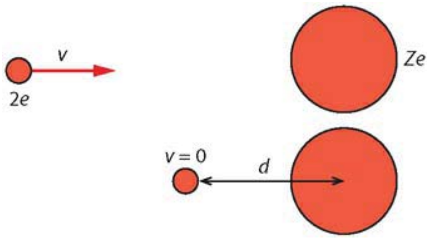


Figure 12.13

We see that the kinetic energy of the alpha particle gets transformed entirely to electrical potential energy at the point of closest approach.

Model Answer 12.3

Experiments show that the radius ***R* of a nucleus with nucleon number *A* is approximately 1.2 × 10^{-15} × $A^{1/3}$ m. Show that this implies that all nuclei have the same density and estimate this density.**

The density is $\rho = \frac{\text{mass}}{\text{volume}} \propto \frac{A}{(A^{1/3})^3} = \frac{A}{A}$, which is independent of A. Hence all nuclei have the same density. A nucleus of mass number *A* has an approximate mass of *A* u, so the density is $\rho = \frac{Au}{\frac{4\pi}{3}(1.2\times10^{-15}\times A^{1/3})^3} = \frac{3Au}{4\pi\times(1.2\times10^{-15})^3 A} = \frac{3\times1.661\times10^{-27}}{4\pi\times(1.2\times10^{-15})^3} \approx 2\times10^{17}\,\text{kgm}^{-3}$.

TEST YOURSELF 12.8

A proton (to be treated here as a point particle) is accelerated from rest by a potential difference *V*. The accelerated proton is directed at a nucleus of molybdenum $\left({}^{96}_{42}\text{Mo}\right)$. Given that the radius of a nucleus of mass number *A* is $R \approx 1.2 \times 10^{-15} \times A^{1/3}$ m, calculate the value of the accelerating potential if this proton just reaches the surface of the nucleus of molybdenum.

hint

What is the kinetic energy of the proton after acceleration? What becomes of this kinetic energy when the proton just stops on the surface of the nucleus?

Determining nuclear radii by electron scattering

It is possible to obtain estimates of nuclear radii by scattering electrons or neutrons from nuclei. The advantage of electrons is that the strong force does not act upon them and so they can approach the nucleus very closely. Neutrons also have an advantage because, being neutral, they can penetrate deep into matter and get very close to the nucleus. The scattering of electrons and neutrons from nuclei shows diffraction minima – that is, angles at which the number of scattered particles is small. These angles are given by $\sin\theta \approx \frac{\lambda}{b}$, where *b* is the diameter of the nucleus and *λ* is the de Broglie wavelength of the scattered particle.

Worked Example 12.2

In a neutron diffraction experiment, a beam of neutrons of energy 85 MeV is incident on a foil made out of lead, and the beam is diffracted. The first diffraction minimum is observed at an angle of 13° relative to the central position where most of the neutrons are observed. Estimate the radius of the lead nucleus.

The neutrons are diffracted from the lead nuclei, which act as ‘obstacles’ of size *b*. From our knowledge of diffraction, the first minimum is given by $\sin\theta \approx \frac{\lambda}{b}$ where *λ* is the de Broglie wavelength of the neutron. The mass of a neutron is $m = 1.67 \times 10^{-27}$ kg and, since its kinetic energy is 85 MeV, its wavelength is $\lambda = \frac{h}{p}$ where

$$p = \sqrt{2E_k m}$$
$$= \sqrt{2 \times 85 \times 10^6 \times 1.6 \times 10^{-19} \times 1.67 \times 10^{-27}}$$
$$= 2.13 \times 10^{-19}\,\mathrm{Ns}$$

Hence

$$\lambda = \frac{6.6 \times 10^{-34}}{2.13 \times 10^{-19}}\,\mathrm{m}$$
$$= 3.1 \times 10^{-15}\,\mathrm{m}$$

Therefore the diameter of the nucleus is estimated to be

$$b = \frac{3.1 \times 10^{-15}}{\sin 13^\circ}\,\mathrm{m}$$
$$= 14 \times 10^{-15}\,\mathrm{m}$$

This corresponds to a radius of 7×10^{-15} m.

12.8 Nuclear energy levels

Alpha and gamma particles produced in radioactive decay have discrete energies. This is evidence in support of the existence of nuclear energy levels. The protons and neutrons in a nucleus exist in discrete energy levels just as the electrons in atoms do. In a transition from one energy level to a lower one, the emitted alpha or gamma particle has a discrete energy equal to the energy difference between the levels involved.

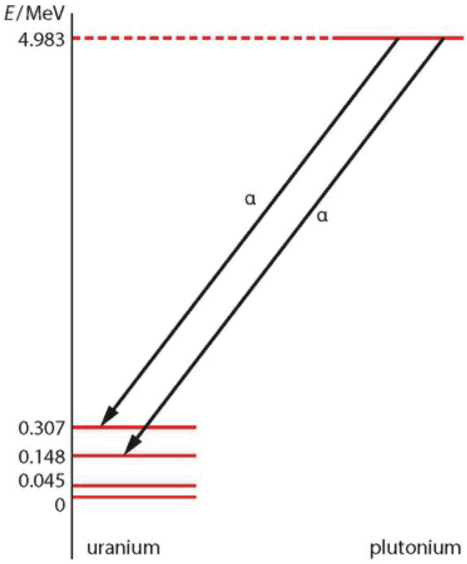


Figure 12.14

Two alpha decays are shown in Figure 12.14, both of them for the reaction $^{242}_{94}\mathrm{Pu} \rightarrow ^{4}_{2}\alpha + ^{238}_{92}\mathrm{U}$.

The energies released are 4.983 − 0.307 = 4.676 MeV and 4.983 − 0.148 = 4.835 MeV

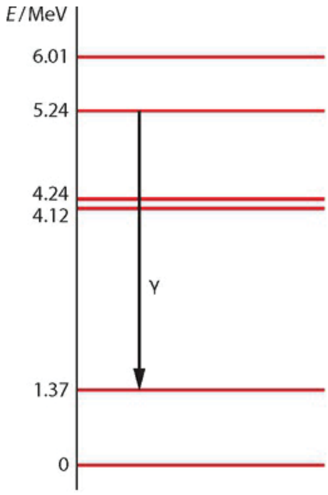


Figure 12.15

Figure 12.15 shows a gamma ray photon emitted in the decay of a magnesium nucleus $\left(^{24}_{12}\mathrm{Mg} \rightarrow ^{0}_{0}\gamma + ^{24}_{12}\mathrm{Mg}\right)$. The photon has energy 5.24 − 1.37 = 3.87 MeV.

In contrast to alpha and gamma energies, the energies of beta particles (i.e. electrons) are continuous. Shown in Figure 12.16 are two decays:

$$^{211}_{83}\mathrm{Bi} \rightarrow ^{0}_{-1}e + ^{211}_{84}\mathrm{Po} + ^{0}_{0}\overline{v}$$
$$^{211}_{84}\mathrm{Po} \rightarrow ^{4}_{2}\alpha + ^{207}_{82}\mathrm{Pb}$$

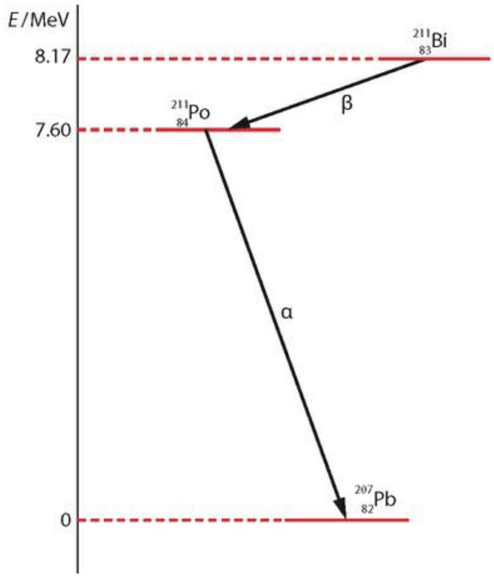


Figure 12.16

In β^- decay, an additional particle is produced: an anti-neutrino. The three reaction products then share the available energy – in many different ways, depending on their direction of motion.

The electron (the β^- particle) in Figure 12.16 would have an energy of approximately 8.17 − 7.60 = 0.57 MeV if there were no antineutrino produced, but in these decays the product electrons have energies that range continuously from 0 to a maximum of 0.57 MeV (Figure 12.17).

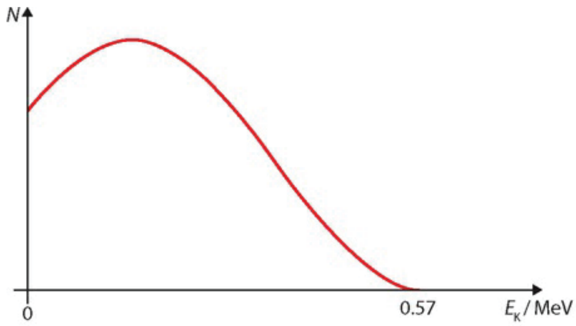


Figure 12.17

Annotated Exemplar Answer 12.3

a State the experimental evidence in support of nuclear energy levels. **[1]**

b Use the energy level diagram in Figure 12.14 to explain why, in the alpha decay $^{242}_{94}Pu \rightarrow {}^{4}_{2}\alpha + {}^{238}_{92}U$, photons are also expected to be emitted. **[2]**

a The energies of alpha, beta and gamma particles emitted in radioactive decay are discrete.

You are correct about alpha and gamma particles, but beta emission does not occur with discrete energy values.

b The alpha decays end up at an excited state of uranium which means that these alpha decays may be accompanied by gamma ray emission when the nucleus of uranium returns to a lower energy state.

A good answer here.

2/3

< Back to chapter

Annotated Exemplar Answer 12.3

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You are correct about alpha and gamma particles, but beta emission does not occur with discrete energy values.

b The alpha decays end up at an excited state of uranium which means that these alpha decays may be accompanied by gamma ray emission when the nucleus of uranium returns to a lower energy state.

A good answer here.

2/3

12.9 Deviations from Rutherford scattering

Rutherford scattering assumes that the only force acting between an incoming alpha particle and the nucleus is the electric force. This implies that as the energy of the alpha particles increases the number of particles scattered at a particular angle will decrease according to the red curve in Figure 12.18.

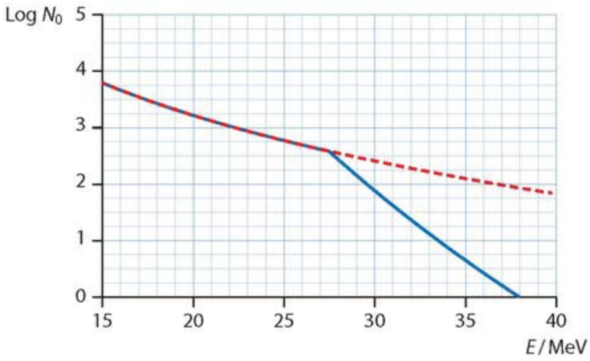

Figure 12.18

In reality, above a certain energy the number of scattered particles drops sharply, according to the blue curve. This is because the alpha particles are now so close to the nucleus that the strong nuclear force begins to act on the alpha particles, drawing some of them into the nucleus and therefore reducing the number that are scattered. Thus the presence of these deviations from perfect Rutherford scattering is evidence for the existence of the strong nuclear force.

12.10 The radioactive decay law

DEFINITIONS

RADIOACTIVE DECAY LAW The rate of decay of a nucleus is proportional to the number N of nuclei present that have not yet decayed: $\frac{dN}{dt} = -\lambda N$.

DECAY CONSTANT, λ The probability of decay per unit time.

ACTIVITY, A The number of decays per second. The initial activity is $A_0 = \lambda N_0$.

HALF-LIFE, $T_{1/2}$ The time taken for the activity of a sample to halve.

The radioactive decay law implies that $N = N_0 e^{-\lambda t}$ and $A = \lambda N_0 e^{-\lambda t}$, thus the number of nuclei that have not yet decayed after time t and the activity after time t are both given by exponential decay laws.

Recall that, after one half-life, the initial activity of a sample is reduced by a factor of 2. The decay constant and half-life are related by $\lambda T_{1/2} = \ln 2$, which can be derived as follows (you may be asked to derive this in an exam).

$$A = A_0 e^{-\lambda t}$$
$$\frac{A_0}{2} = A_0 e^{-\lambda T_{1/2}}$$
$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$
$$\lambda T_{1/2} = \ln 2$$

If we know the molar mass of a pure radioactive sample, we can determine N_0, the number of nuclei present, by measuring the mass of the sample. Measuring the initial activity, A_0, then allows determination of the decay constant (through $A_0 = \lambda N_0$) and hence the half-life (through $\lambda T_{1/2} = \ln 2$). (This method is useful for determining very long half-lives.)

Worked Example 12.3

The activity of a freshly prepared 50 g sample of strontium $\left({}^{90}_{38}\mathrm{Sr}\right)$ is 2.5×10^{14} Bq. Estimate the half-life of strontium.

The molar mass of strontium is 90 g mol^{-1} and so 50 g corresponds to $\frac{50}{90} = 0.556\,\mathrm{mol}$, and hence $0.556 \times 6.02 \times 10^{23} = 3.34 \times 10^{23}$ atoms of strontium. From $A_0 = \lambda N_0$ we find $2.5 \times 10^{14} = \lambda \times 3.34 \times 10^{23}$ or $\lambda = \frac{2.5\times10^{14}}{3.34\times10^{23}} = 7.48 \times 10^{-10}\,s^{-1}$. Finally, from $\lambda T_{1/2} = \ln 2$ we find $T_{1/2} = \frac{\ln 2}{7.48\times10^{-10}} = 9.27 \times 10^{8}\,s \approx 29\,\text{years}$.

TEST YOURSELF 12.9

The half-life of a particular isotope is 2.45 min. Calculate the fraction of the original activity of the isotope 1.00 min after it has been prepared.

TEST YOURSELF 12.10

An isotope X has a half-life of 2.0 min. It decays into isotope Y, which is stable. Initially no quantity of isotope Y is present. Determine the time interval after which the ratio of the numbers of Y nuclei to X nuclei is equal to 4.

TEST YOURSELF 12.11

An isotope X decays into an isotope Y that is itself unstable. Isotope Y decays to a stable isotope Z. Figure 12.19 shows the variation with time of the numbers of the nuclei of isotopes X and Y that have not yet decayed. Initially, the number of nuclei of isotope Y is zero.

hint

Why should you look beyond 2.5 min to calculate the half-life of Y?

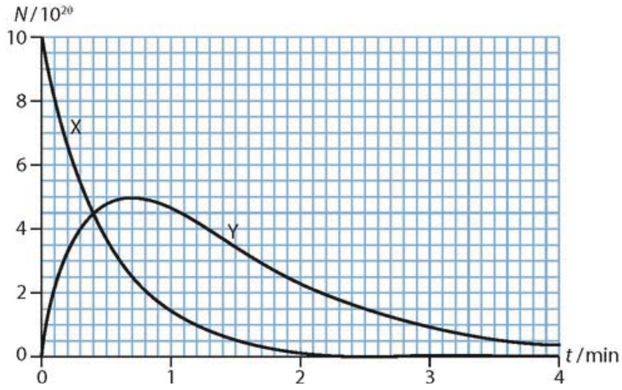

Figure 12.19

Estimate the half-lives of isotopes X and Y, explaining your method.

A RELATIVITY

This option covers the following topics:

- □ Frames of reference
- □ Lorentz transformations
- □ Electromagnetic puzzles
- □ Spacetime diagrams
- □ Relativistic mechanics
- □ The equivalence principle and its consequences
- □ Black holes

A.1 Frames of reference

To specify the times and positions of events we use reference frames.

DEFINITIONS

REFERENCE FRAME A coordinate system (three dimensions for space and one for time) and clocks at every point in space, used to record the time and position of an event. An event thus has four coordinates (x, y, z, t). For clarity we deal in this course with just a single space coordinate, x.

INERTIAL REFERENCE FRAME A reference frame that is not accelerating.

REST FRAME OF AN OBJECT The frame of reference in which the object is at rest.

PROPER LENGTH The length of an object as measured in its rest frame.

PROPER TIME INTERVAL The interval of time between two events that occur at the same point in space.

INVARIANT A quantity that has the same value in all reference frames.

An event is given the coordinates (x, t) in reference frame S. An observer in reference frame S′, which is moving at speed v relative to frame S, assigns coordinates (x', t') to the same event. A separate moving object (represented by the bird in this sketch) is measured to have speed u by an observer in S, and u' by an observer in S′ (Figure A.1).

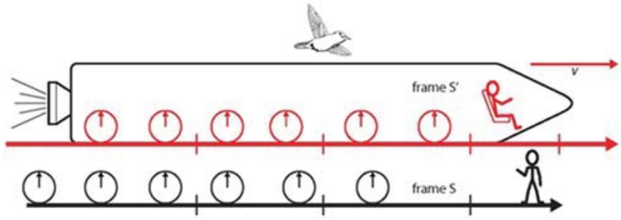

Figure A.1

How are the two sets of measurements related? In pre-relativity physics, the answer would be given by the **Galilean transformation** equations:

$t' = t$

$x' = x - vt$ or $x = x' + vt$

$u' = u - v$ or $u = u' + v$

According to the third of these equations, a light beam would be observed by observers in S and in S′ to be moving at different speeds. If S measured a speed $u = c$ for light, then S′ would measure a speed $u' = c - v$. Conversely, if S′ measured a speed c for light, S would measure $u = c + v$.

However, Maxwell’s laws of electromagnetism showed that c, the speed of light in a vacuum, is a universal constant, independent of the speed of its source.

Einstein solved the problem by trusting the laws of electromagnetism and changing the Galilean transformation equations. And this implied great changes in physics!

Einstein based his theory on two postulates:

- The speed of light in a vacuum is the same for *all inertial observers*.
- The laws of physics are the same for *all inertial observers*.

A consequence of these two postulates is that the Galilean transformation equations must be changed to the following, known as the Lorentz (or Lorentz–Einstein) transformation equations:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad \text{or} \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$x' = \gamma(x - vt) \quad \text{or} \quad x = \gamma(x' + vt')$$

$$u' = \frac{u-v}{1-\frac{uv}{c^2}} \quad \text{or} \quad u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is known as the **Lorentz gamma factor.**

Consider two events with coordinates (x_1, t_1) and (x_2, t_2) in S, and (x'_1, t'_1) and (x'_2, t'_2) in S′. Then the *differences* in these coordinates ($\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$ and so on) obey:

$$\begin{aligned} \Delta t' &= \gamma\left(\Delta t - \tfrac{v}{c^2}\Delta x\right) \\ \Delta x' &= \gamma(\Delta x - v\Delta t) \end{aligned} \quad \text{or} \quad \begin{aligned} \Delta t &= \gamma\left(\Delta t' + \tfrac{v}{c^2}\Delta x'\right) \\ \Delta x &= \gamma(\Delta x' + v\Delta t') \end{aligned}$$

Worked Example A.1

A frame S′ moves past frame S with constant velocity of 0.800 c. At the moment when the origins of the two frames coincide, clocks in both frames are set to zero. An event is determined to have coordinates (x = 480m, t = 6.00 s) in frame S. Determine the coordinates of this event in frame S′.

This is a straightforward application of the transformation equations:

$$\begin{aligned} t' &= \gamma\left(t - \frac{v}{c^2}x\right) \\ &= \frac{5}{3}\left(6.00 - \frac{0.80c}{c^2} \times 480\right) \\ &= \frac{5}{3}\left(6.00 - \frac{0.80}{3.0\times10^8} \times 480\right) \\ t' &= 10.0\,\text{s} \end{aligned} \quad \text{and} \quad \begin{aligned} x' &= \gamma(x - vt) \\ &= \frac{5}{3}(480 - 0.80c \times 6.00) \\ &= \frac{5}{3}(480 - 0.80 \times 3.0 \times 10^8 \times 6.00) \\ x' &= -2.4 \times 10^9\ \text{m} \end{aligned}$$

Worked Example A.2

A train of proper length 120 m moves to the right with speed 0.98c relative to the ground. A photon is emitted from the left end of the train and arrives at the right end. Determine the time this takes according to ground observers.

Here we have two events: event 1 is the emission of the photon from the left end and event 2 is the arrival of the photon at the right end. For an observer in the train, $\Delta x' = 120m,\ \Delta t' = \frac{120}{c} = 4.0 \times 10^{-7}$ s. The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.98^2}} = 5.0$, so for the ground observers:

$$\begin{aligned} \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) \\ &= 5.0 \times \left(4.0 \times 10^{-7} + \frac{0.98}{3.0\times10^8} \times 120\right) \\ &= 4.0 \times 10^{-6}\ \text{s} \end{aligned}$$

TEST YOURSELF A.1

In a certain inertial reference frame S, lightning strikes at a position $x = 3 \times 10^8$m. An observer is standing at the position $x = 6 \times 10^8$m. Light from the strike arrives at this observer when his clock shows t = 3s. What are the coordinates of the event ‘lightning strikes’ according to observers in this frame?

Time dilation

Consider a clock that is at rest at position x' = 0, the origin of the frame S′. Take event 1 to be the first tick of the clock and event 2 to be the second tick of the clock. The time between ticks is Δt in the S frame and $\Delta t'$ in frame S′. Then:

$$\begin{aligned} \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) \\ &= \gamma\left(\Delta t' + \frac{v}{c^2} \times 0\right) \\ \Delta t &= \gamma \Delta t' \end{aligned}$$

So if $\Delta t'$ = 1.00 s and γ = 5.0, then Δt = 5.00 s: ‘moving clocks run slow’. Here, the interval $\Delta t'$ is a proper time interval because the two events happen at the same point in space, $\Delta x'$ = 0. In general, (time interval) = γ × (proper time interval).

Length contraction

Consider a rod of length L_0 as measured by an observer in frame S′. What length does an observer in S measure for this rod? To measure the length of the moving rod, the positions of its ends must be recorded at the *same time* by an observer in S. Hence, using $\Delta x' = \gamma(\Delta x - v\Delta t)$ with $\Delta x' = L_0$ and Δt = 0, we find

$$\begin{aligned} \Delta x' &= \gamma(\Delta x - v\Delta t) \\ L_0 &= \gamma(\Delta x - v \times 0) \\ \Delta x &= \frac{L_0}{\gamma} \end{aligned}$$

So if L_0 = 1.00 m and γ = 5.00, then Δx = 0.20m: 'moving rods get shorter'.

Model Answer A.1

A spacecraft moving at 0.80c relative to Earth passes the Earth on its way to a planet which is a distance of 40 ly away as measured by Earth observers. Calculate the time taken to arrive at the planet according to a Earth observers and b spacecraft observers. The spacecraft continues to move past the planet. As it passes the planet it sends a radio signal to Earth. Calculate the time taken for the signal to arrive at Earth according to c Earth observers and d spacecraft observers.

The gamma factor for a speed of 0.80c is 5/3.

a The time for an Earth observer is simply $\frac{40\text{ly}}{0.80c} = 50\text{ yr}$

b We provide two solutions here. According to a spacecraft observer, the distance separating it and the planet is 40 ly contracted to $\frac{40\text{ ly}}{\frac{5}{3}} = 24\text{ ly}$. Hence the time for the planet to arrive at the spacecraft is $\frac{24\text{ ly}}{0.80c} = 30\text{ yr}$. Alternatively, the time interval between leaving Earth and arriving at the planet is a proper time interval for spacecraft observers. Therefore, $\text{proper time} = \frac{50\text{ ly}}{\frac{5}{3}} = 30\text{ yr}$

c For Earth observers the time is $\frac{40\text{ ly}}{c} = 40\text{ yr}$

d Let T be the time for the signal to arrive at Earth according to spacecraft clocks. According to spacecraft observers the Earth is moving away at 0.80c. The signal (moving at c) has to cover the distance originally separating the Earth and the spacecraft (24 ly according to spacecraft observers) plus the additional distance that the Earth will move away in time T(i.e. 0.80cT). Therefore, $cT = 24 + 0.80cT \Rightarrow T = 120$ yr.

Electromagnetic puzzles

Figure A.2 shows part of a wire at rest in a lab in which a current flows to the left (electrons move to the right). A positive charge outside the wire moves to the right at the same speed as the electrons. Consider the force that will be experienced by this positive charge.

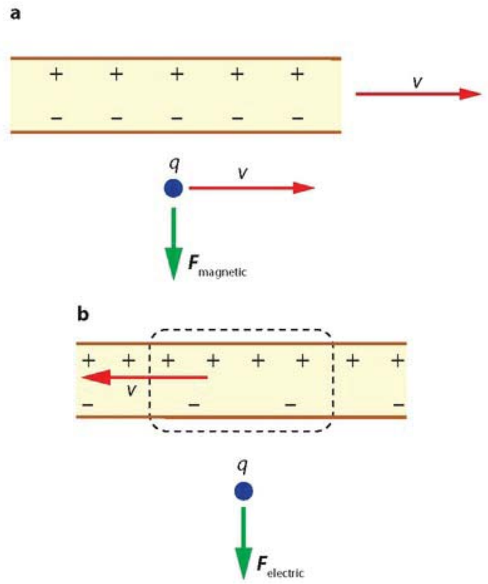

Figure A.2

According to an observer at rest in the lab:

- There is a magnetic field created by the current.
- The charge moves with speed v.
- So there is a magnetic force $F = qvB$ on this charge, directed as shown in Figure A.2a.

According to an observer moving along with the charge:

- The charge q and the electrons in the rod are at rest.
- The positive charges in the wire move to the left with speed v.
- There is a magnetic field created by the positive charges but q is at rest, as shown in Figure A.2b, so there can be no magnetic force since the charge does not move.

So is there a force on the charge? In Figure A.2b the distance between positive charges is smaller and the distance between electrons larger than the distances in the first diagram. This has to be the case if length contraction holds. The separation between the positive charges is a length that moves past the observer who is moving in the frame of q, and so it must contract. The distance between the electrons, which is a moving length in the lab frame, is at rest in this frame, so it must be bigger. The effect of this is that, as far as the observer in the frame of q is concerned, there is a higher density of positive charge in the nearby wire than of negative charge. Thus this observer observes an *electric* force of repulsion.

So, because of relativity, a field that appears to be magnetic in one frame may be wholly or partly electric in another frame, and vice-versa.

Invariants

Consider an event that is measured to have coordinates (x, t) in S and (x', t') in S′. As we know, these coordinates are different in the two frames; the observers disagree about the space and time coordinates. But it can be shown that all observers agree that $x^2 - c^2t^2 = x'^2 - c^2t'^2$ (see the textbook for a proof of this). The quantity $x^2 - c^2 t^2$ is known as an **invariant:** it is the same in all frames. Other invariants include mass and electric charge.

A.2 Experimental evidence for relativity

Muons are unstable particles and decay to electrons. Muons may be created in the atmosphere when cosmic rays from the Sun interact with air molecules.

Consider muons created at a height of 1 km from the Earth's surface (a height measured by Earth observers). Assume that the speed of the muons is 0.95*c* towards the Earth's surface.

The *proper* lifetime of a muon is 2.2×10^{-6} s. The gamma factor for a speed of 0.95*c* is $\gamma = 3.2$.

Without relativity, the distance travelled by muons before most of them would have decayed would be $0.95 \times 3 \times 10 \times 2.2 \times 10^{-6} = 630$ m. So most of them would have decayed before reaching the Earth's surface.

But the picture is more complex when relativity is taken into account.

According to an Earth observer:

The muon lifetime is time-dilated to $3.2 \times 2.2 \times 10^{-6} = 7.0 \times 10^{-6}$ s.

In this time muons travel a distance of $0.95 \times 3.0 \times 10^{8} \times 7.0 \times 10^{-6} \approx 2.0$ km.

Hence most muons make it to the Earth's surface before decaying. This is evidence for time dilation.

According to an observer moving with the muons:

The Earth's surface is moving upward towards the muons at a speed of 0.95*c*. The distance of 1 km is length-contracted to $\frac{1000}{3.2} = 310\text{m}$. In the course of a muon lifetime (2.2×10^{-6} s), the Earth's surface moves a distance of $0.95 \times 3 \times 10^{8} \times 2.2 \times 10^{-6} = 630$ m.

Hence by the time the Earth surface arrives at the position of the muons, most muons have not decayed. This is evidence for length contraction.

Velocity addition and the speed of light

Consider a frame of reference S′ that moves with speed *v* relative to another frame of reference S. The speed of an object is measured to be *u* by an observer in S, and *u*′ by an observer in S′. The two speeds are related by $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ or $u = \frac{u'-v}{1+\frac{u'v}{c^2}}$.

If the 'object' is a ray of light, moving at $u = c$, then both S and S′ must measure the same speed. Indeed, $u' = \frac{c-v}{1-\frac{cv}{c^2}} = c$, as it should be.

TEST YOURSELF A.2

Two spacecraft, A and B, are approaching each other head-on. The speed of A as measured by an observer on the ground is 0.80 c. The speed of B relative to the ground is 0.40 c. Calculate the speed of B as measured by an observer on A.

A.3 Spacetime diagrams

DEFINITIONS

SPACETIME The four-dimensional continuum (three space dimensions and one time dimension) in which physical phenomena take place.

SPACETIME DIAGRAM A graph with time *t* on the vertical axis (usually scaled to ct, giving it units of distance) and distance *x* on the horizontal axis, useful for representing motion and events.

WORLDLINE A curve on a spacetime diagram, consisting of a sequence of events showing the position of a particle as time passes.

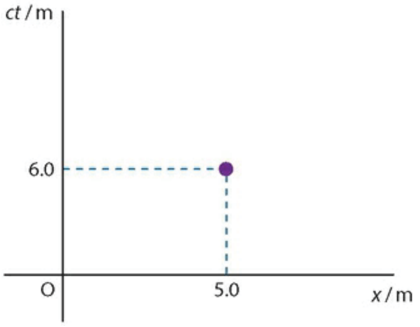

Figure A.3

The dot in Figure A.3 represents an event with coordinates x = 5.0m and ct = 6.0m (which gives

$$t = \frac{6.0}{3.0 \times 10^8} = 2.0 \times 10^{-8}\mathrm{s}).$$

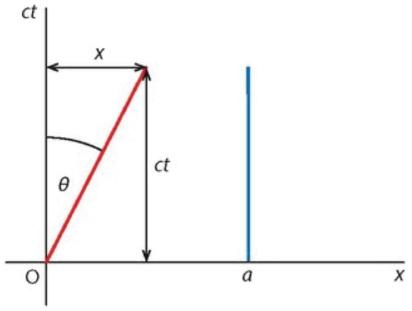

Figure A.4

In Figure A.4, think of the red line as a sequence of points, each representing the position of a particle at a different time. As time increases, the position also increases, so this worldline indicates that this particle is moving. The speed of the particle is given by

$\nu = \frac{\text{distance}}{\text{time}} = \frac{x}{t} = c\frac{c}{ct} = c\tan\theta,\ so\, tan\theta = \frac{\nu}{c}.$

The blue line shows that at different times the position of the particle is the same, so this worldline indicates that this particle is at rest.

Photons travel at the speed of light, so a photon has a worldline that makes an angle of 45° with the axes (Figure A.5).

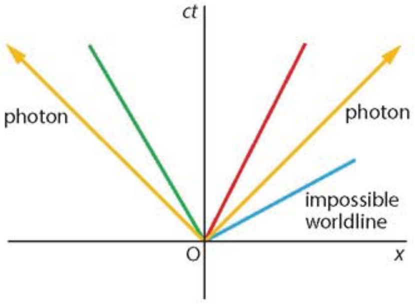

Figure A.5

An accelerating particle would have a curved worldline. The blue worldline in this figure is impossible, because it corresponds to a speed greater than that of light.

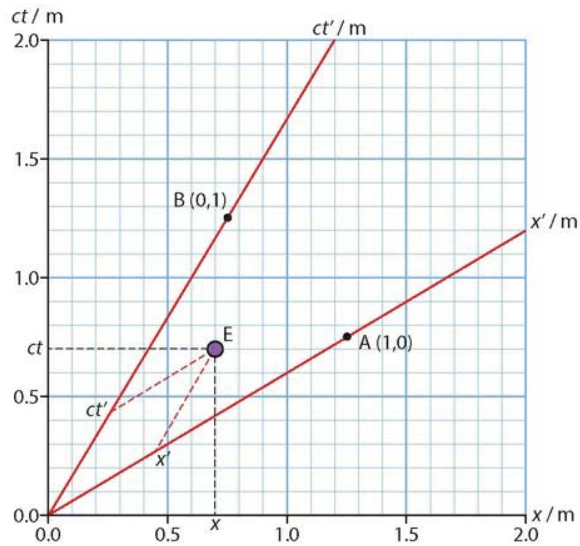

Figure A.6

The spacetime diagram in Figure A.6 shows two reference frames: the black axes correspond to frame S, and the red axes to frame S′ which is moving to the right. Notice that the scales on the S and S′ axes are different from one another.

The purple dot represents an event E. Its coordinates in the S frame are indicated by the dashed black lines parallel to the black axes. The coordinates in S′ are indicated by the red dashed lines parallel to the red axes.

The worldline of the origin shows how fast S′ is moving relative to the S frame. From the slope of this worldline, it travels 0.90 m in

$$ct = 1.50\,\mathrm{m\ or}\ t = \frac{1.50}{3.0 \times 10^{8}} = 0.50 \times 10^{-8}\,\mathrm{s,\ so}\ \nu = \frac{0.90}{0.50 \times 10^{-9}} = 1.8 \times 10^{8}\,\mathrm{ms}^{-1} = 0.60c.$$

Creative and critical thinking

Nature of Science. The theory of general relativity is an almost magical theory in which the energy and mass content of spacetime determines the geometry of spacetime. In turn, the kind of geometry spacetime has determines how the mass and energy of the spacetime move about. To connect these ideas in the theory of general relativity, Einstein used intuition, creative thinking and imagination. Initial solutions of Einstein's equations suggested that no light could escape from a black hole. Then a further imaginative leap – the development of quantum theory – showed an unexpected connection between black holes and thermodynamics.

Worked Example A.3

Using the Lorentz transformations, write expressions for the S-frame coordinates of events A and B in Figure A.6, using the S′-frame coordinates shown in the figure.

For A, $x' = 1\mathrm{m}$ and $ct' = 0$. Then $x = \gamma(x' + vt') = \gamma$ and $ct = \gamma\left(ct' + \frac{v}{c}x'\right) = \frac{\gamma v}{c}$.

For B, $x' = 0$ and $ct' = 1\mathrm{m}$. Then $x = \gamma(x' + vt') = \frac{\gamma v}{c}$ and $ct = \gamma\left(ct' + \frac{v}{c}x'\right) = \gamma$.

Model Answer A.2

a Using two copies of the spacetime diagram in Figure A.6, explain why:

i a rod of proper length 1 m at rest in frame S' is measured to be shorter than 1 m in frame S

ii a rod of proper length 1 m at rest in frame S is measured to be shorter than 1 m in frame S′.

b By using another two copies of the same diagram, explain why:

i a clock at rest at the origin of frame S′ is measured to be ticking slower in S

ii a clock at rest in S is measured to be ticking slower in S′

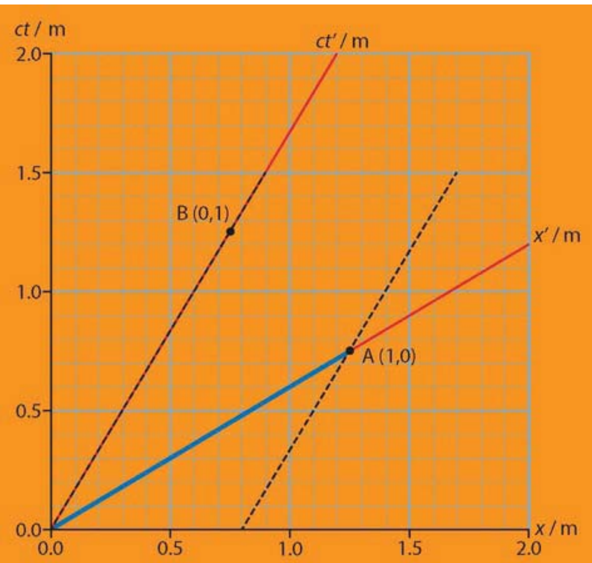

Figure A.7

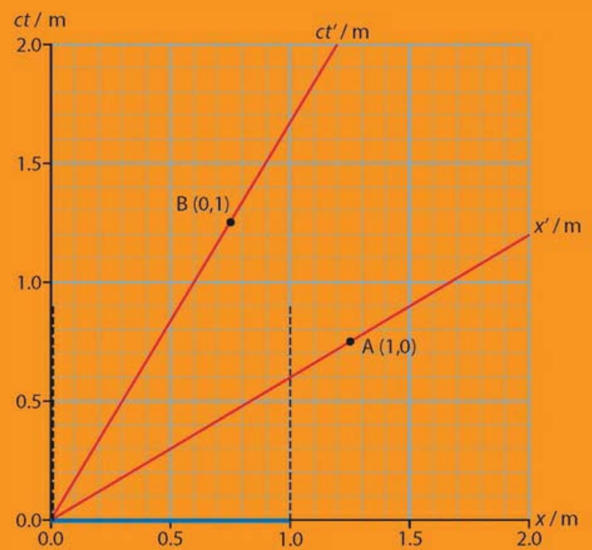

Figure A.8

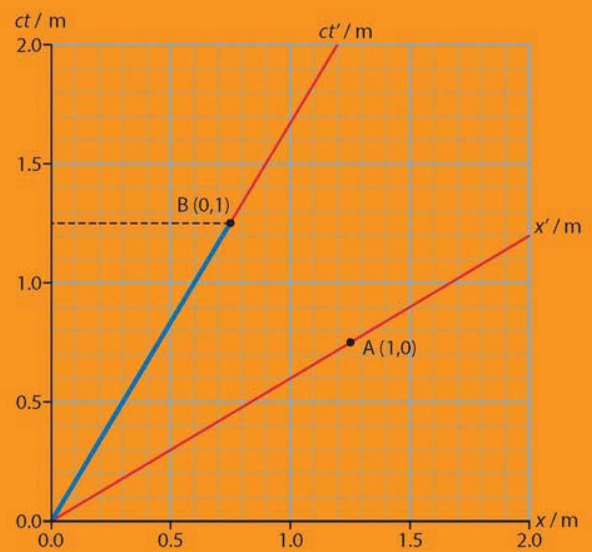

Figure A.9

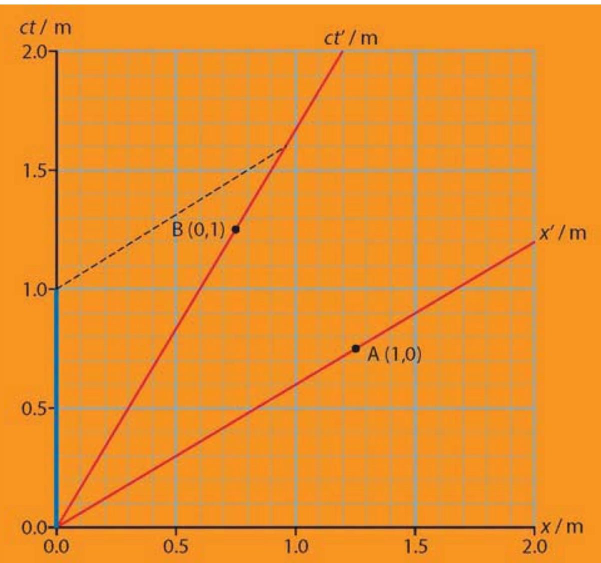

Figure A.10

Simultaneity

Two events are said to be *simultaneous* if they occur at the same time as measured by a particular observer. But if these events occur at different points in space, they will *not* be simultaneous for all other observers. Only if they occur simultaneously *and at the same point in space* will they be simultaneous for all observers.

Here is a *gedanken* (thought) experiment. A train moves to the right with speed v relative to the ground. Light signals are emitted from each end of the train, towards the middle of the train (Figure A.11).

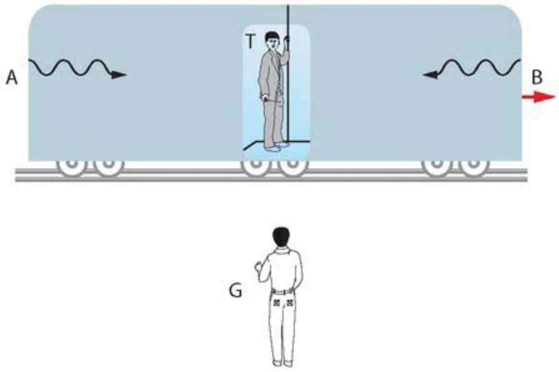

Figure A.11

These signals are emitted simultaneously according to an observer T standing in the middle of the train. Are they emitted simultaneously according to observer G on the ground?

Since the two signals are emitted at different points in space, they will not be simultaneous for the ground observer. To determine which happened first, we can use three different methods: with a spacetime diagram, or with or without the Lorentz transformations.

Spacetime diagram

This is the easiest way to answer the question. The emissions from A and B are denoted by events 1 and 2 respectively. Draw lines parallel to the x-axis and see where they intersect the ct-axis (Figure A.12). Clearly $t_1 < t_2$ so event 1 occurs first.

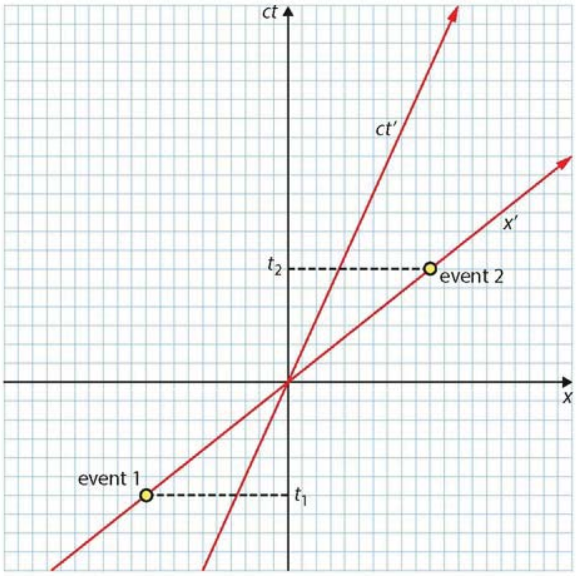

Figure A.12

Using Lorentz transformations

The two events are simultaneous in frame S′ ($\Delta t' = 0$) and are separated by a distance $\Delta x' = L_0$, the proper length of the train. Then, in frame S:

$$\Delta t = t_2 - t_1 = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)$$
$$= \gamma\left(0 + \frac{v}{c^2}\Delta x'\right) = \frac{\gamma v}{c^2}L_0 > 0$$

So we see that $t_2 > t_1$ – that is, event 1 occurs before event 2.

Without equations or spacetime diagram

For observer T in the train:

- Both light signals travel towards T at the same speed, *c*.
- Both distances to T are the same.
- The signals were emitted at the same time and hence will arrive at the same time.

For observer G on the ground:

- Both light signals travel towards T at the same speed, *c*.
- Because the arrivals of the signals are simultaneous for T and occur at the same point in space, they are also simultaneous for G.
- Because T is moving away from the signal source at the left end of the train, in order for the signals to arrive at the same time the signal from the left end must have been emitted first.

Annotated Exemplar Answer A.1

A ground observer G is halfway between two buildings. According to G, a lightning strike hits each building at the same time, and at exactly the moment when a rocket passes G moving to the right. Determine, using a spacetime diagram, which building was hit first according to the rocket observer R. Confirm your conclusion with physical reasoning. **[5]**

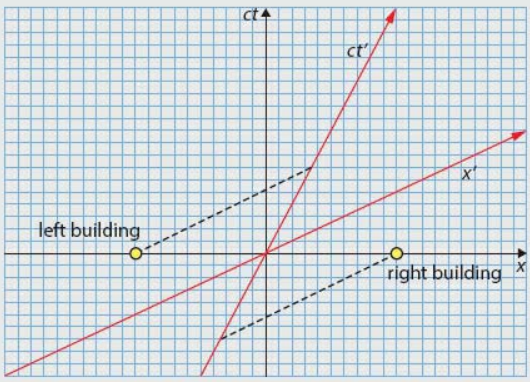

Figure A.13

Draw lines parallel to the primed *x*-axis to get the answer.

The right building was hit first.

Physical reasoning: the rocket approaches the building to the right and so will receive the light first. This means the right building was hit first.

The simultaneous events are the reception by G of the two light signals, and because in addition these occur at the same place (G's location) they are also simultaneous according to R. We are not concerned about when the light reaches R. As seen by R, observer G is moving to the left, away from the strike at the right building. R concludes that, since light from both strikes moves at the same speed, for the light to arrive at G simultaneously the right building must have been struck first.

You need to know how to work with spacetime diagrams. This diagram is clearly drawn and well labelled, and the right lines have been drawn to answer the question.

The interpretation of the diagram is correct. Make sure you know why the lines show that the right building was hit first. You don't need to give more detail to gain the marks, as the question doesn't ask for an explanation.

2/5

TEST YOURSELF A.3

The buildings in Figure A.13 are separated by 900 m (as seen by observer G). Determine how much earlier the right building was hit by lightning according to the rocket observer R, who is moving to the right at a speed of 0.80c.

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Annotated Exemplar Answer A.1

A ground observer G is halfway between two buildings. According to G, a lightning strike hits each building at the same time, and at exactly the moment when a rocket passes G moving to the right. Determine, using a spacetime diagram, which building was hit first according to the rocket observer R. Confirm your conclusion with physical reasoning. **[5]**

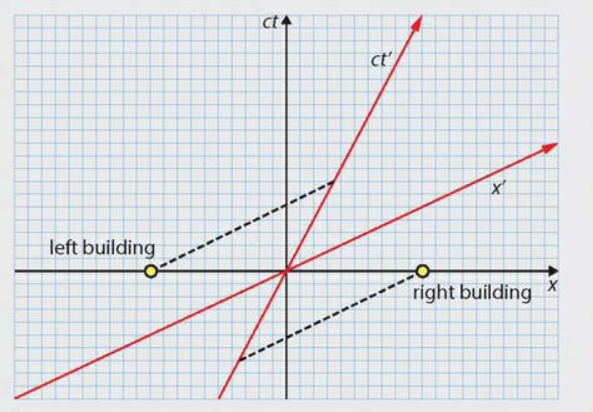

Figure A.13

Draw Lines parallel to the primed x-axis to get the answer.	*You need to know how to work with spacetime diagrams. This diagram is clearly drawn and well labelled, and the right Lines have been drawn to answer the question.*
The right building was hit first.	*The interpretation of the diagram is correct. Make sure you know why the Lines show that the right building was hit first. You don’t need to give more detail to gain the marks, as the question doesn’t ask for an explanation.*
Physical reasoning: the rocket approaches the building to the right and so will receive the light first. This means the right building was hit first.	*The simultaneous events are the reception by G of the two light signals, and because in addition these occur at the same place (G’s Location) they are also simultaneous according to R. We are not concerned about when the Light reaches R. As seen by R, observer G is moving to the Left, away from the strike at the right building. R concludes that, since Light from both strikes moves at the same speed, for the Light to arrive at G simultaneously the right building must have been struck first.*

A.4 The twin paradox

We have seen that the effect of time dilation is a symmetric effect: if you move relative to me I say that your clocks are running slow compared to mine but you say that my clocks are running slow compared to yours.

But a new issue arises when two clocks that are initially at the same place and showing the same time – say, zero – are separated and then returned to the same place. Suppose one clock moves away at a relativistic speed, then suddenly reverses direction and comes back to its starting place next to the clock that stayed behind, and the readings of the two clocks are compared. What do they show?

In the 'twin paradox' version of this story, the clocks are replaced by twins. Jane stays on Earth and claims that since her twin brother Joe is the one who moved away, time must have moved more slowly for him and so he must be younger on his return. But Joe can claim that it is Jane and the Earth that moved away and then came back so, Jane should be younger. Who is actually the younger of the two when the twins are reunited?

The key to resolving this paradox is that Jane has been in the same *inertial* frame throughout. Joe had to accelerate at some point in the trip, either by stopping to turn around or by going in a circle. This means that he was in a *different* inertial frame on his return. During the changeover from one frame to the other he must have experienced acceleration and forces which Jane never did. Joe's *acceleration* makes the situation asymmetric, and it is Joe who is younger on his return to Earth.

To understand this quantitatively, suppose that Joe leaves Earth in a rocket at a speed of 0.60c. He travels to a distant planet a distance of 3.0ly away (as measured by Earth observers) and returns.

The gamma factor corresponding to this relative speed is 1.25. From Jane's point of view, Joe's outbound trip will take $\frac{3.0}{0.6c} = 5\ \mathrm{yr}$ and another 5 yr to return, so she will have aged by 10 years when her brother returns. For Joe, time is running slowly by the gamma factor, so he will age by $\frac{5.0}{1.25} = 4\ \mathrm{yr}$ on the outward trip and another 4 on the way back. He will have aged by 8 years.

We can show all of this on the rather complicated spacetime diagram in Figure A.14.

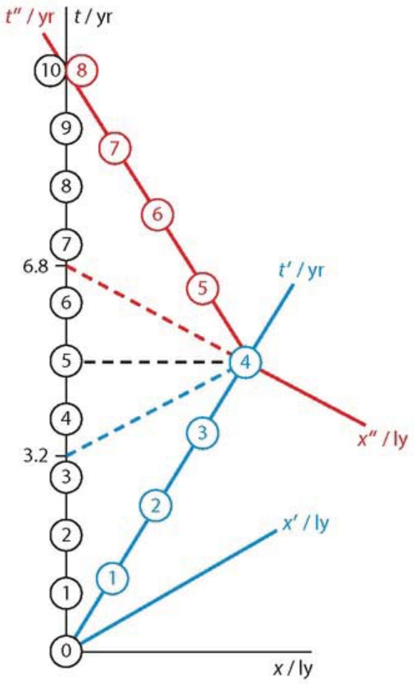

Figure A.14

The black frame, S, is Jane's. The blue frame, S′, is Joe's on the way out, and the red frame, S″, is Joe's on the way back. The circles show clock readings in these frames.

We see that just before Joe turns around and changes frame, the Earth clock shows 5 years and Joe's clock shows 4 years. When Joe's clock shows 4 years, *he* measures (by drawing a line parallel to the x'-axis) that Earth clocks show 3.2 years. (remember that time dilation is symmetric.)

As soon as Joe changes frame, he measures (by drawing a line parallel to the x''-axis) that Earth clocks show 6.8 years.

The sudden switch from to another frame seems to have created an additional time interval of $10 - 2 \times 3.2 = 3.6$ years. This is not mysterious and nothing strange has happened to either Jane or Joe during this time. Rather it is a consequence of the change of frame: the 'now' of frame S′ is found by drawing lines parallel to the x'-axis. The 'now' of frame S″ is found by drawing lines parallel to the x''-axis. That is, the meaning of 'now' is different in each frame.

A.5 Relativistic mechanics

DEFINITIONS

RELATIVISTIC MOMENTUM $p = \gamma mv$. At low speeds, $\gamma = 1$ and the definition then agrees with Newtonian physics.

RELATIVISTIC TOTAL ENERGY The famous formula that can be interpreted as showing the equivalence of mass and energy: $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ or $E = \gamma mc^2$.

REST ENERGY The energy needed to create a particle from the vacuum. In the frame of reference where the particle is at rest (its rest frame) there is still energy in the particle: $E_0 = mc^2$, since $\gamma \approx 1$.

RELATIVISTIC KINETIC ENERGY The total energy minus the rest energy, $E_k = E - mc^2 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$. For low speeds, this formula approximates to $E_k \approx \frac{1}{2}mv^2$.

REST MASS The mass of a particle as measured in its rest frame.

Relativistic energy and momentum are related by $E^2 = p^2c^2 + (mc^2)^2$. Notice that combining $p = \gamma mv$ with $E = \gamma mc^2$ gives $v = \frac{pc^2}{E}$.

We seldom use the numerical value of c in calculations of energy and momentum, instead expressing mass in units of MeVc^{-2} and momentum in units of MeVc^{-1}. For example, the mass of a proton is $m = \frac{E_0}{c^2} = 938\ \text{MeV}c^{-2}$ and the momentum of a proton of total energy E = 1250 MeV is (using $E^2 = p^2c^2 + (mc^2)^2$) $pc = \sqrt{1250^2 - 938^2}\ \text{MeV} = 826\ \text{MeV}$, giving p = 826 MeVc^{-1}.

The speed of light as an upper limit to speed

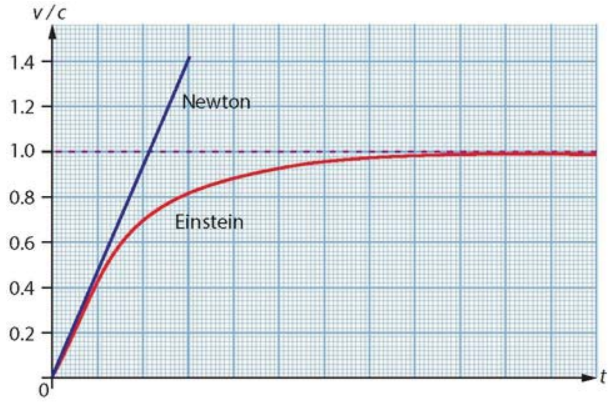

Figure A.15

Suppose a constant force is applied to a body initially at rest. As Figure A.15 shows, in Newtonian mechanics the speed will increase without limit, while in relativistic mechanics the body never reaches the speed of light (dashed line). Notice that the curves are identical at low speeds.

It may be useful to say more about this in an exam. You should be able to explain why a particle with non-zero rest mass cannot reach the speed of light by referring to the infinite amount of energy required $E \to \infty$ as $v \to c$), which makes it impossible; see Figure A.16.

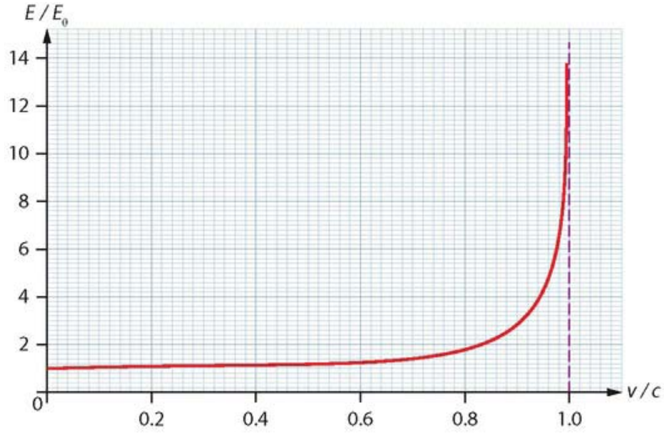

Figure A.16

Accelerating potentials

If a charge q is accelerated by a potential difference V, then $qV = \Delta E_k$, where ΔE_k is the change in the particle's kinetic energy. Following are a few standard relativistic mechanics questions.

TEST YOURSELF A.4

Calculate the speed of a proton (rest mass 938 MeVc^{-2}) whose momentum is p = 2800 MeVc^{-1}.

TEST YOURSELF A.5

Calculate the potential difference required to accelerate an electron (rest mass 0.511 MeVc^{-2}) to a speed of 0.98c.

TEST YOURSELF A.6

Calculate the kinetic energy of a proton (rest mass 938 MeVc^{-2}) of speed 0.950c.

TEST YOURSELF A.7

Calculate the momentum of a muon (μ^{-}, rest mass 106 MeVc^{-2}) of total energy 540 MeV.

TEST YOURSELF A.8

Calculate the speed of a tau particle (τ^{-}, rest mass 1800 MeVc^{-2}) whose total energy is 6500 MeV.

Annotated Exemplar Answer A.2

The rest mass of the ϕ meson is about double that of the η meson. The speed of a ϕ meson is half that of an η meson. Compare the momenta of the two particles. **[2]**

Since momentum is the product of mass times velocity, the two particles have the same momenta.

0/2

The student would be correct in pre-relativity physics. But in relativity there is the gamma factor to worry about in the definition of momentum.

TEST YOURSELF A.9

a Show that the speed of a particle of total energy E and momentum p is given by $v = \frac{pc^2}{E}$. Use this formula to deduce that a massless particle always moves at the speed of light. A particular particle has total energy E = 654 MeV and momentum p = 640 MeVc^{-1}. Calculate:

b the speed of this particle

c the rest mass of the particle.

TEST YOURSELF A.10

A proton is accelerated from rest by a potential difference 3.2×10^9 V. Calculate the speed and momentum of the accelerated proton.

hint

Recall that $qV = \Delta E$. You will have to perform this calculation in most exam papers.

TEST YOURSELF A.11

Calculate the speed of a proton (rest mass 938 MeVc^{-2}) whose momentum is p = 2.40 GeVc^{-1}.

Model Answer A.3

A ψ meson has rest mass 3100 MeVc^{-2}. It decays at rest in a lab into a muon–anti-muon pair. The rest mass of both the muon and the anti-muon is 106 MeVc^{-2}. Determine the momentum of the muon relative to the lab.

The ψ meson is at rest, so its momentum is zero. Therefore the muon and anti-muon have equal and opposite momenta. Let the magnitude of the momentum of the muon be p. The energy of the muon is $\sqrt{106^2 + p^2c^2}$ and conservation of energy states that:

$$3100\text{MeV} = 2\sqrt{106^2 + p^2c^2}$$

$$3100^2\text{MeV}^2 = 4(106^2 + p^2c^2)$$

$$p^2c^2 = \left(\frac{3100^2}{4} - 106^2\right)\text{MeV}^2$$

$$p = 1.55 \times 10^3\text{MeVc}^{-1} = 1.55\text{GeVc}^{-1}$$

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Annotated Exemplar Answer A.2

The rest mass of the ϕ meson is about double that of the η meson. The speed of a ϕ meson is half that of an η meson. Compare the momenta of the two particles. **[2]**

Since momentum is the product of mass times velocity, the two particles have the same momenta.

The student would be correct in pre-relativity physics. But in relativity there is the gamma factor to worry about in the definition of momentum.

0/2

A.6 General relativity

The equivalence principle

The equivalence principle states that it is impossible to distinguish the effect of acceleration from the effect of gravitation. There are two ways to state this principle (you should be able to state both versions):

- **Version 1:** A frame of reference moving at constant velocity in outer space (i.e. far from all masses) is equivalent to a frame of reference that is falling freely in a gravitational field (Figure A.17).

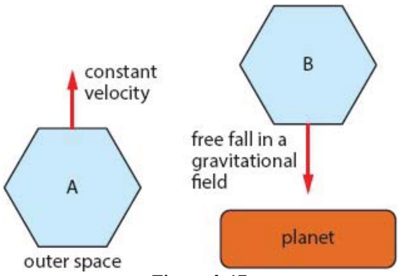

Figure A.17

- **Version 2:** A frame of reference undergoing an acceleration a in outer space (i.e. far from all masses) is equivalent to a frame of reference that is at rest in a gravitational field of field strength $g = a$ (Figure A.18).

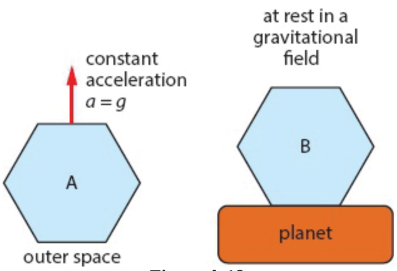

Figure A.18

In version 2, an object released in an accelerating frame of reference in outer space will *appear* to fall to the floor because the floor is accelerating towards it.

Consequences of the equivalence principle

Bending of light

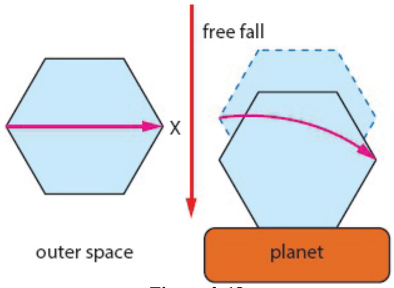

Figure A.19

Consider an inertial frame in outer space (Figure A.19). According to an observer inside the box, a ray of light emitted from the left wall will travel on a straight line and hit the right wall at a point X.

But this frame is equivalent to a freely falling frame in a gravitational field. By the time the light ray goes across, the frame has accelerated downwards. The ray must still hit at X and so it appears to an observer outside the box to be following a curved path. An outside observer would conclude that, in a gravitational field, light bends towards the mass causing this field.

Gravitational red-shift

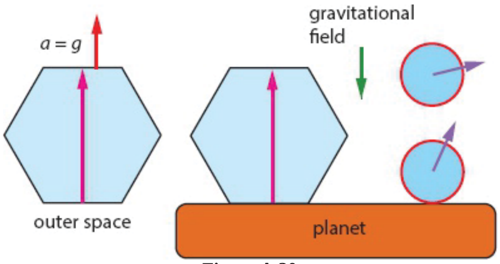

Figure A.20

In the left panel of Figure A.20, a frame is accelerating in outer space. A ray of light is emitted from the base of the frame.

To an observer outside, the top of the frame is moving away from the light and, because of the Doppler effect, the frequency measured at the top will be lower than that emitted. But because this frame is equivalent to a frame at rest in a gravitational field, an observer inside will conclude that a ray of light rising in a gravitational field has its frequency reduced. This is referred to as **gravitational red-shift**.

In the Pound–Rebka–Snider gravitational red-shift experiment, light of frequency f emitted on the surface of the Earth is to be observed at a height H. Gravitational red-shift is predicted to reduce the frequency of this light, according to $\frac{\Delta f}{f} = \frac{gH}{c^2}$. If the experiment is done on a tower of height 30 m, $\frac{\Delta f}{f} = \frac{9.8 \times 30}{9 \times 10^{16}} \approx 3 \times 10^{-15}$ so the experiment must be able to measure frequency to 15 decimal places!

Gravitational time dilation

We have learned that a ray of light travelling upwards in a gravitational field will have its frequency reduced. This means that the period of the light will increase.

Imagine a clock whose ticks are separated by a time equal to the period of this light. An observer using such clocks in this frame will see that when 1 s goes by according to a clock at the base, more than 1 s goes by according to a clock at the top.

Thus the equivalence principle predicts that the time between two events in a gravitational field is longer when measured by a clock far from the source of the field than by a clock near the source of the field. The observer far away sees the clock near the source to be running slow. This is referred to as **gravitational time dilation**.

TEST YOURSELF A.12

A rocket accelerates in outer space far from all masses. A ray of light is emitted across the rocket in a direction that is initially at right angles to the direction of acceleration, from a point a height h from the base. State and explain whether the ray will meet the opposite side of the rocket at a height that is less than, equal to or greater than h.

hint

Use the equivalence principle to find an equivalent situation where you know the answer.

TEST YOURSELF A.13

A ray of light is emitted, initially horizontally, in a lab of length 300 m on Earth. Estimate the amount by which the ray will bend downwards after travelling the length of the lab.

hint

Use the equivalence principle to find an equivalent situation where you know the answer.

Annotated Exemplar Answer A.3

A box is in free fall in the gravitational field of a planet. A ray of light is emitted from the base of the box and is observed at the top. Compare the frequency of light measured at the top with that at emission. **[3]**

The frequency will be less. The ray of light has to rise in a gravitational field and so will suffer gravitational time dilation.

You are correct in saying that there will gravitational red-shift. But there will also be a Doppler blue-shift because the observer at the top is moving towards the light. This cancels the red-shift, so the frequency is unchanged.

1/3

Another way to look at this is given by the equivalence principle, according to which this situation is equivalent to a frame of reference moving at constant velocity far from all masses. In such a frame there would be no change in the light frequency.

Curved spacetime

DEFINITIONS

CURVED SPACETIME The theory of general relativity states that four-dimensional spacetime gets distorted by the mass and energy it contains. Curved spacetime means that the rules of classical geometry do not hold and have to be replaced by other rules.

GEODESIC The path followed by an object on which no net force acts. It is the path of least length in spacetime. Light travels along geodesics.

GRAVITATIONAL LENSING Multiple images of an object caused by the bending of light by a very massive object in or near the path of the light.

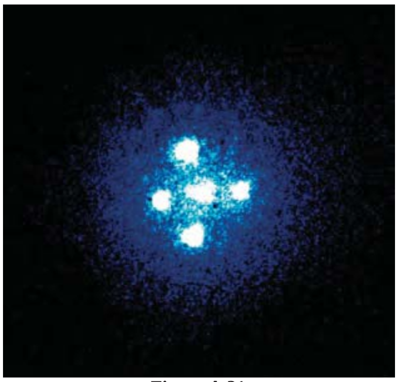

Figure A.21

Figure A.21 shows four images of the same distant quasar. The images have been formed by light that has bent around a massive galaxy that is directly in front of the quasar and appears at the centre of this picture.

This also occurs to a lesser extent with light from other stars. The mass of the Sun causes a curvature of the space around it. Light from a star bends on its way to the Earth and this makes the star appear to be in a slightly different position from its actual position. Figure A.22 illustrates this effect, though the scale in the figure is greatly exaggerated.

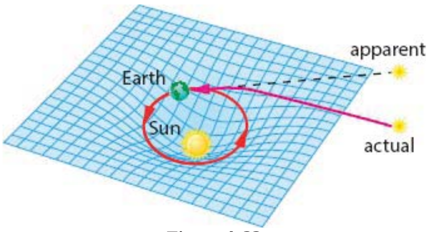

Figure A.22

A ray of light bends in the gravitational field of the Sun so that an observer on Earth sees the star at a position that is different from the true position of the star.

TEST YOURSELF A.14

Explain how Newton and Einstein would have accounted for the motion of a planet around the Sun.

Black holes: an extreme prediction of general relativity

DEFINITIONS

BLACK HOLE A 'singularity of spacetime', a point of infinite spacetime curvature.

SCHWARZSCHILD RADIUS, R_S The distance from a black hole at which the escape speed is equal to the speed of light: $R_s = \frac{2GM}{c^2}$.

EVENT HORIZON A surface enclosing a black hole such that nothing from within the surface can be communicated to the outside. Nothing can move from inside the event horizon to the outside. Its radius is the Schwarzschild radius.

Since nearby mass constantly falls into a black hole, its Schwarzschild radius constantly increases. This means that the area of the event horizon also increases.

A black hole with a mass equal to that of the Sun would have a radius of $R_s = \frac{2\times6.67\times10^{-11}\times2.0\times10^{30}}{(3.0\times10^{8})^2} \approx 3\text{km}$.

Time dilation near a black hole

Mass bends spacetime. Here is an example of how mass bends not just space but time.

The time interval between two events is measured by a clock near a black hole as Δt_0, and by another clock very far from the black hole as Δt. General relativity predicts that:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$

where r is the distance of the clock from the centre of the black hole and R_S is the Schwarzschild radius of the black hole. As this clock approaches the event horizon, $r \to R_S$ and therefore $\Delta f \to \infty$, so it appears to the faraway observer that time near the event horizon is slowing down.

TEST YOURSELF A.15

A probe near a black hole of Schwarzschild radius $R_S = 5.0 \times 10^8$ m emits one pulse every second. The pulses are received every 5.0 seconds by a spacecraft far away. How far from the event horizon of the hole is the probe?

Checklist

After studying this Option you should be able to:

- ☐ understand what is meant by a frame of reference
- ☐ solve problems with Lorentz transformations, time dilation and length contraction
- ☐ solve simple problems in electromagnetism under relativistic conditions
- ☐ explain why muons can reach the Earth's surface from space
- ☐ solve relativistic mechanics problems
- ☐ understand the equivalence principle and explain its consequences
- ☐ solve problems with the Schwarzschild radius and time dilation near a black hole.

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Annotated Exemplar Answer A.3

A box is in free fall in the gravitational field of a planet. A ray of light is emitted from the base of the box and is observed at the top. Compare the frequency of light measured at the top with that at emission. **[3]**

The frequency will be Less. The ray of Light has to rise in a gravitational field and so will suffer gravitational time dilation.

1/3

You are correct in saying that there will gravitational red-shift. But there will also be a Doppler blue-shift because the observer at the top is moving towards the light. This cancels the red-shift, so the frequency is unchanged.

Another way to look at this is given by the equivalence principle, according to which this situation is equivalent to a frame of reference moving at constant velocity far from all masses. In such a frame there would be no change in the light frequency.

B ENGINEERING PHYSICS

This option covers the following topics:

- □ Rotational mechanics
- □ Fluid mechanics
- □ Thermodynamics
- □ Oscillations and damping

B.1 Rotational mechanics

Rotational kinematics

DEFINITIONS

ANGULAR POSITION θ, the angle swept by a rotating body relative to a reference line.

ANGULAR SPEED ω, the rate of change with time of the angular position.

ANGULAR ACCELERATION α, the rate of change with time of the angular speed.

The definitions make it clear that we will have the obvious equations derived from linear motion:

$$\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$$

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right) t$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

When a particle rotates with angular speed ω about some axis, the particle's linear sped is given by $v = \omega r$, where r is the distance of the particle from the axis of rotation.

In a graph of θ versus time t, the gradient is the angular speed ω.

In a graph of ω versus time t, the gradient is the angular acceleration α and the area under the curve is the change in angular position.

In a graph of α versus time t, the area under the curve is the change in angular speed.

TEST YOURSELF B.1

A body rotates with initial angular speed 36 rad s^{-1} and angular acceleration −0.54 rad s^{-2}. Determine how many full revolutions the body will complete before it comes to rest.

Annotated Exemplar Answer B.1

Figure B.1 shows the variation with time t of the angular acceleration of a rotating body. The body was initially at rest. Determine the angular speed at 24 s. **[2]**

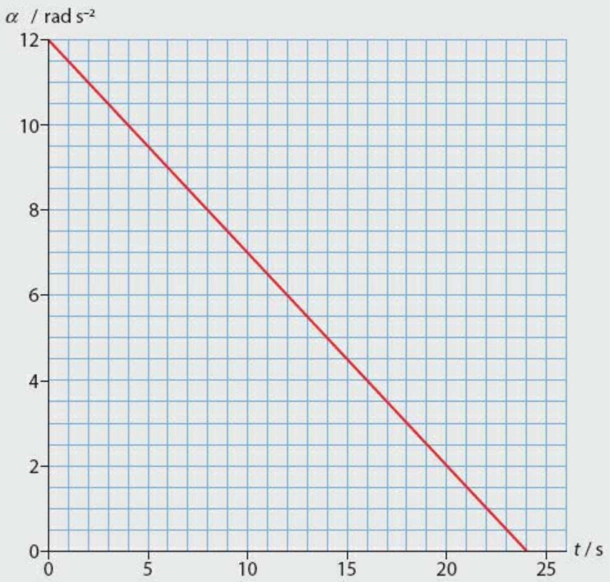


Figure B.1

0/2

From the graph, at $t = 0$, $\alpha = 12\ \text{rad s}^{-2}$

using $\omega = \omega_0 + \alpha t$

gives $\omega = 0 + (12 \times 24) = 288\ \text{rad s}^{-1}$.

You have read the value correctly from the graph. But you need to look carefully at the graph to see what else it tells you. What is the shape of the graph?

This equation does relate angular speed at time t to initial angular speed and angular acceleration, but it is only valid for constant acceleration, α. The graph shows that $\alpha = 12\ \text{rad s}^{-2}$ at $t = 0\ \text{s}$ and $\alpha = 0\ \text{rad s}^{-2}$ at $t = 24\ \text{s}$. The angular acceleration is not constant.

The value for α is incorrect, so the calculated value is wrong. The angular speed at 24 s is the area under the graph, which is $\frac{1}{2} \times 12 \times 24 = 144\ \text{rad s}^{-1}$.

Torque

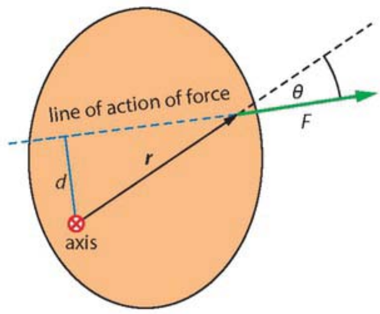


Figure B.2

A net force acting on a point particle will accelerate that particle. But a net force on an extended body will not only accelerate the centre of mass of the body but may also cause the body to rotate about some axis. The torque Γ of a force F about a given axis is the product of the magnitude of the force times the perpendicular distance d of the line of the force from the axis: $\Gamma = Fd$. This is equivalent to $\Gamma = Fr \sin\theta$ (see Figure B2.2). Torque measures the ability of a force to rotate a body. It is measured in N m.

The two forces shown in Figure B.3 have zero torque about the given axis.

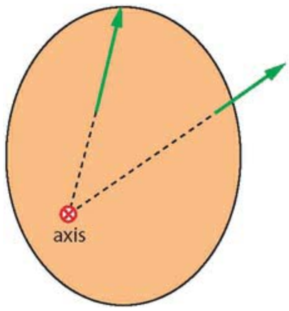


Figure B.3

TEST YOURSELF B.2

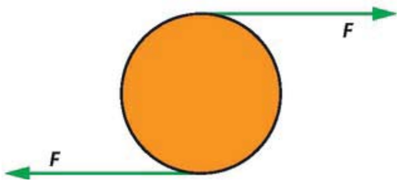
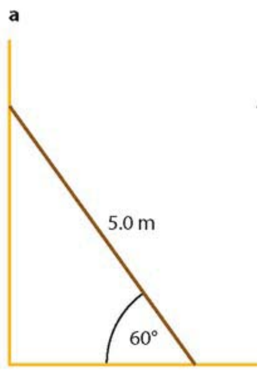


Figure B.4

Figure B.4 shows a disc of radius R. Two forces of equal magnitude F are applied at the circumference of the disc at diametrically opposite points. Calculate the net force and net torque about an axis normal to the disc and through its centre.

Equilibrium

An extended body is in translational equilibrium if the net force is zero and it is in rotational equilibrium if the net torque is zero. As an example consider a rod leaning against a vertical wall, as shown in Figure B.5a.



We can assume there is no friction between the wall and the rod. But there has to be friction between the rod and the floor if the rod is to remain in equilibrium in this position.

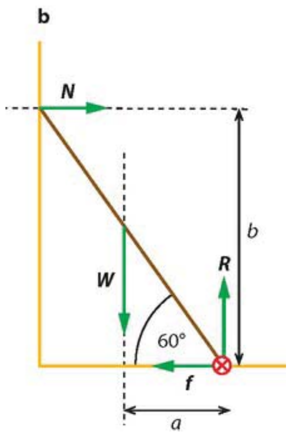


Figure B.5

Figure B.5b shows the forces on the rod. What is the coefficient of static friction μ_s between the floor and the rod, given that the weight of the rod is 280 N and 60° is the largest angle at which we can have equilibrium?

We begin by demanding that the net force be zero. This is the condition for translational equilibrium:

$N = f$ and $R = W$

So right away R = 280 N.

To make progress we must now apply the condition for rotational equilibrium. The net torque must be zero about any axis. Choose an axis through which as many forces as possible pass; this will simplify the analysis because we will not have to deal with their torques. A sensible choice is the axis into the plane of the page at the bottom of the rod.

Then the forces with non-zero torques are W and N. These two forces have opposing torques, so

$$Wa = Nb$$
$$280 \times 2.5 \times \cos 60^\circ = N \times 5.0 \times \sin 60^\circ$$
$$N = 80.8 \approx 81\text{N}$$

Now that we have found N, we know that $f = 80.8 \approx 81$ N also. Since 60° is the largest angle at which we can have equilibrium, it follows that the frictional force is the largest possible, so

$$f = \mu_s R \Rightarrow \mu_s = \frac{f}{R} = \frac{80.8}{280} \approx 0.29.$$

Kinetic energy of a rotating body

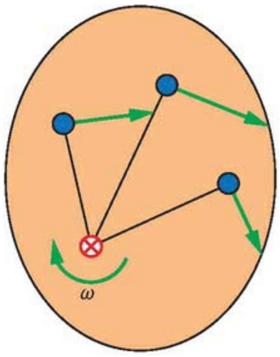


Figure B.6

Figure B.6 shows an extended body rotating about an axis with constant angular speed ω. Each particle of the body rotates with the same angular speed, but each has a different linear speed $v_i = \omega r_i$, where r_i is the distance from the axis of the ith particle.

The total kinetic energy is the sum of the kinetic energies of all the particles of the body:

$$E_\text{k} = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 + \cdots + \tfrac{1}{2}m_N v_N^2$$
$$= \tfrac{1}{2}\left(\sum_{i=1}^{N} m_i r_i^2\right)\omega^2$$

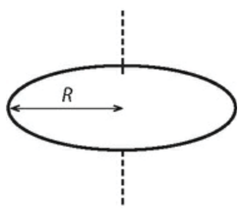
The quantity $I = \sum_{i=1}^{N} m_i r_i^2$ is known as the **moment of inertia** of the body about the axis of rotation, so $E_\text{k} = \tfrac{1}{2}I\omega^2$.

The moment of inertia depends on:

- the axis of rotation
- the mass of the body
- the way the mass is distributed i.e the shape of the body.

Figure B.7 gives expressions for the moments of inertia of masses of various shapes, relative to a vertical axis (dashed line).

a



$I = MR^2$

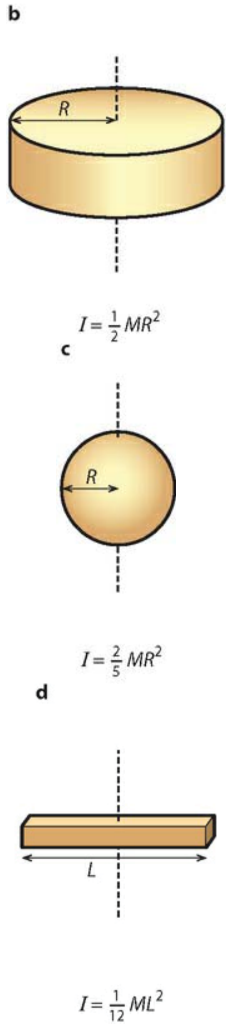


Figure B.7 Moments of inertia for **a** horizontal ring; **b** cyclinder; **c** sphere; **d** rod.

Rolling without slipping

Imagine a wheel that rolls on horizontal ground. As the wheel rotates it moves forward. If the point on the wheel in contact with the ground does not slip, then the angular speed ω and the linear velocity v of the centre of mass must be related by $v = \omega r$, and similarly for the angular and linear accelerations, $a = \alpha r$.

Conservation of energy

For a body of mass M that rolls as it rotates, the total energy is given by:

$$E = \frac{1}{2}Mu^2 + \frac{1}{2}I\omega^2 + Mgh$$

where u is the velocity of the centre of mass, ω is the angular speed, I is the moment of inertia about the axis of rotation and h is the height of the centre of mass from some arbitrary horizontal level. In the absence of frictional forces and torques, the total energy is conserved.

Worked Example B.1

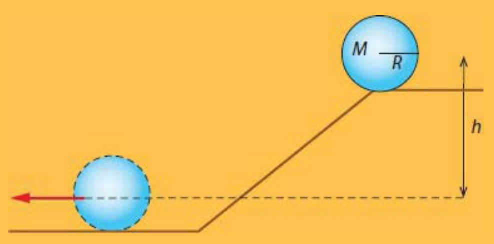


Figure B.8

A sphere of radius R and mass M is released down an inclined plane (Figure B.8).

Calculate the speed of the sphere after its centre of mass has descended a height h. The sphere is assumed to roll without slipping.

The total energy of the sphere at the point of release is $E = Mgh$. If the lower level is taken as the reference level for gravitational potential energy, the total energy at this level is $E = \frac{1}{2}Mu^2 + \frac{1}{2}I\omega^2 + 0$. Because we have rolling without slipping, $u = \omega R$ or $\omega = \frac{u}{R}$. The moment of inertia is $I = \frac{2}{5}MR^2$ (see Figure B.7).

Hence $\frac{1}{2}Mu^2 + \frac{1}{2}\frac{2}{5}MR^2\frac{u^2}{R^2} + 0 = Mgh$, which gives $u = \sqrt{\frac{10gh}{7}}$.

A point mass would have a greater speed, $u = \sqrt{2gh}$, because it has no rotational kinetic energy.

Newton's second law

Newton's second law for linear motion, $F_{net} = Ma$, has its counterpart for rotational motion, $\Gamma_{net} = I\alpha$. This is applied to a sphere rolling down a hill in the following worked example.

Worked Example B.2

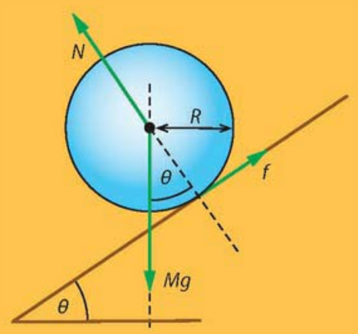


Figure B.9

Figure B.9 shows the forces on a sphere of mass *M* rolling down an inclined plane without slipping. Determine:

a the acceleration of the centre of mass

b the time for its centre of mass to descend a vertical distance *h*.

a We must find both the net force and the net torque on the sphere.
The net force in the direction of the inclined plane is $Mg \sin\theta - f$, so Newton's second law for the translational motion is:

$$\begin{aligned} F_{net} &= Ma \\ Mg\sin\theta - f &= Ma \end{aligned} \quad \text{Newton's second law for translational motion}$$

(Perpendicular to the plane, $N = Mg \cos\theta$ because there is no acceleration in this direction.)
The net torque about a horizontal axis through the centre of mass is just fR (N and Mg have zero torques about this point), and the moment of inertia of the sphere about its axis is $I = \frac{2}{5}MR^2$. So Newton's second law for the rotational motion is:

$$\begin{aligned} \Gamma &= I\alpha \\ fR &= \frac{2}{5}MR^2\alpha \end{aligned} \quad \text{Newton's second law for rotational motion}$$

We assume rolling without slipping and so $\alpha = \frac{a}{R}$, so the equation for rotational motion becomes $f = \frac{2}{5}Ma$. Plugging this back into the equation for linear motion, we have $Mg\sin\theta - \frac{2}{5}Ma = Ma$, or $a = \frac{5g\sin\theta}{7}$.
(Note that after descending a vertical distance h the sphere would acquire a speed of $v^2 = 2as = 2 \times \frac{5g\sin\theta}{7} \times \frac{h}{\sin\theta} = \frac{10gh}{7}$, just as we found in Worked example B.1.)

b The time taken to descend a vertical distance h is found from

$$v = at \Rightarrow t = \frac{v}{a} = \frac{\sqrt{\frac{10gh}{7}}}{\frac{5g\sin\theta}{7}} = \frac{1}{\sin\theta}\sqrt{\frac{70h}{25g}}.$$

TEST YOURSELF B.3

Calculate the acceleration of a cylinder rolling down an inclined plane that makes an angle θ to the horizontal.

Angular momentum

A body with moment of inertia I about an axis, and rotating with angular speed ω about that axis, has angular momentum $L = I\omega$. By analogy with linear motion, the rate of change of angular momentum is equal to the net torque on the body: $\Gamma = \frac{\Delta L}{\Delta t}$. For a point particle of mass m rotating about an axis with angular speed ω, the angular momentum is $L = mr^2\omega$, since the moment of inertia of a point particle is just mr^2. Since $v = \omega r$, it follows that, for a point particle, $L = mvr$.

If the net torque on the body is zero, then $\Delta L = 0$; this means that the angular momentum stays constant.

This is the law of **conservation of angular momentum**.

TEST YOURSELF B.4

A spherical star of mass M and radius R rotates about its axis, making one revolution per day. The star explodes radially and symmetrically, leaving a spherical star of mass $\frac{1}{50}M$ and radius $\frac{1}{200}R$. Determine the new number of revolutions per second.

Model Answer B.1

A horizontal disc rotates about a vertical axis through its centre with angular speed ω. A second disc of the same radius but double the mass is dropped horizontally onto the rotating disc. Calculate:

a the new angular speed of the system

b the fraction of the original rotational kinetic energy lost.

After τ seconds the second disc has acquired the angular speed calculated in a.

c Derive an expression for the torque that acted on the second disc.

d Suggest what became of the lost rotational kinetic energy.

a The initial angular momentum is $L = \frac{1}{2}MR^2\omega$. After the second disc lands, the new moment of inertia is $\frac{1}{2}MR^2 + \frac{1}{2}(2M)R^2 = \frac{3}{2}MR^2$. Hence $\frac{1}{2}MR^2\omega = \frac{3}{2}MR^2\omega'$, so $\omega' = \frac{\omega}{3}$

b The initial rotational kinetic energy is $E_k = \frac{1}{2}I\omega^2 = \frac{MR^2\omega^2}{4}$. After the second disc lands, it is $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{3}{2}MR^2\left(\frac{\omega}{3}\right)^2 = \frac{MR^2\omega^2}{12}$. The energy lost is then $\frac{MR^2\omega^2}{4} - \frac{MR^2\omega^2}{12} = \frac{MR^2\omega^2}{6}$, so the required fraction is $\frac{\frac{MR^2\omega^2}{6}}{\frac{MR^2\omega^2}{4}} = \frac{2}{3}$

c The angular momentum of the second disc changed by $MR^2\frac{\omega}{3}$. Hence

$$\Gamma = \frac{\Delta L}{\Delta t} = \frac{MR^2\frac{\omega}{3}}{\tau} = \frac{MR^2\omega}{3\tau}$$

d The lost energy was converted by friction into thermal energy as the second disc slid over the first.

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Annotated Exemplar Answer B.1

Figure B.1 shows the variation with time t of the angular acceleration of a rotating body. The body was initially at rest. Determine the angular speed at 24 s. **[2]**

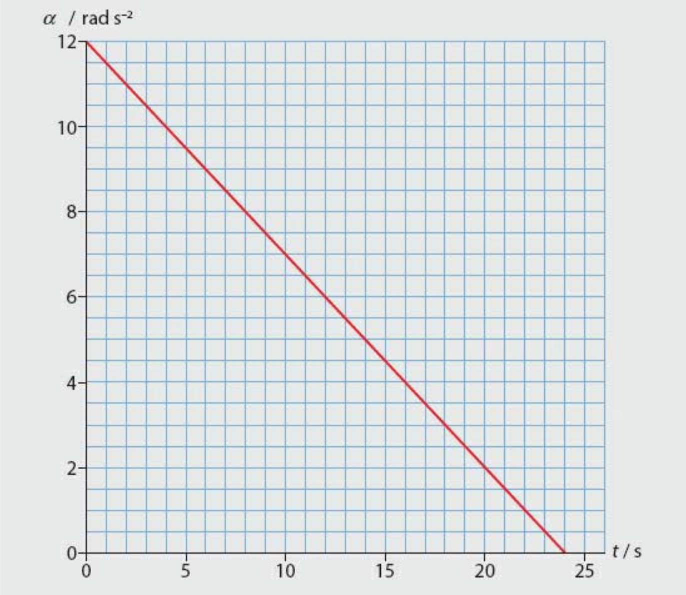


Figure B.1

From the graph, at $t = 0$, $\alpha = 12$ rad s^{-2} using $\omega = \omega_0 + \alpha t$ gives $\omega = 0 + (12 \times 24) = 288$ rad s^{-1}.

You have read the value correctly from the graph. But you need to look carefully at the graph to see what else it tells you. What is the shape of the graph?

The value for α is incorrect, so the calculated value is wrong. The angular speed at 24 s is the area under the graph, which is $\frac{1}{2} \times 12 \times 24 = 144\, rad s^{-1}$.

This equation does relate angular speed at time t to initial angular speed and angular acceleration, but it is only valid for constant acceleration, α. The graph shows that $\alpha = 12$ rad s^{-2} at $t = 0$ s and $\alpha = 0$ rad s^{-2} at $t = 24$ s. The angular acceleration is not constant.

B.2 Thermodynamics

DEFINITIONS

CLOSED SYSTEM A system in which no mass can enter or leave.

OPEN SYSTEM A system in which mass can enter or leave.

ISOLATED SYSTEM A system in which no energy can enter or leave.

ISOTHERMAL EXPANSION OR COMPRESSION A change in the state of a gas in which its temperature stays constant.

ADIABATIC COMPRESSION OR EXPANSION A change in the state of a gas in which no energy enters or leaves.

ISOBARIC EXPANSION OR COMPRESSION A change in the state of a gas in which its pressure stays constant.

ISOVOLUMETRIC CHANGE A change in the state of a gas in which its volume stays constant.

Adiabatic changes

For any change of state of an ideal gas, the ideal gas law $pV = RnT$ applies, but for an adiabatic change we have the additional result that $pV^{5/3}$ = constant. This means that when an ideal gas changes adiabatically from a state with (p_1, V_1) to one with (p_2, V_2), we have that $p_1V_1^{5/3} = p_2V_2^{5/3}$.

Worked Example B.3

An ideal gas expands adiabatically from an initial state with $p_1 = 6.2 \times 10^5$ Pa, $V_1 = 4.5 \times 10^{-3}$ m^3, T_1 = 310K to a state with pressure $p_2 = 3.4 \times 10^5$ Pa. Calculate the new volume and temperature of the gas.

We use $p_1V_2^{5/3} = p_2V_2^{5/3}$ to find $6.2 \times 10^5 \times (4.5 \times 10^{-3})^{5/3} = 3.4 \times 10^5 \times V_2^{5/3}$. This gives $V_2^{5/3} = \frac{6.2\times10^5}{3.4\times10^5} \times \left(4.5 \times 10^{-3}\right)^{5/3} = 2.236 \times 10^{-4}$ and so $V_2 = (2.236 \times 10^{-4})^{3/5} = 6.5 \times 10^{-3}$ m^3.

To find the new temperature use the ideal gas law to find that $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow T_2 = \frac{p_2V_2T_1}{p_1V_1} = \frac{3.4\times10^5\times6.5\times10^{-3}\times310}{6.2\times10^5\times4.5\times10^{-3}} = 260\text{K}$.

Internal energy of an ideal gas

For a monatomic gas, the average kinetic energy of its molecules is $\overline{E}_\text{k} = \frac{3}{2}kT$, where k is the Boltzmann constant, $k = \frac{R}{N_\text{A}} = \frac{8.31}{6.02\times10^{23}} = 1.38 \times \text{JK}^{-1}$.

For an ideal gas, the internal energy is just the total random kinetic energy of the molecules, and so

$$U = N\overline{E}_\text{k} = \frac{3}{2}NkT = \frac{3}{2}nRT$$

(The last equality follows from $Nk = N\frac{R}{N_\text{A}} = \frac{N}{N_\text{A}}R = nR$, n being the number of moles.) This implies that $\Delta U \frac{3}{2}nR\Delta T$. But $pV = nRT$ for an ideal gas, so an additional and equivalent formula for its internal energy is $U = \frac{3}{2}pV$.

This implies $\Delta U = \frac{3}{2}p\Delta V$ if pressure is kept constant, or $\Delta U = \frac{3}{2}V\Delta p$ if volume is kept constant.

Worked Example B.4

An ideal gas at an initial temperature of 260 K expands at a constant pressure of 1.20×10^6 Pa from a volume of 3.10×10^{-3} m^3 to a volume of 5.80×10^{-3} m^3. Calculate the change in its internal energy.

$\Delta U = \frac{3}{2}p\Delta V$ and so $\Delta U = \frac{3}{2} \times 1.20 \times 10^6 \times (5.80 \times 10^{-3} \times 10^{-3}) = 4860\text{J}$.

It is convenient to use this version for the change in internal energy, rather than $\Delta U = \frac{3}{2}nR\Delta T$.

TEST YOURSELF B.5

The pressure of an ideal gas in a container of fixed volume 3.1×10^{-3} m^3 is increased from 2.2×10^5 Pa to 4.6×10^5 Pa. Calculate the change in the internal energy of the gas.

Work done by a gas

When a gas at pressure p pushes out a piston by a small distance Δx it does a small amount of work, given by $W = F\Delta x$. Because $p = \frac{F}{A}$, where A is the cross-sectional area of the piston, we have that $F = pA$ and so $W = pA\Delta x$.

But $A\Delta x$ is the change in volume and so $W = p\Delta V$.

This formula for work done can only be used if the pressure stays the same during the change. If the pressure varies, we must find the area under the graph of pressure versus volume (Figure B.10).

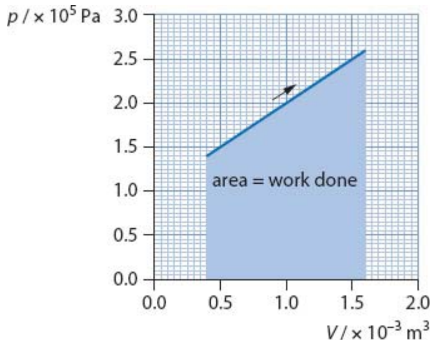


Figure B.10

The first law of thermodynamics

The first law of thermodynamics states that the heat energy Q supplied to a system such as a gas is equal to the change in the system's internal energy U, plus the work done: $Q = \Delta U + W$.

This law is a statement of the conservation of energy.

There are sign conventions for this law that you should know: for a gas, $W > 0$ means that work is done by the gas and it expands, while $Q > 0$ means that heat is supplied to the gas.

TEST YOURSELF B.6

An ideal gas has an initial temperature of 300 K. It expands at constant pressure of 4.0×10^5 Pa from an initial volume of 2.0×10^{-3} m^3 to a volume of 5.0×10^{-3} m^3.

a Calculate the new temperature of the gas and the work done.

b Has thermal energy been provided to the gas or has it been taken out?

TEST YOURSELF B.7

For an ideal gas:

a define internal energy

b state and explain how the internal energy and the absolute temperature are related.

hint

The gas is ideal so does it have potential energy? What is absolute temperature related to?

TEST YOURSELF B.8

A gas is compressed adiabatically by a piston. Explain, using the molecular model of a gas, why the temperature of the gas will increase.

TEST YOURSELF B.9

Equal amounts of energy Q are supplied to two equal quantities of ideal gas. The first absorbs the energy at constant volume whereas the second absorbs it at constant pressure. Determine which one shows the larger temperature increase.

hint

To determine the temperature change we must see what happens to internal energy – why?

Pressure–volume diagrams for various changes of state

We can represent the change in the state of a gas with a pressure–volume diagram. Figures B.11 and B.12 show the four basic changes: isobaric, isovolumetric, isothermal and adiabatic.

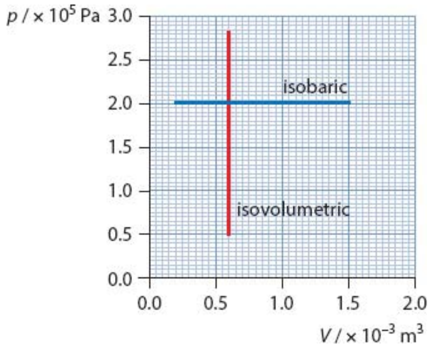


Figure B.11

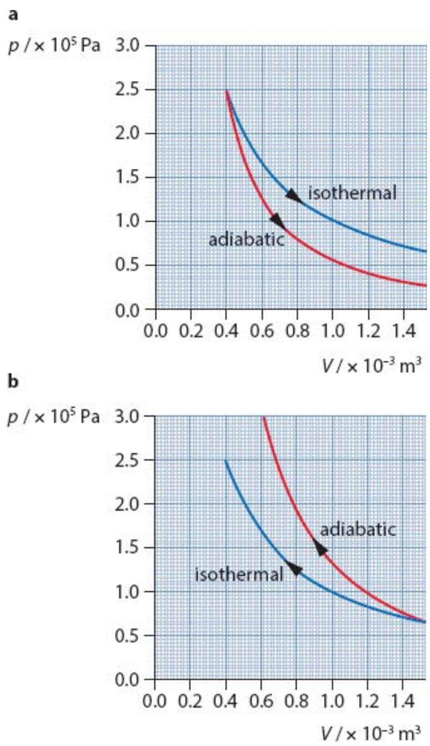


Figure B.12

Notice that, for both compressions and expansions, the adiabatic curve is steeper than the isothermal curve.

You should be able to use the first law of thermodynamics (along with the ideal gas law, if necessary) to determine Q, ΔU and W for various processes. Following are a few standard examples.

- **Isovolumetric change:** The pressure of a gas is decreased at constant volume. Determine whether heat is supplied to or removed from the gas. There is no work done since there is no change in volume. If the pressure is reduced at constant volume, the ideal gas law says that the temperature must drop. Hence the internal energy decreases, $\Delta U < 0$. Hence $Q = \Delta U + 0 < 0$ and so heat is removed.
- **Isothermal expansion:** A gas expands at constant temperature. Determine whether heat is supplied to or removed from the gas. The work done is positive since the gas expands and the volume increases. The internal energy stays constant since the temperature is constant.

Hence $Q = 0 + W > 0$ and so heat is supplied to the gas.

- **Adiabatic compression:** A gas is compressed adiabatically. Determine whether the internal energy increases or decreases.
  The process is adiabatic and so $Q = 0$. Work done is negative since the gas is compressed. Hence $0 = \Delta U + W$ and so $\Delta U = -W > 0$. The internal energy and temperature increase.
- **Isobaric expansion:** A gas expands isobarically. Determine whether heat is supplied to or removed from the gas.
  The volume increases at constant pressure and the ideal gas law says that the temperature and hence the internal energy must increase, $\Delta U > 0$. Work done is positive since the gas expands, $W > 0$.
  Hence $Q = \Delta U + W > 0$. Heat must be supplied to the gas.

The pressure–volume diagrams in Test yourself B.10–B.12 show **cyclical changes** – changes that start and end at the same point. Looking, for example, at Figure B.13, we see that work is done *by* the gas along AB, while work is done *on* the gas along BCA. The **net** work (work done by the gas minus work done on the gas) is the shaded area within the loop.

TEST YOURSELF B.10

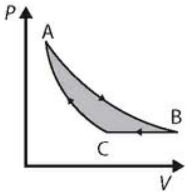


Figure B.13

The cycle in Figure B.13 includes an isothermal change and an adiabatic change.

a State which is the isothermal process and which is the adiabatic process.

b In which leg of the cycle is thermal energy extracted from the gas?

c In which leg is the magnitude of the work done the greatest?

TEST YOURSELF B.11

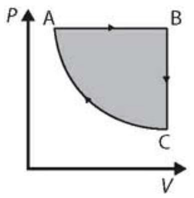


Figure B.14

The curved part of the cycle in Figure B.14 is an isothermal process.

Thermal energy Q_1 is supplied to the gas along AB, and Q_2 is removed along BC and CA.
Determine which is greater, Q_1 or Q_2.

TEST YOURSELF B.12

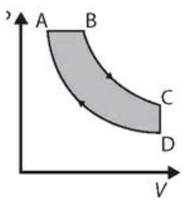


Figure B.15

The curved parts of the cycle in Figure B.15 are both adiabatic processes.

Show that thermal energy is supplied to the gas along AB and taken out along CD.

The second law of thermodynamics

Molecules in a solid vibrate about fixed equilibrium positions, whereas molecules in a gas move randomly about. We know approximately where the solid's molecules are but we have little information about the positions of the molecules of the gas. We say that the state of the solid is *ordered* while the state of the gas is *disordered*.

It is a common observation that isolated physical systems tend to move to states of higher disorder. **Entropy** is a concept that quantifies disorder, and the second law of thermodynamics states that the total entropy of the universe always increases. Entropy will increase when heat is added to a system, and will decrease if heat is removed from it. The change in entropy is given by $\Delta S = \frac{\Delta Q}{T}$, where T is the temperature of the system at which heat is added or removed.

Consider a volume of liquid water surrounded by air at −10°C. The water will lose heat to the surrounding air, freeze and become solid ice. The entropy of the water will decrease and the entropy of the surroundings will increase, but the overall change in the entropy of the universe will have increased, consistent with the second law of thermodynamics.

Model Answer B.2

Two bodies at temperatures 300 K and 400 K are put in thermal contact. During a short interval of time, 1J of heat flows from the hot to the cold body. This amount of heat is so small that the temperatures of the bodies do not change appreciably. Calculate the change in entropy of each body and the overall entropy change for the system consisting of the two bodies.

The hot body loses heat so its entropy **decreases** by $\Delta S = \frac{\Delta Q}{T} = \frac{1}{400}\mathrm{JK^{-1}}$. The cold body receives heat so its entropy **increases** by $\Delta S = \frac{1}{300}\mathrm{JK^{-1}}$. The overall change of entropy in the system is $\Delta S = \left(\frac{1}{300} - \frac{1}{400}\right) = \left(\frac{1}{1200}\right)\mathrm{JK^{-1}}$. This is a positive quantity; thus the entropy of the total system has increased.

Heat engines

DEFINITIONS

HEAT ENGINE A device that uses heat to perform mechanical work (Figure B.16). To be practical, a heat engine must work on a cycle, so that at the end of the cycle the engine is returned to its initial state.

A heat engine run in reverse works as a refrigerator – that is, work is done on the engine to force heat to move from a cold reservoir to a hotter reservoir (Figure B.17).

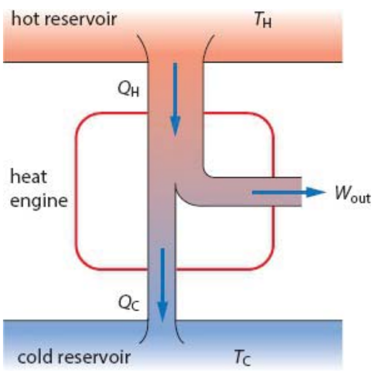


Figure B.16

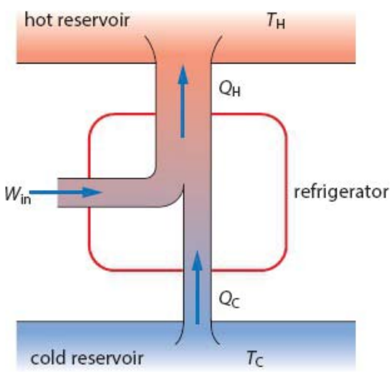


Figure B.17

EFFICIENCY, e For a heat engine, the ratio of work done to input heat: $e = \frac{W}{Q_H}$.

CARNOT EFFICIENCY, e_c $e_C = 1 - \frac{T_C}{T_H}$.

The second law can be formulated in several different ways, including in terms of heat engines, and it puts limits on how efficient a heat engine can be.

- **Kelvin version:** It is impossible to completely convert heat into mechanical work in a cyclic process.
- **Clausius version:** It is impossible for heat to flow from a cold to a hot reservoir without work being performed.
- **Carnot version:** The most efficient heat engine operating between two given temperatures is the Carnot cycle, shown in Figure B.18.

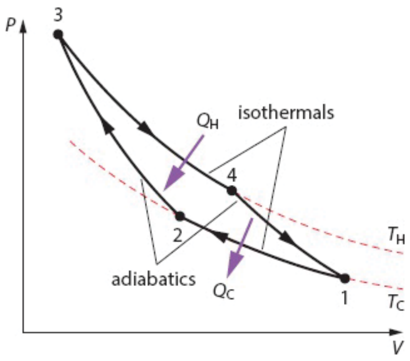


Figure B.18 Pressure – volume diagram for the Carnot cycle that consists of two isothermals and two adiabatics.

TEST YOURSELF B.13

A Carnot engine operates between the temperatures of 600 K and 400 K. The engine has an input heat of 1200 J. Determine the work done.

B.3 Fluids

Fluids at rest

DEFINITIONS

PRESSURE The normal force per unit area on a surface.

ARCHIMEDES' PRINCIPLE A body that is immersed in a fluid will experience an upward buoyant force given by $B = \rho_{fluid}\, gV_{imm}$, where V_{imm} is the immersed volume.

PASCAL'S PRINCIPLE The pressure applied to any part of an enclosed incompressible fluid will be transmitted to all other parts of the liquid.

Figure B.19 shows a partial volume of liquid, marked with a dashed line.

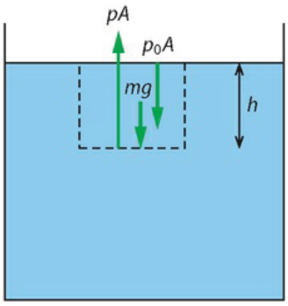


Figure B.19

The marked volume has an area of A at its top surface. The vertical forces on this volume are its weight and the forces due to the pressure P_0 of the atmosphere and the pressure p of the liquid at depth h. Equilibrium demands that they balance – that is, $mg + P_0A = pA$ or $\rho_{fluid}\, Ahg + P_0A = pA$. This can be rearranged to $p = \rho_{fluid}\, gh + P_0$, an expression for the liquid pressure as a function of depth h.

All horizontal forces on the marked volume are also balanced. Note that, at any given depth, the pressure is the same in all directions.

Because the pressure depends only on depth, all points at the same depth have the same pressure (Figure B.20).

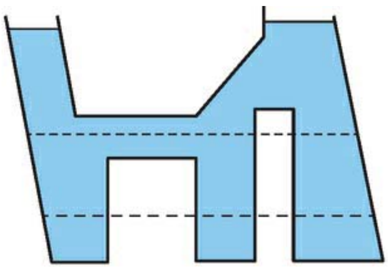


Figure B.20

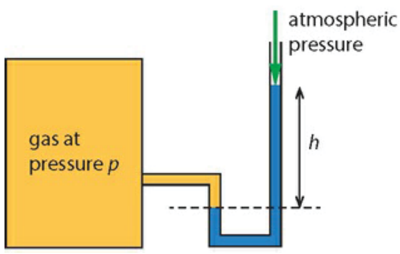


Figure B.21

This is the principle behind the **manometer**, an instrument for measuring gas pressure. In Figure B.21, the left side of the U-shaped column of fluid experiences a pressure p equal to that of the gas in the reservoir. On the right side at the same level, the fluid is subject to atmospheric pressure plus the pressure exerted by the column of fluid above that level – that is, $\rho_{fluid}\, gh + P_0$.

If the fluid column is in equilibrium, $F = \rho_{fluid}\, gh + P_0$, and from the value of h the pressure of the gas can be calculated.

TEST YOURSELF B.14

A cube of solid material of density 850 kg m^{-3} floats in a liquid of density 950 kg m^{-3}. Determine the fraction of the volume of the body that is submerged in the liquid.

The ideal fluid

DEFINITIONS

LAMINAR FLOW Fluid flow such that the fluid velocity at each point does not change with time.

INCOMPRESSIBLE FLOW Fluid flow such that fluid density is everywhere the same.

NON-VISCOUS FLOW Fluid flow without resistance forces.

STREAMLINE The path followed by a small bit of a fluid as it moves.

FLOWTUBE A set of neighbouring streamlines (Figure B.22).

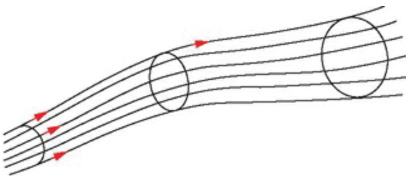


Figure B.22

IDEAL FLUID A fluid whose flow is laminar, incompressible and non-viscous.

When an ideal fluid flows in a pipe of varying cross-sectional area, its velocity will also vary: the fluid will move faster where the area is smaller (Figure B.23).

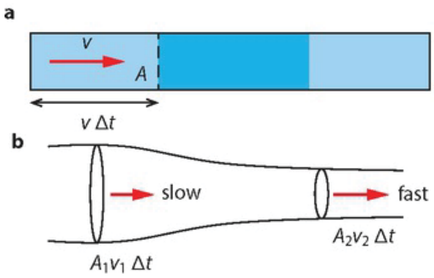


Figure B.23

This is expressed by the equation of continuity: $A_1v_1 = A_2v_2$.

Model Answer B.3

Water comes out of a faucet of cross-sectional area A_1 with a speed of 0.85 ms^{-1}. Calculate the vertical distance after which the cross-sectional area of the water is reduced to half its initial value.

After falling a distance h the water will acquire a speed given by $v^2{}_2 = v_1{}^2 + 2gh$. Hence, from the continuity equation $A_1v_1 = A_2v_2$ we find that:

$$\begin{aligned} A_2\sqrt{v_1^2 + 2gh} &= A_1v_1 \\ \frac{v_1}{\sqrt{v_1^2 + 2gh}} &= \frac{1}{2} \\ 4v_1^2 &= v_1^2 + 2gh \\ h &= \frac{3v_1^2}{2g} = \frac{3\times0.85^2}{2\times9.8} = 11\text{cm} \end{aligned}$$

The Bernoulli equation

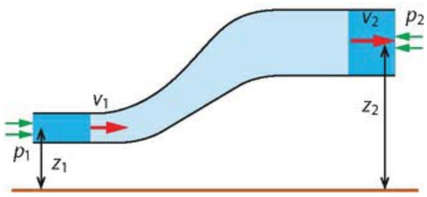


Figure B.24

The Bernoulli equation relates the fluid pressure p, flow speed v and height z at different points on a particular streamline (see Figure B.24):

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

Model Answer B.4

A reservoir of water has a depth of 48 m. Water can leave the reservoir through a pipe whose opening is 18 m from the ground (see Figure B.25).

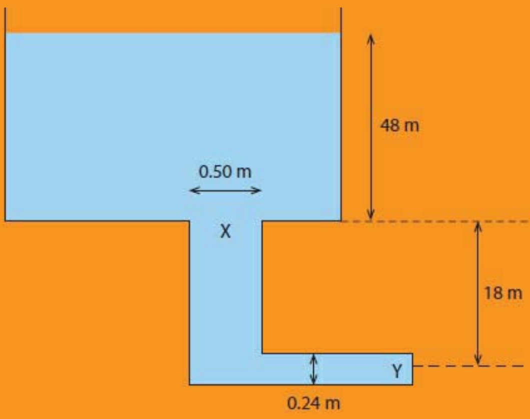


Figure B.25

a Calculate the pressure at X when no water flows.

b The exit at Y is opened and water now flows. Calculate the speed at X.

c Calculate the pressure at X now that the water is flowing. (Assume no appreciable change in the water level in the reservoir; take water density as 1000 kg m^{-3} and atmospheric pressure as 1.0 × 10^5 Pa.)

a The pressure is found from $p = p_{atm} + \rho gh$, so $p = 1.0 \times 10^5 + 1000 \times 9.8 \times 48 = 5.7 \times 10^5$ Pa.

b Applying Bernoulli's equation, $p_{atm} + 0 + 1000 \times 9.8 \times (48 + 18) = p_{atm} + \frac{1}{2} \times 1000 \times v_Y^2 + 0$, which gives $V_Y = 36\ ms^{-1}$. From the equation of continuity, the water speed at X obeys $V_X \times 0.50^2 = V_Y \times 0.24^2$ and so $V_X = 8.3\ ms^{-1}$.

c Applying Bernoulli's equation again we find $p_{atm} + 0 + 1000 \times 9.8 \times 48 = p_X + \frac{1}{2} \times 1000 \times 8.3^2$ and so $p_X = 5.4 \times 10^5$ Pa.

TEST YOURSELF B.15

Water is stored in a container. A hole of diameter 4.0cm is made in the container at a depth of 1.50m and the water flows out. Calculate the initial flow rate (volume per second) of the water.

Measuring fluid speed

The Bernoulli equation may be exploited in a number of situations to measure fluid speed.

Pitot tube

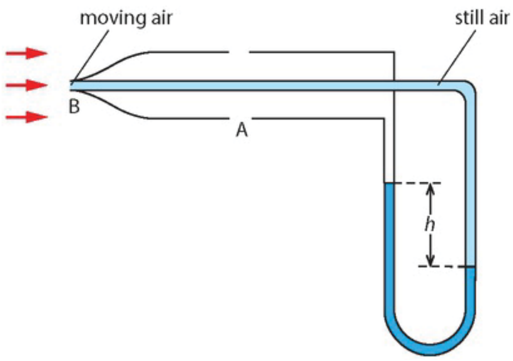


Figure B.26

In a Pitot tube (Figure B.26), a fluid – typically air – of density p is moving fast along the outside of the tube and so at hole A its speed is large and hence the pressure is low. Air at B, at the front of the tube, has been brought to rest and so the pressure here is high. The inner and outer spaces are connected to separate arms of a manometer, as shown.

Applying the Bernoulli equation to the streamline BA along the side of the Pitot tube, we find

$$p_{\mathrm{B}} + \tfrac{1}{2}\rho v_{\mathrm{B}}^2 + \rho g z_{\mathrm{B}} = p_{\mathrm{A}} + \tfrac{1}{2}\rho v_{\mathrm{A}}^2 + \rho g z_{\mathrm{A}}$$
$$p_{\mathrm{B}} + 0 + 0 = p_{\mathrm{A}} + \tfrac{1}{2}\rho v_{\mathrm{A}}^2 + 0$$
$$v_{\mathrm{A}} = \sqrt{\frac{2(p_{\mathrm{B}} - p_{\mathrm{A}})}{\rho}}$$

Venturi tube

In a Venturi tube (Figure B.27), a fluid flows from a tube with a large cross-sectional area A_1 to one with a small cross-sectional area A_2, and its speed changes from v_1 to v_2.

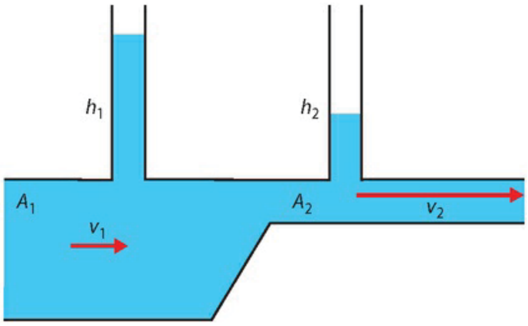


Figure B.27

The speeds are related by the equation of continuity:

$$A_1 v_1 = A_2 v_2, \text{ or } v_2 = v_1 \frac{A_1}{A_2}.$$

Bernoulli's equation says that $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$.

Hence $p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right)$.

But $p_1 - p_2 = \rho g h_1 - \rho g h_2$, so $\mathrm{v}_1 = \sqrt{\frac{2g(h_1 - h_2)}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}$

Model Answer B.5

You should not remember complicated formulae like the ones above. You can derive everything yourself! In figure B27 the following data apply: A_1 = 45 cm^2, A_2 = 25 cm^2, h_1 = 68 cm and h_2 = 52 cm. The liquid is water of density 1000 kg m^{-3}. Calculate V_1.

Applying the continuity equation we find: $45 \times v_1 = 25 \times v_2 \Rightarrow v_2 = \frac{45}{25} v_1$.

Applying Bernoulli's law to a horizontal streamline at the base of the tubes we fin d:

$\frac{1}{2} \times 1000 \times v_1^2 + 1000 \times 9.8 \times 0.68 + 0 = \frac{1}{2} \times 1000 \times \left(\frac{45}{25} v_1\right)^2 + 1000 \times 9.8 \times 0.52 + 0 \Rightarrow v_1 \approx 1.2 \mathrm{ms}^{-1}$

.

The airfoil

A very important application of the Bernoulli equation is the airfoil (Figure B.28).

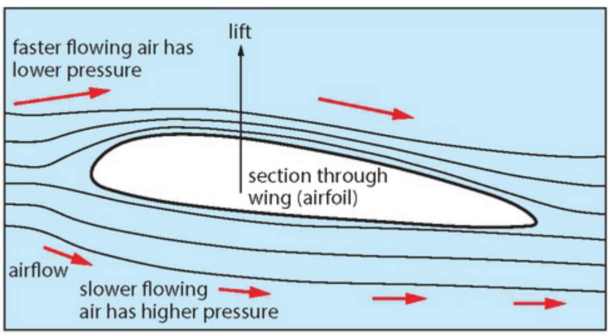


Figure B.28

Consider two identical flowtubes in front of the airfoil. One will go over the airfoil and the other under it. The one over the airfoil is thinner, indicating that the flow speed there is higher. Hence the pressure is lower, resulting in an upward force on the airfoil.

Stokes' law

Stokes' law gives the resistive drag force *F* on a small sphere of radius *r* that moves through a fluid with speed *v*:

$$F = 6\pi\eta r v$$

The coefficient *η* is known as the **viscosity** of the liquid, and is measured in Pa s. It is a measure of how the motion of one fluid layer affects a neighbouring layer.

Terminal speed

A ball falling through a fluid (Figure B.29) will experience the force of gravity downwards and the buoyant and drag forces upwards.

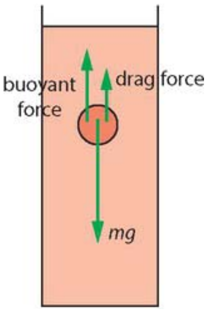


Figure B.29

As the sphere's speed increases, the drag force increases as well, and eventually the net force will become zero. The sphere will then fall at constant speed, its **terminal speed**.

The terminal speed is found by setting the net force equal to zero:

$$mg - 6\pi\eta r v - \rho_{\text{fluid}} V g = 0 \Rightarrow v = \frac{mg - \rho_{\text{fluid}} g}{6\pi\eta r}$$

Using $m = \rho_{\text{body}} V = \frac{4\pi r^3 \rho_{\text{body}}}{3}$, we find $\nu = \frac{2r^2 g}{9\eta}(\rho_{\text{body}} - \rho_{\text{fluid}})$.

Turbulence

Figure B.30a illustrates the laminar flow of a fluid with zero viscosity. Figure B.30b shows laminar flow of a viscous fluid: the fluid moves more slowly where it touches the pipe because of viscosity. In Figure B.30c the flow is turbulent: the magnitude and direction of the velocity vectors at different parts of the liquid have become chaotic and unpredictable.

An indication of the onset of turbulence in fluid flow in a pipe of radius *r* is given by the **Reynolds number**, a unit-less number given by $R = \frac{v\rho_{\text{fluid}} r}{\eta}$. Flow is non-turbulent when *R* is less than about 1000, and becomes turbulent for larger Reynolds numbers.

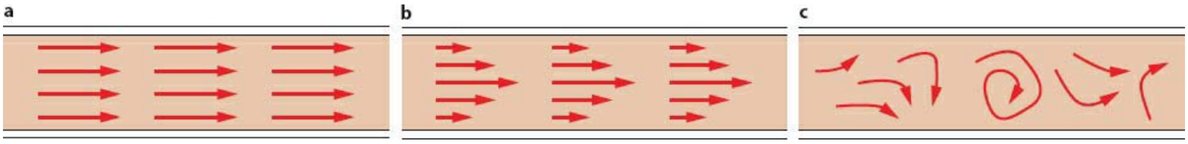


Figure B.30

B.4 Oscillations with damping

DEFINITIONS

DAMPED OSCILLATIONS Oscillations in which mechanical energy is reduced because of the presence of frictional forces, so that amplitude decreases.

DRIVEN OSCILLATIONS Oscillations in which an external periodic force acts on a system.

LIGHT DAMPING Oscillations in which a small frictional force leads to a gradual decrease in amplitude (see Figure B.31).

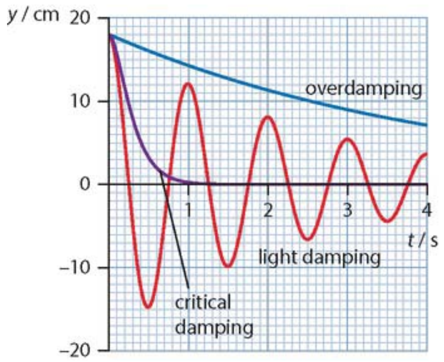


Figure B.31

CRITICAL DAMPING A degree of damping such that the system returns to its equilibrium position as fast as possible but without performing oscillations (see Figure B.31).

OVERDAMPING A degree of damping such that the system returns to equilibrium without oscillations in a time longer than that for critical damping (see Figure B.31).

RESONANCE A condition in which the driving frequency is equal to the natural frequency of the system, leading to a maximum amplitude of oscillation.

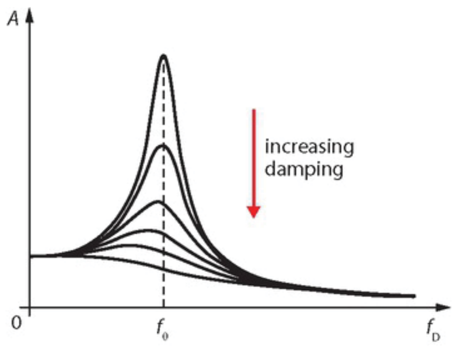


Figure B.32

Figure B.32 is graph of amplitude versus driving frequency for driven oscillations with various degrees of damping. For small damping (top curve), the amplitude is largest when the driving frequency f_D is equal to the natural frequency f_0 (we have resonance). For heavier damping (lower curves), the frequency at which the amplitude is largest is slightly lower than the natural frequency.

Risk assessment

Nature of Science. The phenomenon of resonance is ubiquitous in physics and engineering. Microwave ovens emit electromagnetic radiation in the microwave region of wavelengths that match the vibration frequencies of water molecules so they can be absorbed and warm food. In the medical technique known as MRI, the patient is exposed to a strong magnetic field and resonance is used to force absorption of radio frequency radiation by protons in the patient's body. This technique provides detailed images of body organs and cellular functions. Just as with buildings and bridges, detailed studies are necessary to avoid the possibility of unwanted resonances creating catastrophic large-amplitude oscillations.

The *Q* factor

The *Q* factor is a unit-less quantity that indicates how quickly an underdamped oscillation will die out. It is defined as:

$$Q = 2\pi \frac{\text{energy stored in each cycle}}{\text{energy dissipated in each cycle}}$$

For the red (light damping) curve in Figure B.31, the amplitude drops from 18 to 12 in the first cycle. Since stored energy is proportional to the square of the amplitude, the Q factor in this case is $Q = 2\pi\frac{18^2}{18^2-12^2} \approx 11$. In rough practical terms, the system will perform about 11 oscillations before dissipating most of its energy. An alternative expression for the Q factor is:

$$Q = 2\pi f\frac{E_{\text{stored}}}{P}$$

where E_{stored} is the energy stored in one oscillation, f is the oscillation frequency and P is the average power dissipated per cycle.

Response of a system to being driven

When a system is driven, there is a phase difference – a difference in timing – between the displacement of the system and the displacement of the driver. This phase difference depends on the relation between the natural and driving frequencies, as shown in Figure B.33.

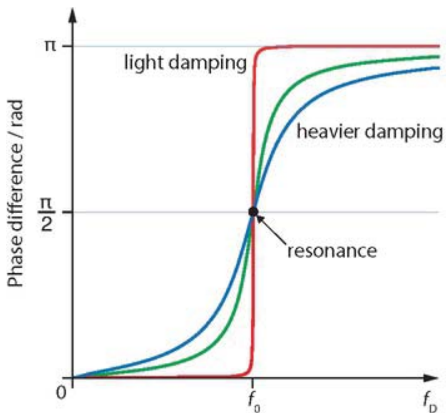


Figure B.33

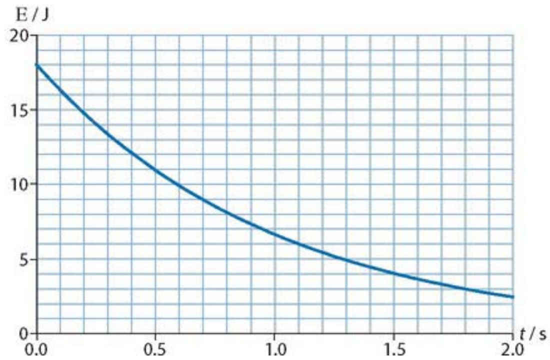
For very light damping the phase difference is zero if $f_D < f_0$ and π if $f_D > f_0$.

At resonance, $f_D = f_0$, the phase difference is $\frac{\pi}{2}$.

TEST YOURSELF B.16

The graph shows how the total energy of lightly damped system varies with time.

At $t = 0$ the system is at maximum displacement; the period of oscillations is 1.0 s. On a copy of this graph sketch a graph to show how the kinetic energy of the system varies with time.



Checklist

After studying this Option you should be able to:

- □ solve kinematic and mechanics problems for rotating rigid bodies
- □ calculate the work done on and by a gas
- □ describe various processes on a pressure-volume diagram
- □ apply the first law of thermodynamics
- □ understand the various versions of the second law of thermodynamics and the restrictions this imposes on heat engines
- □ calculate pressure changes in a fluid
- □ apply Archimedes' principle
- □ describe an ideal fluid and solve problems using the continuity and Bernoulli equations
- □ discuss the Bernoulli effect
- □ discuss damped and forced oscillations
- □ calculate the Q factor for a system.

C IMAGING

This option covers the following topics:

- □ Lenses and mirrors
- □ Lens and mirror defects
- □ Compound microscopes and telescopes
- □ Radio telescopes and interferometry
- □ Optic fibres
- □ Dispersion and attenuation
- □ Imaging techniques in medical diagnosis

C.1 Introduction to imaging

DEFINITIONS

PRINCIPAL AXIS The line through the centre of a lens and at right angles to its face.

FOCAL POINT OR FOCUS (PLURAL FOCI) The point on the principal axis through which all rays parallel to the principal axis pass, or appear to pass, after refraction through a lens.

FOCAL LENGTH, *f* The distance from the focal point to the centre of the lens.

REAL IMAGE An image formed by actual light rays.

VIRTUAL IMAGE An image formed by mathematical extensions of light rays. It cannot be displayed on a screen.

There is a focal point associated with each side of a lens, and the two foci are at equal distances from the centre of the lens.

A ray of light will, upon entering a lens, refract at both faces of the lens. In practice, for simplicity we draw a lens as a simple plane and show the bending taking place only at this plane. To construct an image we must know how at least two rays will refract through the lens.

Converging lenses

Determining the image by construction

Rays parallel to the principal axis which enter from one side of a converging lens refract through the focal point on the other side of the lens, as in Figure C.1a. Rays that are parallel to each other but not parallel to the principal axis refract as shown in Figure C.1b.

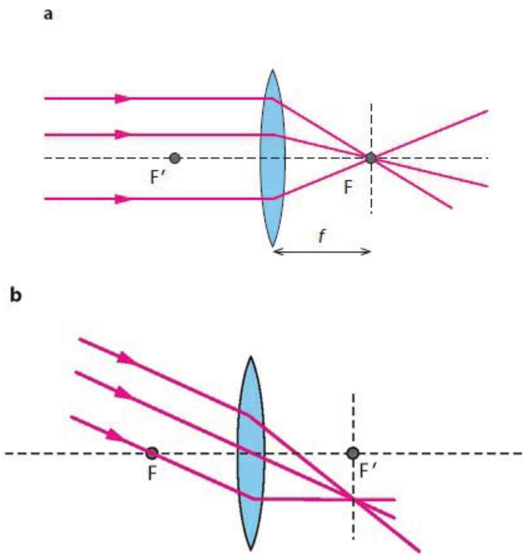


Figure C.1

To construct an image we can draw several **standard rays** from a point on the object and see where they converge, as shown in Figure C.2. Three standard rays refract through a converging lens: a ray entering parallel to the principal axis will exit through the far focal point, F; a ray entering through the near focal point F′ will exit parallel to the principal axis; a ray entering through the centre of the lens will exit with no change in direction.

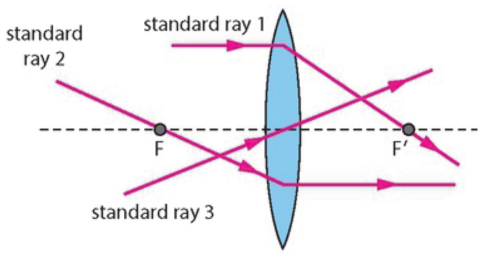


Figure C.2

A converging lens will form a real image if the object distance is greater than the focal length (Figure C.3) and a virtual image if the object distance is less than the focal length (Figure C.4).

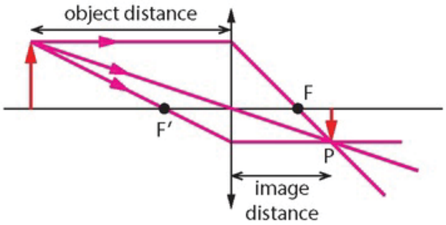


Figure C.3

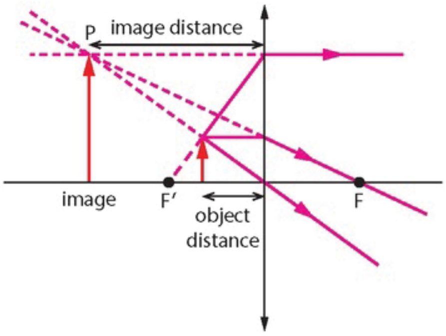


Figure C.4

Determining the image by calculation

The position, type and size of an image may also be determined by using the lens equation:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where:

u = object distance from lens

v = image distance from lens

f = focal length.

This equation is used to find the image distance v.

We can also use the formula for magnification:

$$M = \frac{h'}{h} = -\frac{v}{u}$$

where h' is the image height and h is the object height.

These formulas use the convention of positive v for a real image and negative v for a virtual image. Positive M represents an upright image and negative M an inverted image.

Worked Example C.1

An object of height 3.0 cm is placed 12 cm in front of a converging lens of focal length 4.0 cm. Determine the characteristics of the image.

We know that u = 12 cm and f = 4.0 cm, so $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{4.0} - \frac{1}{12} = \frac{1}{6} \Rightarrow v = 6.0\,\text{cm}$.

Since $v > 0$, the image is real.

The magnification is $M = -\frac{6.0}{12} = -0.50$, implying that the image is inverted ($M < 0$) and reduced ($|M| < 1$).

The image size is $Mh = -0.50 \times 3.0 = -1.5$ cm.

The diagram in Figure C.3 roughly corresponds to this situation.

TEST YOURSELF C.1

Determine the image characteristics for an object 3.0 cm tall that is placed 8.0 cm from a converging lens of focal length 6.0 cm.

TEST YOURSELF C.2

Copy and complete the ray diagram in Figure C.5 to form the image of the object.

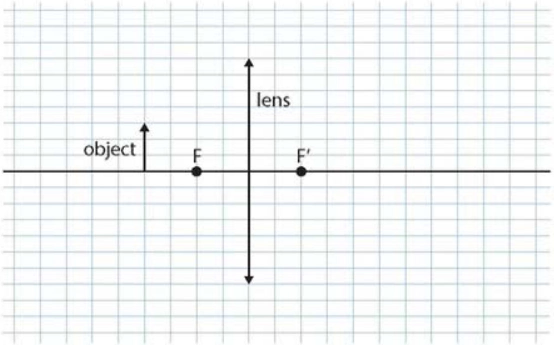


Figure C.5

TEST YOURSELF C.3

Copy and complete the ray diagram in Figure C.6 that will form the image of the object.

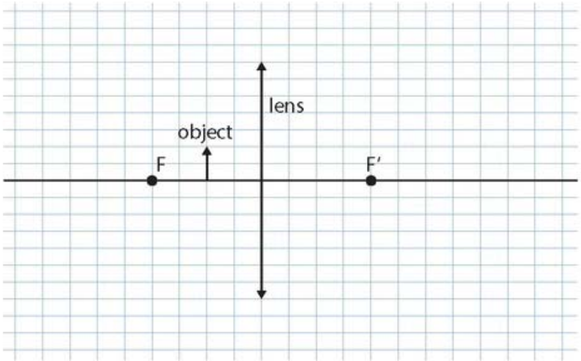


Figure C.6

Worked Example C.2

An object is placed 4.0 cm from a converging lens of focal length 12 cm. Determine the characteristics of the image.

We know that: u = 4.0 cm and f = 12 cm, so

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{12} - \frac{1}{4.0} = -\frac{1}{6} \Rightarrow v = -6.0\,\text{cm}.$$

Since $v < 0$, the image is virtual.

The magnification is $M = -\frac{-6.0}{4.0} = +1.50$, implying that the image is upright ($M > 0$) and larger ($|M| > 1$).

The diagram in Figure C.4 roughly corresponds to this situation.

TEST YOURSELF C.4

An object is placed in front of a converging lens and an image is formed. The top half of the lens is now covered with opaque paper. Discuss the changes, if any, in the appearance of the image.

Diverging lenses

Determining the image by construction

In a diverging lens, a beam of rays parallel to the principal axis will, after refraction in the lens, emerge in a direction away from the principal axis (Figure C.7). Figure C.8 shows the refraction of the three standard rays for a diverging lens.

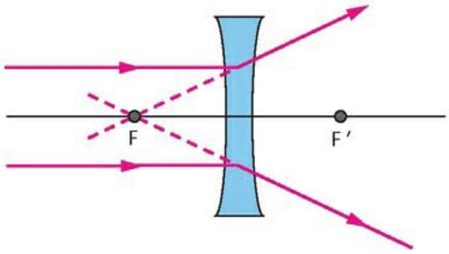


Figure C.7

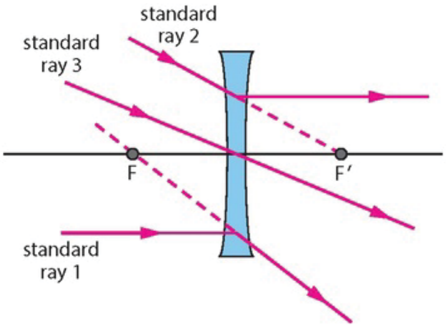


Figure C.8

Figure C.9 shows how to use the three standard rays to form the image for a diverging lens of focal length 10 cm. Viewed from the right, the image will be reduced and upright. Because it is virtual, it will appear to be on the left side of the lens.

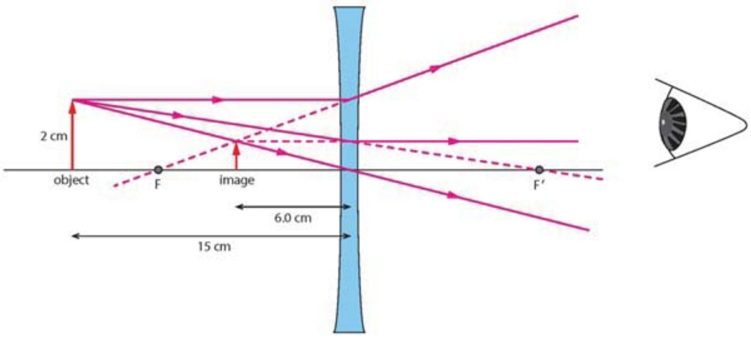


Figure C.9

Determining the image by calculation

The image characteristics can also be obtained using the lens equation. This is the same equation as for a converging lens except that the focal length for a diverging lens enters the formula as a negative number (−10 cm in this case). Since u = 15 cm, we have that

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{-10} - \frac{1}{15}$$

$$v = -6.0\,\text{cm}$$

The negative sign for v implies a virtual image. The magnification is $M = -\left(\frac{-6.0}{15}\right) = +0.40$, implying that the image is upright and 2.0 × 0.40 = 0.80 cm tall.

Lens combinations

Figure C.10 shows an object placed in front of two converging lenses that are 12 cm apart. The first lens creates an image of the object at point P, and in turn this image serves as the object for the second lens, forming the final image at Q.

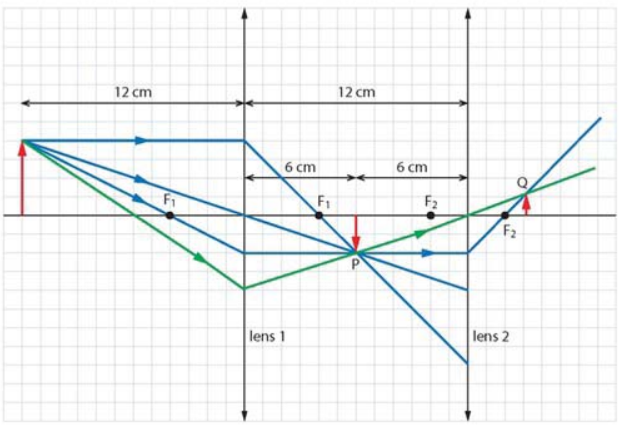


Figure C.10

The image in the first lens is easily constructed using the three standard rays (shown in blue).

However, only one of these blue rays is a standard ray for the second lens. In order to form the image in the second lens, we add the green ray, which leaves the top of the object at P and goes through the middle of the second lens to form the final image at Q.

(For completeness we could extend the green ray backwards through the first lens and to the top of the original object.)

TEST YOURSELF C.5

An object is placed in front of a pair of converging lenses, as shown in Figure C.11. This is the general arrangement of lenses in a compound microscope (see Section C.2).

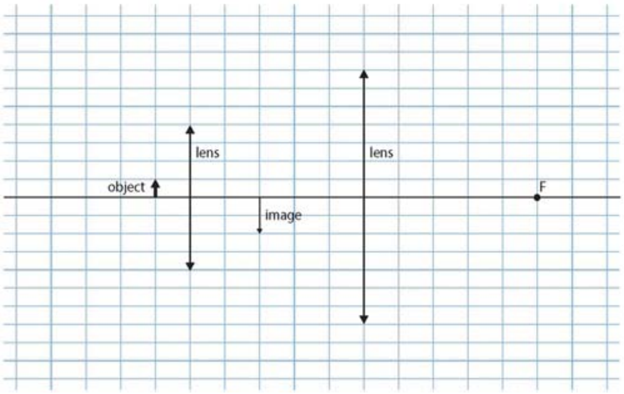


Figure C.11

The left-hand lens forms a real image of this object. In turn this serves as the object for the right-hand lens. The resulting image is virtual.

Using a copy of the diagram, draw rays to locate the final image.

The simple magnifier

DEFINITIONS

NEAR POINT The closest point to the eye where the eye can focus without strain. The distance to the near point is denoted by D and is about 25 cm for a healthy eye.

FAR POINT The furthest point from the eye where the eye can focus without strain. The distance to the far point is taken to be infinite for a healthy eye.

ANGULAR MAGNIFICATION The ratio of the angle subtended at the eye by the image to the angle subtended by the object.

How large an object appears to the eye depends on the angle the object subtends at the eye (Figure C.12). A larger, virtual image is produced when a small object is placed closer to a converging lens than the focal length (Figure C.13). The lens acts as a magnifier because the angle which the image subtends at the eye is greater than the angle without the lens.

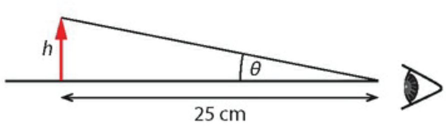


Figure C.12

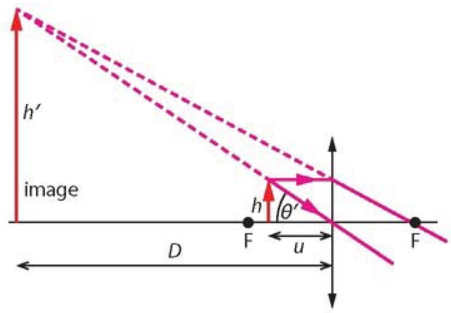


Figure C.13

The angular magnification is $M = \frac{\theta'}{\theta}$. Now, $\theta = \frac{h}{D}$ and $\theta' = \frac{h'}{D}$, so $M = \frac{h'}{h}$.

This is the same as the linear magnification, given by $\frac{-v}{u}$.

From the lens formula, $\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$, so $\frac{-v}{u} = 1 - \frac{v}{f}$. But in this case the image is virtual and formed at the near point, so $v = -D$, giving $M = 1 + \frac{D}{f}$.

This formula should be used when the image is at the near point; D is the near point distance.

A simple magnifier with an image formed at infinity

In Figure C.14, an object is placed at the focal point of the lens and so the image is formed at infinity.

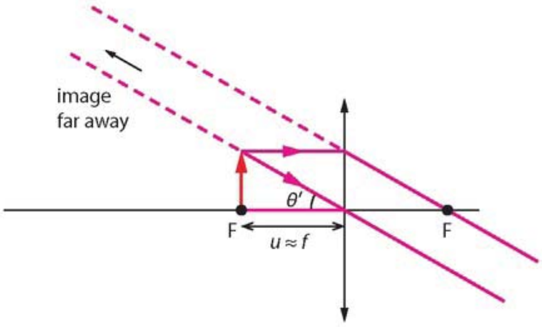


Figure C.14

In this case, $v = \infty$ and so $u = f$. Thus, $M = \frac{\theta'}{\theta} = \frac{h/f}{h/D} = \frac{D}{f}$.

This formula should be used when the image is at infinity.

Lens aberrations

Lenses suffer from two types of defects, also known as aberrations.

Chromatic aberration

Because glass has a slightly different refractive index for different wavelengths of light, light of different wavelengths will come to focus at slightly different points. This results in a coloured, blurred image (Figure C.15a).

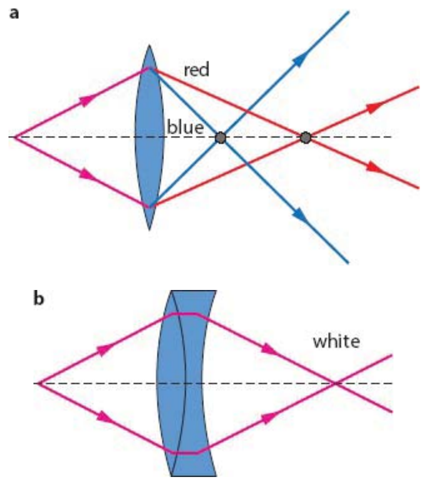


Figure C.15

The situation may be partially corrected by using the lens in combination with a second diverging lens (Figure C.15b). The chromatic aberration caused by the second lens is opposite to that caused by the first.

Spherical aberration

This type of aberration arises because rays far from the principal axis come to a focus closer to the lens than rays close to the axis. This results in an image with less detail, as well as blurring and curving at the edges (Figure C.16).

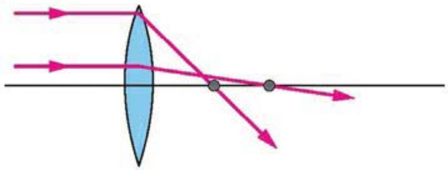


Figure C.16

Because the rays far from the axis also have a different magnification, an image of a straight edge object will appear curved (Figure C.17).

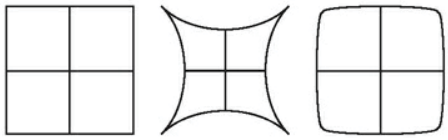


Figure C.17

The problem is reduced by blocking those rays that are far from the principal axis of the lens, in a procedure known as **stopping down.** This reduces the amount of light through the lens and so reduces the brightness of the image.

Wavefronts and lenses

A ray of light incident on a lens at a non-zero angle of incidence changes direction upon refraction. Since rays are normal to wavefronts, this means that the wavefronts will also change direction.

In the material of a lens, the speed of light is less than what is in air, and since the frequency stays the same, the wavelength is smaller – that is, the wavefronts inside the lens are closer together. When these waves cross a boundary with a curved surface, they will change shape. This is illustrated for plane waves refracted by a converging lens in Figure C.18, and for plane waves refracted by a diverging lens in Figure C.19.

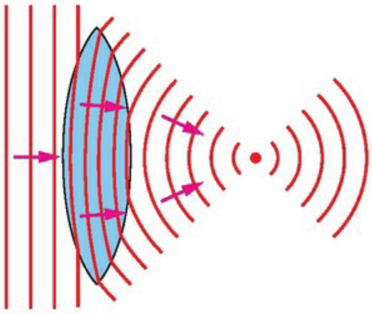


Figure C.18

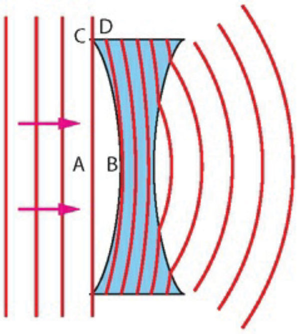


Figure C.19

Model Answer C.1

Planar wavefronts approach a diverging lens.

a Draw the wavefronts inside the lens and after they have refracted through the lens.
b Explain the shape of the wavefronts inside the lens.

a The shape of the wavefronts is given by Figure C.19.

b The diagram shows a wavefront AC that has just reached the lens. Point A will propagate in air to point B. Point C, however, will propagate inside the lens. The speed of light inside the glass is less than that in air and so the wavefront will move a smaller distance to D. Hence the new wavefront inside the lens will be bent.

Mirrors

Mirrors reflect light incident on them. Figure C.20a shows the reflection of three standard rays in a converging (or concave) mirror. Figure C.20b shows reflection by a diverging (or convex) mirror.

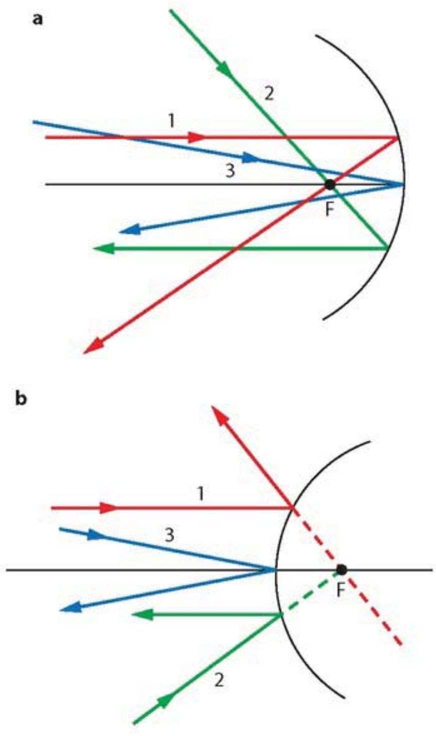


Figure C.20

The same equation used for lenses can also be applied to mirrors. The focal length of a concave mirror is taken to be positive, while that for a convex mirror is taken to be negative. In addition, we have here that the focal length *f* is half the radius *R* of the sphere from which the mirror was cut.

Worked Example C.3

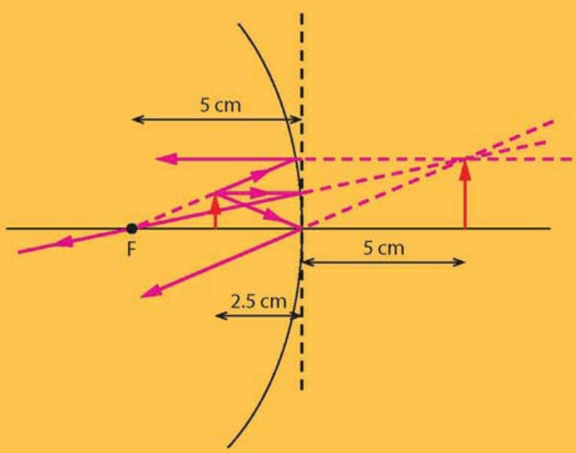


Figure C.21

An object of height 1.5 cm is placed 2.5 cm in front of a concave mirror of radius 10 cm. Determine the details of the image.

The focal length is half the radius, or 5.0 cm. Hence

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{5.0} - \frac{1}{2.5}$$

$$v = -5.0\,\text{cm}$$

The negative sign implies a virtual image; that is, the image appears as if it were inside the mirror.

The magnification is $M = -\left(\frac{-5.0}{2.5}\right) = +2.0$, implying an image that is upright and 2.0 × 1.5 = 3.0 cm tall. A ray diagram of this situation is shown in Figure C.21.

Mirrors do not suffer from chromatic aberration, but do suffer from spherical aberration just as lenses do.

C.2 Imaging instrumentation

The compound microscope

This is used to obtain an enlarged image of a small object placed very close to the microscope. It consists of two converging lenses: the objective (near the object) and the eyepiece (near the eye).

The (small) object is placed near the objective's focal point. The objective lens forms a real enlarged image of this object. This image acts as the object for the eyepiece and the eyepiece now acts as a magnifier for this object. See also Test yourself C.5.

In Figure C.22, we draw the three standard rays leaving the top of the object and extend these to form an object for the eyepiece lens. Of these, only the blue ray is a standard ray for the eyepiece. To proceed we need another construction ray; the green dashed line from the top of the image to the centre of the eyepiece lens, which will continue straight through that lens. We now extend this green line backwards. Where the green and blue lines meet (at Z) is the top of the final image.

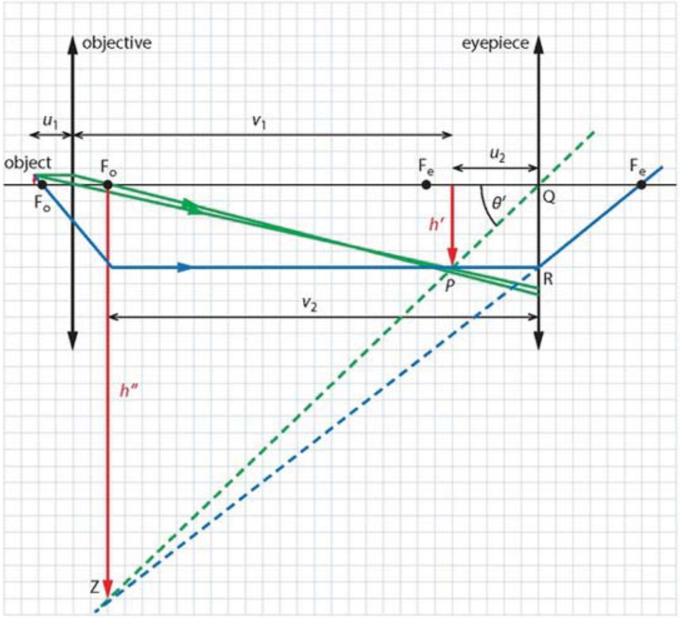


Figure C.22

The microscope is said to be in **normal adjustment** when the final image is formed at the near point of the eye, a distance D = 25 cm from the eyepiece. (In the diagram, each square horizontally corresponds to 1.0 cm.)

The magnification of the compound microscope

The magnification of a microscope is the product of the linear magnification produced by the objective times the angular magnification of the eyepiece. The objective produces a linear magnification of $M_o = -\frac{v_1}{u_1}$. The eyepiece produces an angular magnification of $1 + \frac{D}{f_e}$, where D is the near-point distance. Hence the overall magnification of the microscope is $M_o \times \left(1 + \frac{D}{f_e}\right)$. In the example in Figure C.22, $M_0 = -\frac{22}{2.2} = -10$ and $1 + \frac{D}{f_e} = 1 + \frac{25}{6.5} = 5.2$, for an overall magnification of −52. This means that a small object of size h will appear through the microscope to have a size of 52h as viewed by the naked eye at the near point.

Model Answer C.2

A compound microscope has an objective lens of focal length 1.25 cm and an eyepiece lens of focal length 5.30 cm. A small object is placed at a distance of 1.40 cm from the objective. The final image is formed a distance of 25 cm from the eyepiece. Calculate:

a the distance of the image in the objective from the objective
b the distance of this image from the eyepiece lens
c the magnification of the microscope.

a From $\frac{1}{V} = \frac{1}{f} - \frac{1}{u}$ we find $\frac{1}{V} = \frac{1}{1.25} - \frac{1}{1.40} \Rightarrow V = 11.67\,\text{cm}$

b We know that in the eyepiece v = −25.0 cm and so $\frac{1}{u} = \frac{1}{5.30} - \frac{1}{-25.0} \Rightarrow u = 4.37\,\text{cm}$

c We use $M = M_o \times \left(1 + \frac{D}{f_e}\right)$ to find $M = -\frac{11.67}{1.47} \times \left(1 + \frac{25.0}{5.30}\right) \approx -48$

The refracting telescope

A refracting telescope is constructed so that its two lenses are a distance $f_o + f_e$ apart; (Figure C.23).

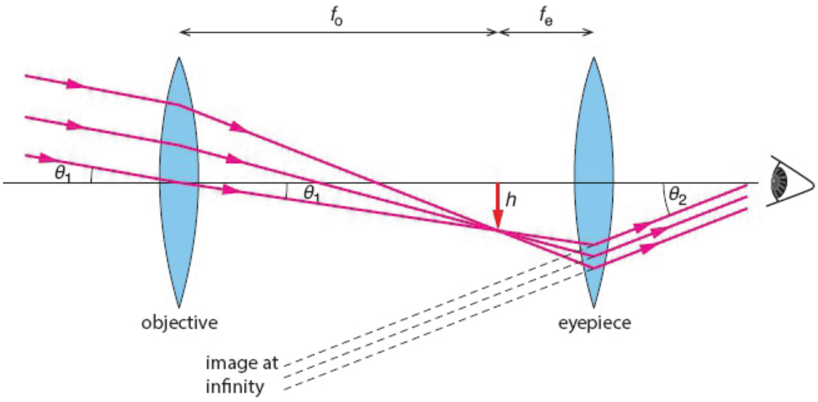


Figure C.23

Since the object is very far away, its image in the objective is formed at at the focal plane of the objective, which is also the focal point of the eyepiece. Therefore the final image is formed at infinity. A distant object subtends a very small angle θ_1 at the eye (much smaller than what is actually illustrated here), and the telescope magnifies that to a larger angle θ_2.

The angular magnification of the telescope is given by $M = \frac{\theta_2}{\theta_1} = \frac{f_o}{f_e}$.

The reflecting telescope

Refracting telescopes suffer from a number of disadvantages:

- To collect lots of light from a distant object requires a very large lens, but such lenses are hard to make and difficult to support.
- Lenses suffer from chromatic aberration.
- A lens has two surfaces which need to be polished to high precision, while a mirror has only one.

Therefore modern large telescopes use mirrors instead. These are reflecting telescopes. Figure C.24 shows two standard types of reflecting telescope, the Newtonian (a) and the Cassegrain (b). The mirror at the back is parabolic.

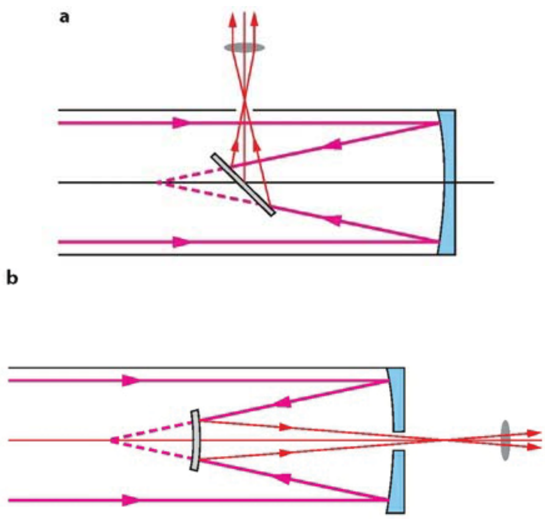


Figure C.24

Satellite-based telescopes

Telescopes based on satellites have several advantages over Earth-based ones:

- They are not subject to light pollution (ambient light makes it difficult to detect light from a distant faint source).
- There is no atmospheric turbulence or convection currents (these make the refractive index of air vary from place to place, resulting in fuzzy images).
- They can operate day and night.
- They can operate at all wavelengths (Earth-based telescopes are limited to wavelengths that are not absorbed by the atmosphere).

On the other hand, space-based telescopes are expensive to put up, repair and maintain.

The radio telescope

Many sources in the universe emit radio waves, and telescopes have been developed that receive radio waves.

The basic radio telescope is a parabolic dish. Good resolution demands small diffraction angles $\left(\theta \approx \frac{\lambda}{b}\right)$, but radio wavelengths are large, which requires that a radio telescope has a very large diameter b. The Arecibo radio telescope in Puerto Rico, shown in Figure C.25, operates at wavelengths of about 21 cm and has a diameter of 300 m.

Figure C.25

Radio interferometry

The resolution of radio telescopes may be improved by using signals collected by a large array of single-dish radio telescopes. The effective diameter of the array is the largest distance separating the individual telescopes in the array. This requires sophisticated techniques and computer power to assemble and combine the individual signals. Figure C.26 shows the VLA (Very Large Array) in New Mexico in the US.

Figure C.26

Radio, infrared and X-ray telescopes have greatly advanced our understanding of objects in the universe by complementing the knowledge derived from optical telescopes. The image in Figure C.27 is a combination of infrared, optical and X-ray images of a dying star in the constellation Cassiopeia.

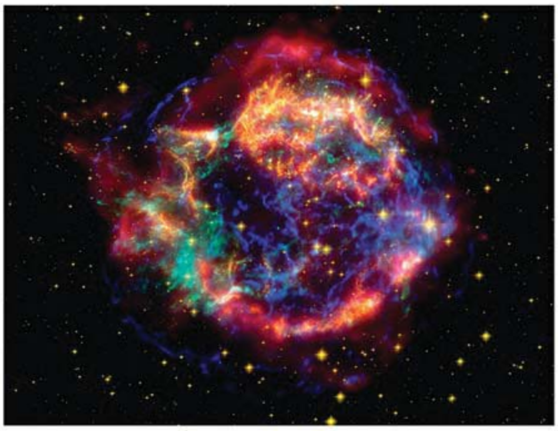


Figure C.27

C.3 Optical fibres

DEFINITIONS

DISPERSION The variation of wave speed with wavelength. Rays of light of different wavelength will, in general, arrive at the end of an optical fibre at different times. This may cause bits of information to arrive at the end of the fibre in the wrong order.

ATTENUATION The loss of power when a signal travels through a fibre, due mainly to impurities in its glass core.

POWER LOSS IN DECIBELS (dB) A logarithmic measure of signal power loss: $(\text{dB}) = 10 \log \frac{P_{\text{final}}}{P_{\text{initial}}}$.

NOISE Power due to unwanted signals that travels through a medium along with the signal of interest.

SIGNAL-TO-NOISE RATIO (SNR) A logarithmic measure of the strength of a signal compared to the accompanying noise: $\text{SNR}=10\log\frac{P_{\text{signal}}}{P_{\text{noise}}}$, where P_{signal} and P_{noise} are, respectively, the power of the signal and of the noise.

Ordinary amplifiers amplify noise as well as the signal, so they have no effect on the SNR.

Figure C.28 shows a ray of light entering a medium of low refractive index from a medium of high refractive index; the refracted ray bends away from the normal. As the angle of incidence is increased, the angle of refraction will eventually become 90°. The angle of incidence in this case is called the **critical angle,** θ_c.

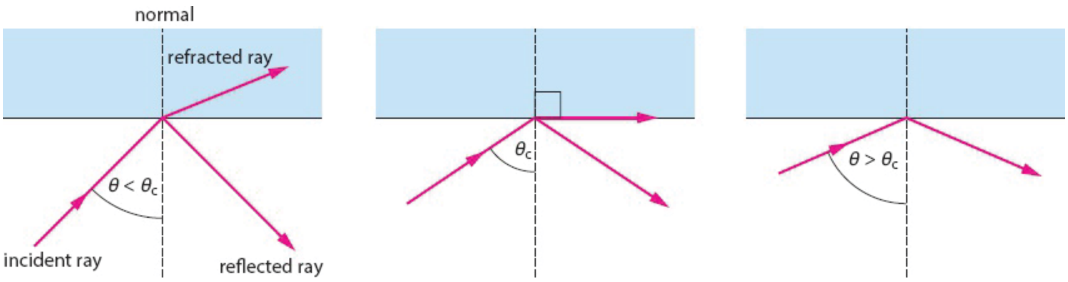


Figure C.28

For an angle of incidence greater than the critical angle, no refraction takes place: the ray simply reflects back into the medium from which it came. This is called **total internal reflection.**

The critical angle can be found from **Snell's law:** $n_1 \sin \theta_c = n_2 \sin 90^\circ$, so $\text{Sin}\,\theta = \frac{n_2}{n_1}$ and $\theta_c = \text{Sin}^{-1}\frac{n_2}{n_1}$.

The phenomenon of total internal reflection is exploited in **optical fibres.** An optical fibre consists of a thin glass core surrounded by a material of lower refractive index called the cladding (Figure C.29). The cladding protects the fibre but also prevents light from leaking from one fibre to a neighbouring one, as fibres are often used in bundles.

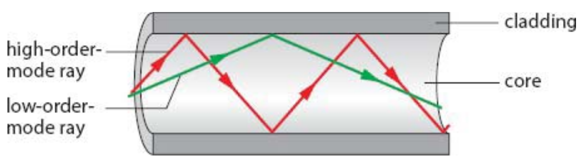


Figure C.29

The diagram shows two rays travelling along the fibre: the green ray undergoes fewer reflections and is called a **low-order** ray. The red ray undergoes many reflections and is called a **high-order** ray. A high-order ray travels a longer distance than a low-order ray as it traverses the same length of fibre.

Such thin fibres are flexible, and light can be sent down the length of the fibre core. For most angles of incidence, total internal reflection occurs so that the light stays within the core.

Applied science

Nature of Science. The development of optical fibres has been one of the main forces behind the revolution in communications that we experience today. The fast, clear and cheap transfer of information in digital form from one part of the world to another, with all that this implies about the free flow of information and immediate access to it, has a lot to do with the capabilities of modern optical fibres.

Model Answer C.3

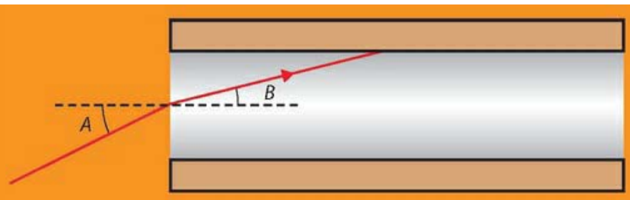


Figure C.30

An optical fibre consists of a glass tube of refractive index 1.50, surrounded by cladding of refractive index 1.40 (Figure C.30).

a Calculate the critical angle for the glass–cladding boundary.

b The diagram shows a ray of light that has entered the glass tube from air. Calculate the largest angle *A* for which the ray will undergo total internal reflection at the glass–cladding boundary.

a Applying Snell's law, we find $1.50 \times \sin \theta_c = 1.40 \times \sin 90°$ and so $\theta_c = \sin^{-1} \frac{1.40}{1.50} \approx 69°$.

b The angle of incidence at the glass–cladding boundary must be 69° or more, so the angle B must be $B = 90° - 69° = 21°$ or less. Hence $1.00 \times \sin A = 1.50 \times \sin B$, giving an acceptance angle of $A = 33°$ or less.

Dispersion and attentuation

There are two kinds of dispersion in an optical fibre:

- **Material dispersion:** Different wavelengths will travel through the glass core of an optical fibre at different speeds because wave speed depends on wavelength. Therefore, light rays of different wavelengths will reach the end of the fibre at different times, even if they follow the same path.
- **Waveguide (modal) dispersion:** Rays of light entering an optical fibre will in general follow different paths. High-order rays undergo very many internal reflections over a given distance while low-order rays undergo fewer reflections.

Dispersion limits the bit rate and the maximum frequency that can be transmitted. Material dispersion may be limited by using single-frequency light.

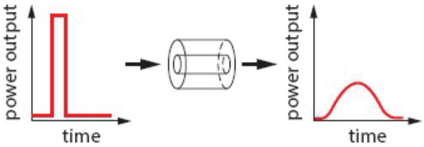


Figure C.31

In Figure C.31 the area under each plot of power versus time is the energy carried by the pulse. In an output pulse the area is somewhat lower because some energy has been lost during transmission.

Since the pulse width increases as a result of dispersion, the associated carrier wave period increases as well, and one consequence is that the maximum frequency that can be transmitted decreases.

Waveguide dispersion may also be minimised by using very thin fibres, called **monomode** fibres. Figure C.32 illustrates a variety of grades of optical fibre.

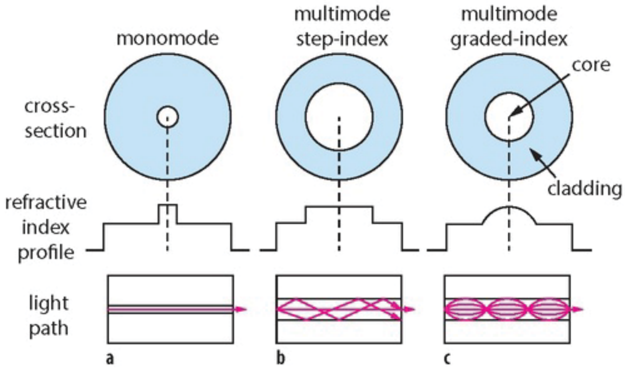


Figure C.32

- In a monomode fibre (panel a), the core is very thin and all rays follow essentially the same path, reducing waveguide dispersion.
- In a step-index fibre (panel b), the core has uniform refractive index. Rays follow paths consisting of straight line segments.
- In a graded-index fibre (panel c), the refractive index of the core decreases as we move away from the axis and towards the cladding. Rays follow curved paths. These fibres have reduced waveguide dispersion because rays on off axis paths cover the longer distances at higher speeds so take essentially the same time as axis rays.

The specific attenuation of an optical fibre depends on the wavelength of the light. It is a minimum for infrared wavelengths, which is why infrared light is used in optical fibre transmission.

TEST YOURSELF C.6

A signal of initial power 8.0 mW suffers a power loss of 16dB. Calculate the output signal power.

TEST YOURSELF C.7

A signal of power 180 mW is input to a cable of specific attenuation 3.0 dB km^{-1}. Calculate the power of the signal after it has travelled 12 km in the cable.

TEST YOURSELF C.8

An amplifier has a gain of 6.8 dB. The signal that is input to the amplifier has power 0.25 mW. Calculate the power of the output signal.

TEST YOURSELF C.9

A signal travels along a monomode fibre of attenuation per unit length 4.0 dB km^{-1}. The signal enters a number of equally spaced amplifiers, each providing a gain of 20 dB. Estimate how many amplifiers must be used so that after travelling a total distance of 400 km the signal emerges from the last amplifier with no power loss.

Advantages of optical fibres

Optical fibres are now used for high-quality telephone communication and the transfer of digital data. They provide a very high bandwidth, low attenuation and high security. These are the main advantages over older media of communication such as wire pairs and coaxial cables.

Wire pairs	Used only for simple links nowadays, for example connections to a loudspeaker. **Advantages:** cheap, easy to handle and install; wire pairs have reduced crosstalk. **Disadvantages:** susceptible to interference from other nearby wires (crosstalk), low bandwidth, needs frequent amplification, signal distortion due to dispersion.
Coaxial cables	Carry cable TV signals and telephone communications. **Advantages:** high bandwidth, reduced attenuation and crosstalk effects. **Disadvantages:** the average distance between amplification varies a lot with frequency. There is still noise.

TEST YOURSELF C.10

The specific attenuation of a coaxial cable is 14 dB km^{-1}. A signal of initial power 200 mW is input to such a cable.

a Calculate the power of the signal after it has travelled a distance of 3.0 km along the cable.

b State whether a signal of similar power to that in **a** but of much higher frequency would suffer a larger or smaller attenuation.

C.4 Medical imaging

DEFINITIONS

X-RAYS Short-wavelength (10^{-10} m) electromagnetic radiation.

ATTENUATION The reduction in the intensity of the X-rays transmitted due to absorption and/or spreading of the X-ray beam.

ATTENUATION COEFFICIENT, μ The probability per unit length that a particular X-ray photon will be absorbed.

HALF-VALUE THICKNESS, $X_{1/2}$ The distance after which the intensity of the transmitted X-rays is reduced to half the original intensity.

MASS ABSORPTION COEFFICIENT The ratio of the attenuation coefficient to the density of the medium.

X-ray imaging

The idea behind X-ray imaging is that different materials absorb different amounts of X-rays. One can therefore 'see' the boundaries between different materials. In general, attenuation coefficients depend on X-ray photon energy. The choice of energy is dictated by, for example, the need to have attenuation coefficients for soft tissue and bone which are as different as possible. In this way, a contrast between the images of bone and tissue is achieved.

Sharpness of image

An X-ray image can be made sharper by using an X-ray sorce that is as point-like as possible. The image is also made sharper by preventing stray X-rays from multiple scattering from reaching the photographic film. This is achieved by placing metal strips, oriented along the direction of the incident X-ray beam, between the patient and the film (Figure C.33).

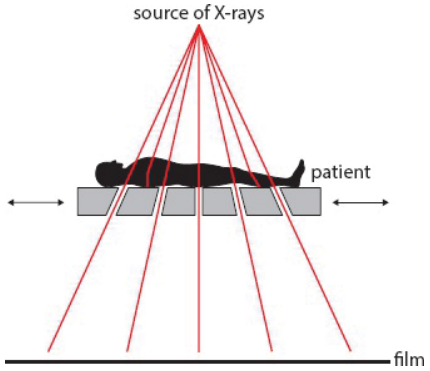


Figure C.33

Attenuation

After they have travelled a distance x through a material, the intensity of X-rays is reduced according to an exponential law: $I = I_0 e^{-\mu x}$ (Figure C.34). I_0 is the initial intensity of the beam.

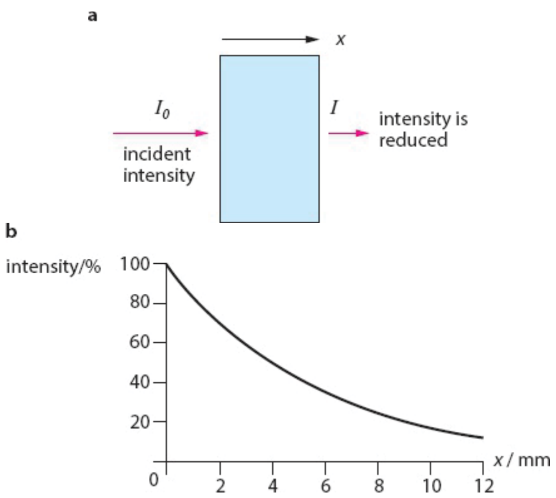


Figure C.34

TEST YOURSELF C.11

The attenuation coefficient for tissue at a particular energy is 0.52 mm^{-1}. Calculate the fraction of X-ray intensity that is transmitted through 1.2 mm of soft tissue.

TEST YOURSELF C.12

Two sources of attenuation of X-rays are the photoelectric effect (photons cause electrons to be emitted) and Compton scattering (photons scatter off electrons). The attenuation coefficient for the photoelectric effect depends on the atomic number Z of a sample ($\mu \propto Z^3$). For Compton scattering, μ is independent of Z. Suggest whether the photoelectric effect or Compton scattering is more useful for medical imaging with X-rays.

Relation between half-value thickness and attenuation coefficient

When $x = x_{1/2}$, $I = \frac{I_0}{2}$. Substituting into $I = I_0 e^{-\mu x}$, we find $\frac{I_0}{2} = I_0 e^{-\mu x_{1/2}}$, so $\frac{1}{2} = e^{-\mu x_{1/2}}$, so $\ln 2 = \mu x_{1/2}$.

(You should be able to derive this relation.)

Model Answer C.4

X-rays are incident on tissue and bone of equal thickness 5.0 cm.

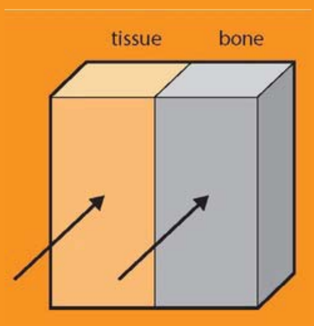


Figure C.35

For the X-ray energy used, the attenuation coefficients for tissue and bone are 0.14 cm^{-1} and 0.82 cm^{-1} respectively.

a Calculate the ratio of the intensity of X-rays transmitted through tissue to that through bone.

b Use the answer to (a) to explain why a good image of a fracture in a bone will be obtained with X-rays of this energy.

a $\frac{I_{tissue}}{I_{bone}} = \frac{I_0 e^{-0.14\times4.0}}{I_0 e^{-0.82\times4.0}} = 29.96 \approx 30$

b There is a clear difference between the intensities transmitted though tissue and bone so there will be a good contrast between the images of tissue and bone.

Intensifying screen

X-rays need a considerably longer time to expose photographic film than visible light does, and this means that the exposure of a patient to X-rays can be long. To reduce this, intensifying screens are used: X-rays incident on the screens cause visible light to be emitted and this light can then be used to expose photographic film (Figure C.36).

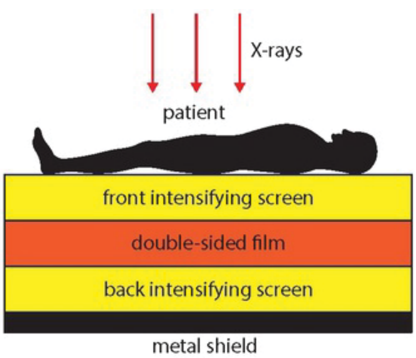


Figure C.36

Barium meal

In situations where there is no substantial difference between the attenuation coefficients of different parts of the body, a contrast may be achieved with a so-called ‘barium meal’. The patient swallows a quantity of a barium compound, which absorbs X-rays more strongly than the tissue surrounding the intestinal tract.

Computed tomography

In computed tomography (CT), X-rays are directed at the patient in a series of horizontal planes, and the transmitted radiation is detected on the opposite side. This horizontal ‘slice’ is divided into a large number of pixels and a computer reconstructs the amount of radiation absorbed in each pixel. In this way an image of the slice is constructed (Figure C.37).

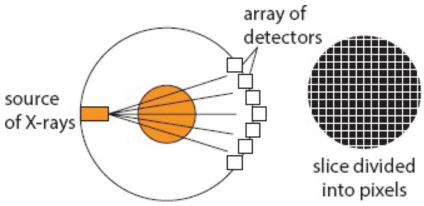


Figure C.37

This procedure is repeated for adjacent ‘slices’ so that a three-dimensional image is built up. A computer can then rotate this image so it can be viewed from various directions.

A CT image is thus superior to the plain two-dimensional images obtained with ordinary X-ray imaging techniques, and also produces images much faster (which can be critical in the case of an accident). However, the patient receives substantially larger quantities of radiation.

Model Answer C.5

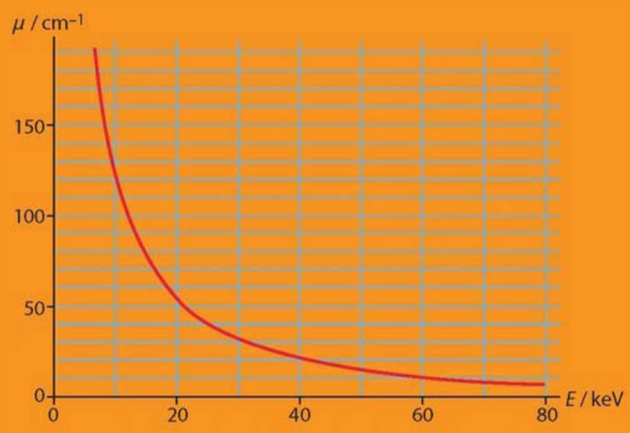


Figure C.38

Figure C.38 shows the variation of the attenuation coefficient μ in soft tissue with X-ray photon energy E.

a Define ‘attenuation coefficient’.

b To obtain an X-ray image in soft tissue, X-rays of energy 50 keV are being used. Use the graph to state and explain whether any significant change will occur to the quality of the image if low-energy X-rays are filtered out.

c Calculate the fraction of the incident intensity that is transmitted through 2.2 mm of soft tissue for X-rays of energy 50 keV.

a This is the probability per unit length that a particular X-ray photon will be absorbed.

b No significant change is expected. The low-energy photons would be absorbed and would not contribute to the image.

c The attenuation coefficient at 50 keV is 1.5 mm^{-1} and so

$$I = I_0 e^{-\mu x} = I_0 e^{-1.5\times 2.2} = 0.037 I_0$$

Ultrasound

Ultrasound is sound of frequency higher than the 20 kHz limit of the human ear. Typically, ultrasound for medical uses has a frequency in the MHz region. Ultrasound is produced by crystals that are subjected to an alternating voltage that forces the crystal to vibrate and emit ultrasound, a phenomenon known as **piezoelectricity.**

DEFINITIONS

IMPEDANCE The product of the density of a medium times the speed of sound in the medium.

IMPEDANCE MATCHING The reduction of reflected intensity when ultrasound travels between two regions, accomplished by making their impedances equal or similar.

A-SCAN (AMPLITUDE MODULATED) A map of the intensity of the reflected ultrasound signal from the skin and various organs inside the body.

B-SCAN (BRIGHTNESS MODULATED) Conversion of an A-scan pattern into a pattern of dots whose brightness is proportional to the signal strength.

An ultrasound pulse is directed at the body by placing a **transducer** (a probe which can emit sound signals and also detect them) at a point on the skin. Pulses will be detected which have reflected from various interfaces in the body, each pulse delayed according to the time it takes for it to reach an interface and be reflected (Figure C.39).

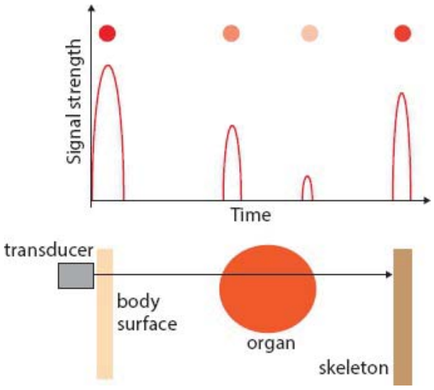


Figure C.39

In the diagram, the first pulse is a reflection from the surface of the skin. The next two are from the front and the back surfaces of an internal organ. The last pulse is from bones in the skeleton. The diagram is a graph of the strength of the reflected signal versus time. The fact that the signal strength decreases is evidence of attenuation (absorption of energy by the body).

Combining a series of images from transducers at various positions, in the form of dots whose brightness indicates the reflected sugnal strength, produces a **two-dimensional image** of the surfaces that have caused the reflections. The result is known as a B-scan (Figure C.40).

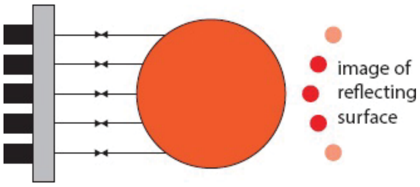


Figure C.40

TEST YOURSELF C.13

Explain why pulses rather than continuous waves of ultrasound are emitted by the transducer in ultrasound imaging.

Annotated Exemplar Answer C.1

a By reference to diffraction, state one advantage of using high-frequency ultrasound in medical diagnosis. **[2]**

b Explain why, in medical diagnosis using ultrasound, a gel-like substance is placed on the patient's skin between the skin and the probe. **[2]**

a *High-frequency ultrasound has a small wavelength (about 0.15 mm). This allows smaller structures to be detected than with ultrasound of a lower frequency, which would have a longer wavelength.*

The answer shows that you understand that the higher the frequency, the shorter the wavelength, and that the size of the object that can be imaged depends on the wavelength. But you lose a mark for not making the link to diffraction. Always check that you have answered the question.

b *The gel has impedance close to that of tissue and so most of the ultrasound gets transmitted into the body. The ultrasound that enters the body gets reflected off organs and can also exit the body so it can be picked up by a detector.*

You need to mention what happens if the impedances do not match. Without the gel, most of the ultrasound would be reflected at the interface between air and skin. Make sure you include all relevant detail in your written answer.

2/4

It is important to understand why a gel is needed in ultrasound diagnosis. The actual formulas for transmitted and reflected intensity are $\frac{I_t}{I_0} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$ and $\frac{I_r}{I_0} = \frac{(Z_1-Z_2)^2}{(Z_1+Z_2)^2}$, but these are not given in the data booklet. If they are needed, they will be provided in the exam materials.

Nuclear magnetic resonance imaging

In the imaging technique known as magnetic resonance imaging (MRI), the patient is not exposed to any harmful radiation, which is a major advantage over CT scanning. Other advantages include the high quality of the images obtained and the ability to view any cross-section of the body. One disadvantage is the high cost of MRI equipment.

The technique is based on a property of protons called **spin.** A proton's spin will align parallel or anti-parallel to an applied magnetic field, and the energy of the proton will depend on whether its spin is up (i.e. parallel to the magnetic field) or down (opposite to the field). The state with spin up has a lower energy than that of spin down. The energy difference between these states depends on the magnetic field strength.

The patient is placed in an enclosure that creates a very uniform magnetic field throughout the body (Figure C.41).

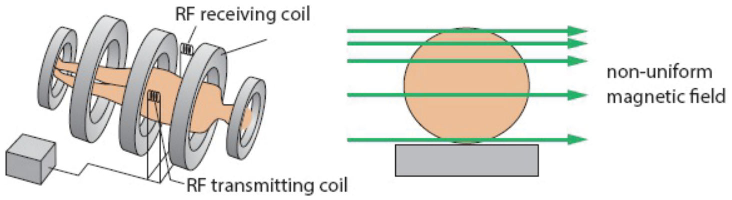


Figure C.41

A source of radiofrequency radiation forces spin-up protons to jump to the higher-energy spin-down state. The protons then make a transition back down to the spin-up state, emitting a photon in the process.

These emitted photons can be correlated with the magnetic environment around the point from which they were emitted. This is done by adding a further, **non-uniform** magnetic field. Since the energy difference between spin-up and spin-down states depends on magnetic field strength, only protons in regions where the magnetic field has a desired value will give rise to photon emission. Techniques similar to CT scanning are then used to create a three-dimensional image, with the detected photons correlated with specific regions of the body.

Checklist

After studying this Option you should be able to:

- ☐ **Construct ray diagrams for lenses and mirrors**
- ☐ **Describe lens and mirror defects**
- ☐ **Construct rays diagrams for the compound microscopes and the refracting telescope**
- ☐ **Describe radio telescopes and interferometry**
- ☐ **Solve problems with dispersion and attenuation in optic fibres**
- ☐ **Describe imaging techniques in medical diagnosis including X-rays, ultrasound and MRI**
- ☐ **Solve problems with X-ray attenuation**

< Back to chapter

Annotated Exemplar Answer C.1

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b The gel has impedance close to that of tissue and so most of the ultrasound gets transmitted into the body. The ultrasound that enters the body gets reflected off organs and can also exit the body so it can be picked up by a detector.

You need to mention what happens if the impedances do not match. Without the gel, most of the ultrasound would be reflected at the interface between air and skin. Make sure you include all relevant detail in your written answer.

2/4

D ASTROPHYSICS

This option covers the following topics:

- □ Stellar quantities
- □ Stellar characteristics and stellar evolution
- □ Cosmology
- □ Further stellar evolution
- □ Further cosmology

D.1 Stellar quantities

DEFINITIONS

THE SOLAR SYSTEM Our solar system consists of eight planets that revolve around a star we call the Sun. The order of the planets, starting from the one closest to the Sun, is: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune (remembered as My Very Elegant Mother Just Served Us Noodles). All of these except the first two have moons orbiting them.

ASTEROID One of very many small, rocky objects in orbit around the Sun in between the orbits of Mars and Jupiter.

COMET A small object of ice, rock and frozen gases whose nucleus is about 10 km across. Those comets that orbit the Sun have very large, very elliptical orbits. Some comets do not orbit the Sun and just appear once within the solar system. When a comet is near the Sun, gases stream out from it for thousands of kilometers in a direction away from the Sun.

ASTRONOMICAL DISTANCES The average distance between the Earth and the Sun is called an **astronomical unit** (AU); 1 AU = 1.50×10^{11} m. The diameter of the solar system is about 80 AU. The distance travelled by light in one year is called a **light year** (ly); 1 ly = 9.46×10^{15} m. Another convenient unit is the **parsec** (pc); 1 pc = 3.09×10^{16} m.

STAR A normal star is a large mass consisting mostly of hydrogen, with a temperature in its core that is high enough for nuclear fusion reactions to take place. The energy released in these reactions produces an outward pressure on the outer layers of the star, thus preventing gravity from collapsing the star. A star is in equilibrium under the action of the opposing pressures of gravity and radiation.

BINARY STAR A pair of stars orbiting a common centre. The stars have a common period of revolution, which means they are always diametrically opposite each other. The common period is determined by the separation of the stars and their total mass. The heavier star is in the inner orbit.

STELLAR CLUSTER A large number of stars relatively close to each other that affect each other through their gravitational forces. Stellar clusters are divided into **globular** and **open** clusters. Stars in globular clusters tend to be closer to one another and are mainly old, evolved stars; open clusters contain young stars further apart than those in globular clusters.

CONSTELLATION A group of stars in a recognisable pattern as seen from Earth, though not necessarily close to each other in space. Stars and constellations appear to revolve around the Earth in the course of the night because of Earth's rotation on its axis. In the course of a year, as the Earth revolves around the Sun, different parts of the night sky are visible from Earth and so different stars and constellations are visible.

NEBULA A cloud of gases and dust extending for light years across space. It is formed by the mass ejected when a red giant or supergiant explodes.

GALAXY A collection of very many stars (of the order of 10^9) that attract one another gravitationally. A galaxy can be spherical, elliptical, spiral or irregular in shape. The average separation of stars in a galaxy is of the order of a few pc, and the average separation of galaxies is of the order of a few hundred kpc.

CLUSTERS AND SUPERCLUSTERS OF GALAXIES Galaxies are often found in groups called clusters, and clusters are grouped together in superclusters.

Radiation from stars

DEFINITIONS

LUMINOSITY, L The total power radiated by a star, given by $L = \sigma A T^4$, where A is the star's surface area and T its surface temperature in kelvin. The constant σ is the **Stefan–Boltzmann constant**, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}. The unit of luminosity is the watt (W).

APPARENT BRIGHTNESS, b The power received per unit area, a measure of how bright a star appears from Earth. It is given by $b = \frac{L}{4\pi d^2}$, where d is the distance to the star from Earth. The unit of apparent brightness is W m^{-2}. A star with low luminosity may appear brighter than another star of higher luminosity because it is closer.

Figure D.1 shows the **proton–proton cycle**, in which hydrogen fuses with hydrogen to produce helium. The outflow of energy from fusion reactions in the core of a star (Figure D.2) produces an outward radiation pressure that balances the gravitational pressure.

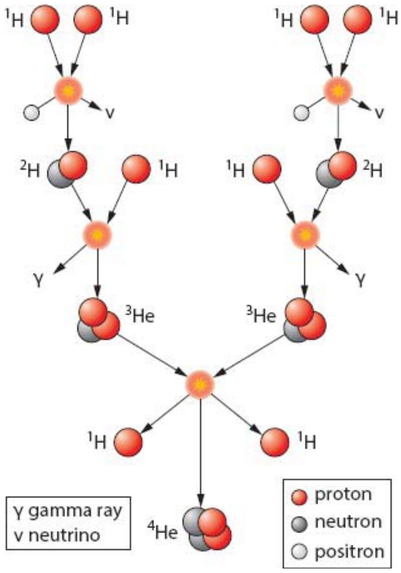


Figure D.1

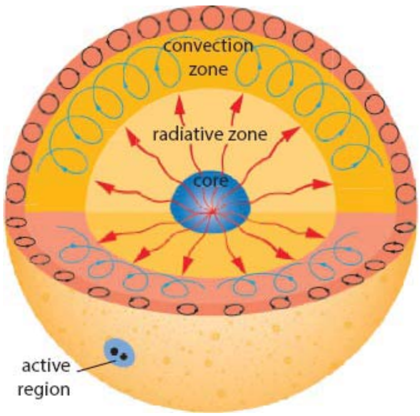


Figure D.2

Distances to stars: the parallax method

The parallax method consists of observing a star at two different times, six months apart. In the six months, the Earth has moved in its orbit around the Sun and so the star will be observed to have shifted relative to the background of the very distant stars (Figure D.3).

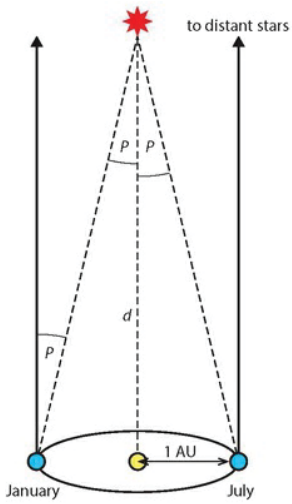


Figure D.3

Typically the parallax angle is very small, so tan $p \approx p = \frac{1\text{AU}}{d} \Rightarrow d = \frac{1\text{AU}}{p}$. Since $d \propto \frac{1}{p}$, we define a new unit of distance, the parsec (pc), as the distance at which the parallax angle is 1 arc second ($1''$ or $\frac{1}{3600}$ of a degree). Thus, if the parallax angle in arc seconds is *p*, the distance in pc is given by $d = \frac{1}{p}$. In metres, $1\text{pc}=\frac{1\text{ AU}}{\tan 1''} = \frac{1.49\times10^{11}}{\tan\frac{1}{3600}} = 3.1 \times 10^{16}\text{m}$.

Worked Example D.1

The distance to the star Wolf-359 is 4.93×10^5 AU and its apparent brightness is 1.97×10^{-12} W m^{-2}. Calculate its luminosity.

This requires direct substitution into the formula $b = \frac{L}{4\pi d^2}$. Solving for luminosity, we get:

$$L = 4\pi d^2 b = 4\pi \times (1.97 \times 10^{-12}) \times (4.93 \times 10^5 \times 1.50 \times 10^{11})^2 = 1.35 \times 10^{23}\,\text{W}$$

Model Answer D.1

Star A is at a distance of 100 pc from Earth. Its luminosity is 4 times greater than that of star B and its apparent brightness is 100 times greater than that of star B. Deduce that the distance of star B from Earth is 500 pc.

We apply $b = \frac{L}{4\pi d^2}$ to star A and star B to get $b_A = \frac{L_A}{4\pi d_A^2}$ and $b_B = \frac{L_B}{4\pi d_B^2}$. Dividing the equations side

by side:

$$\frac{b_A}{b_B} = \frac{\frac{L_A}{4\pi d_A^2}}{\frac{L_B}{4\pi d_B^2}} = \frac{L_A}{L_B}\frac{d_B^2}{d_A^2}, \text{ so } 100 = 4\frac{d_B^2}{d_A^2} \Rightarrow \frac{d_B^2}{d_A^2} = 25 \Rightarrow \frac{d_B}{d_A} = 5 \Rightarrow d_B = 5 \times 100$$

$$= 500\text{pc}$$

(This is a comparison or ratio problem, so units do not have to be changed. The original distance was in pc so the new distance will be in pc as well. Note the very useful techinique of dividing equations side by side – that is, divide the left side of one by the left side of the other, and the right side of one by the right side of the other, which still gives an equality.)

Annotated Exemplar Answer D.1

From Earth, the apparent brightness of the Sun is $1.4 \times 10^3\,\text{W m}^{-2}$. Spica is 260 ly away and has a luminosity that is 6000 times that of the Sun. Deduce the apparent brightness of Spica. **[3]**

Convert the distance from light years to metres:
$260 \times 9.46 \times 10^{15}\,\text{ly} = 2.5 \times 10^{18}\,\text{m}$

Then use $b = \frac{L}{4\pi d^2}$ with $L = 6000$

$b = \frac{6000}{4\pi \times (2.5 \times 10^{18})^2} = 7.6 \times 10^{-35}\,\text{W m}^{-2}$

Good – you have explained what you are doing. The conversion is in the data booklet, so you don't need to remember it.

The luminosity is not just 6000, but 6000 times that of the Sun, which is not given. This is a comparison problem, so you need to find an expression for the Sun's luminosity and use that:

$b_{Sun} = \frac{L_{Sun}}{4\pi(1.50 \times 10^{11})^2} = 1.4 \times 10^3$, so

$L_{Sun} = 1.4 \times 10^3 \times 4\pi\,(1.50 \times 10^{11})^2$

(The value for the Earth–Sun distance is in the data booklet – make sure you are very familiar with this booklet.)

With the right value of L, this would give the correct answer. Don't carry out the calculation until you have all the numbers:

$b_{Spica} = \frac{6000 \times 1.4 \times 10^3 \times 4\pi\,(1.5 \times 10^{11})^2}{4\pi \times (2.5 \times 10^{18})^2}$

Cancel 4π to make the calculation simpler. The result is $3.0 \times 10^{-8}\,\text{W m}^{-2}$. (The value for the Earth–Sun distance is in the data booklet – make sure you are very familiar with this booklet.)

1/3

TEST YOURSELF D.1

A binary star system consists of two stars, A and B. The temperature of A is 5000 K and that of B is 25000 K. The luminosity of A is 36 times that of B. Calculate the ratio of the radius of A to that of B.

< Back to chapter

Annotated Exemplar Answer D.1

From Earth, the apparent brightness of the Sun is 1.4×10^3 W m^{-2}. Spica is 260 ly away and has a luminosity that is 6000 times that of the Sun. Deduce the apparent brightness of Spica. **[3]**

Convert the distance from light years to metres:

$$260 \times 9.46 \times 10^{15} ly = 2.5 \times 10^{18} m$$

$$\text{then use } b = \frac{L}{4\pi d^2} \text{ with } L = 6000$$

$$b = \frac{6000}{4\pi \times (2.5 \times 10^{18})^2} = 7.6 \times 10^{-35} wm^{-2}$$

With the right value of L, this would give the correct answer. Don't carry out the calculation until you have all the numbers:

$$b_{Spica} = \frac{6000 \times 1.4 \times 10^3 \times 4\pi(1.5 \times 10^{11})^2}{4\pi \times (2.5 \times 10^{18})^2}$$

Cancel 4π to make the calculation simpler. The result is 3.0×10^{-8} W m^{-2}. (The value for the Earth–Sun distance is in the data booklet – make sure you are very familiar with this booklet.)

Good – you have explained what you are doing. The conversion is in the data booklet, so you don't need to remember it.
The luminosity is not just 6000, but 6000 times that of the Sun, which is not given. This is a comparison problem, so you need to find an expression for the Sun's luminosity and use that:

$$b_{sun} = \frac{L_{sun}}{4\pi(1.50 \times 10^{11})^2} = 1.4 \times 10^3 \text{ , so}$$

$$L_{sun} = 1.4 \times 10^3 \times 4\pi(1.50 \times 10^{11})^2$$

(The value for the Earth–Sun distance is in the data booklet – make sure you are very familiar with this booklet.)

1/3

D.2 Stellar characteristics and stellar evolution

DEFINITIONS

WIEN'S LAW The wavelength λ_0 at which the emitted energy of a black body is a maximum is related to its surface temperature T by $\lambda_0 T = 2.9 \times 10^{-3}$ m K.

CEPHEID STAR OR CEPHEID VARIABLE A star whose brightness varies with a well-defined period. This period is related to the star's luminosity and so can be used to estimate the distance to the star.

HERTZSPRUNG –RUSSELL (HR) DIAGRAM A plot of the luminosity of stars versus their surface temperature.

MAIN -SEQUENCE STAR A star undergoing fusion of hydrogen into helium. Such stars occupy a diagonal strip on an HR diagram from top left to bottom right.

RED GIANT A large, relatively cool, reddish star which occupies the upper right region of the HR diagram.

RED SUPERGIANT A very large, relatively cool, reddish star which occupies the extreme upper right region of the HR diagram.

WHITE DWARF A very small, hot, dense star occupying the lower left region of the HR diagram. This is the end stage in the evolution of low mass stars.

NEUTRON STAR A very hot, very dense star which is the end stage in the evolution of massive stars.

CHANDRASEKHAR LIMIT The largest mass a white dwarf star can have. It is about 1.4 solar masses.

OPPENHEIMER–VOLKOFF LIMIT The largest mass a neutron star can have. It is about 3 solar masses.

Stars radiate as black bodies. Therefore the variation of their intensity with wavelength is a typical black-body curve. The peak wavelength is related to surface temperature according to Wien's law (Figure D.4).

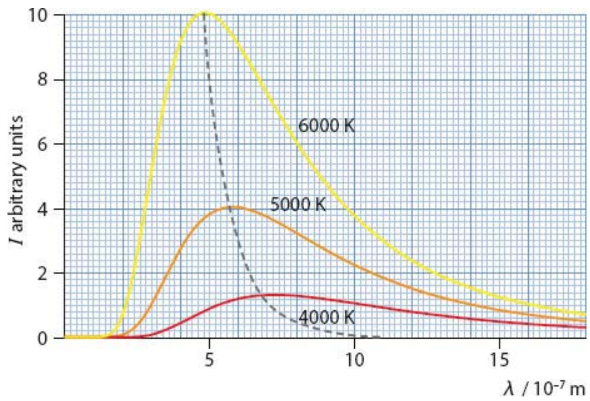


Figure D.4

TEST YOURSELF D.2

1 A star has a surface temperature of 3000 K. Calculate the wavelength at which most of its energy is emitted, and state the colour of the star.

Chemical composition from stellar spectra

We learned in Chapter 7 that when light that has been passed through a gas is analysed, dark lines appear on a background of the colours of the rainbow. The dark lines correspond to photon wavelengths that have been absorbed by the atoms of the gas. A similar thing happens when the light that is produced within a star moves outwards through the outer layers of the star. Specific elements absorb specific wavelengths and so the dark lines give us information about what elements exist in the star.

A real absorption spectrum of a star contains many dark lines. Figure D.5 shows typical absorption spectra for stars of different temperature. Notice the dominance of hydrogen and helium lines, indicating the presence of these elements in these stars. There are many other dark lines, indicating the presence of other elements as well.

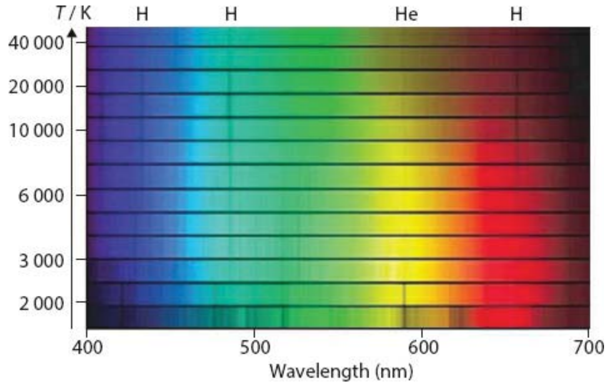


Figure D.5

Model Answer D.2

Figure D.5 shows that dark hydrogen lines are almost absent for low-temperature as well as high-temperature stars. Explain this observation.

The cooler stars have hydrogen atoms in their ground state. Visible light photons do not have enough energy to excite these atoms to higher energy levels and produce absorption lines. Very high-temperature stars would have all their hydrogen atoms ionised, so no absorption could take place.

The Hertzsprung–Russell diagram

Hertzsprung and Russell independently proposed plotting stars according to their luminosity (vertical axis) and surface temperature (horizontal axis), as shown in Figure D.6. Notice that temperature increases as we move to the left.

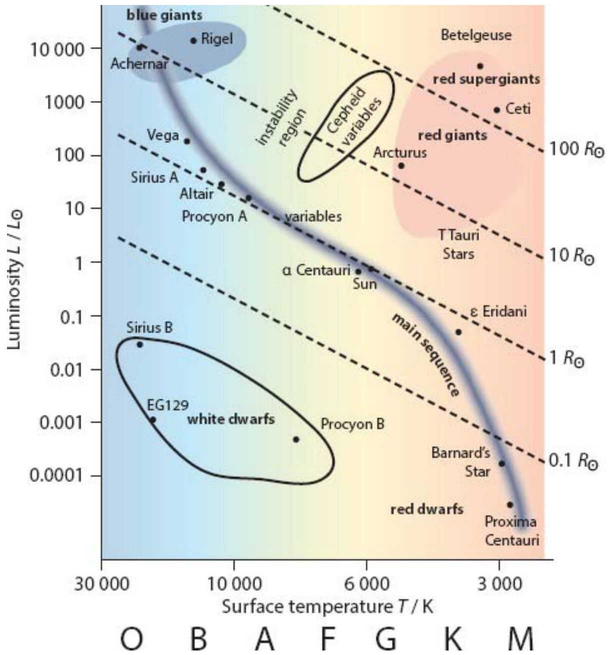


Figure D.6

The dashed lines represent stars of equal radius. Here $R_\odot$ is the radius of our Sun.

In such a plot, stars appear to be grouped into three major areas: the so-called main sequence (the broad curve running from upper left to lower right), white dwarfs (at lower left) and red giants and red supergiants (at upper right).

TEST YOURSELF D.3

Using the HR diagram, calculate the ratio of the radius of Achernar to that of EG 129. Comment on the result.

TEST YOURSELF D.4

Barnard's star is at a distance of 6.0 ly from Earth. Using the HR diagram, estimate its apparent brightness. (Solar luminosity $L_s = 3.8 \times 10^{26}$ W.)

Cepheid stars

Cepheid stars or Cepheid variables are stars whose brightness varies periodically with time (Figure D.7), because the stars expand and contract.

It turns out that the period of variation of a Cepheid's brightness is related to its average luminosity: the longer the period, the higher the luminosity (Figure D.8).

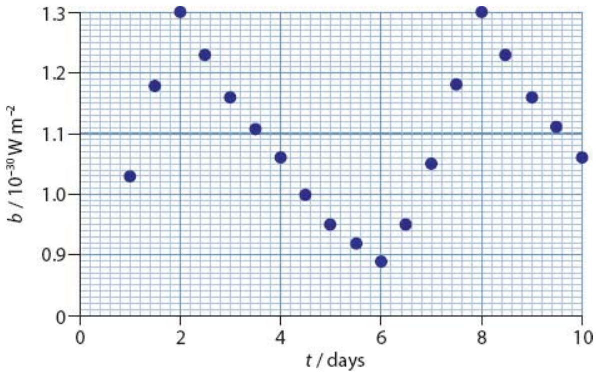


Figure D.7

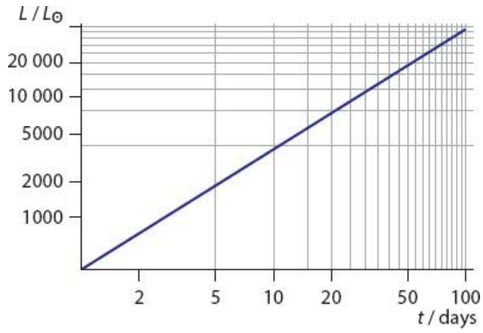


Figure D.8

So by observing a Cepheid over time we can measure its period and thus deduce its average luminosity. This makes Cepheids 'standard candles' – that is, stars of known luminosity.

By also measuring a Cepheid's average apparent brightness we can deduce the distance to it. This allows distances up to 40 Mpc to be measured.

Model Answer D.3

A Cepheid star and another star are located in the same galaxy. Explain how the Cepheid may be used to estimate the luminosity of the other star.

Since the stars are in the same galaxy, their distances are approximately the same. Therefore $b_{\text{star}} = \frac{L_{\text{star}}}{4\pi d^2}$ and $b_{\text{Cephied}} = \frac{L_{\text{Cephied}}}{4\pi d^2}$, so $L_{\text{star}} = \frac{b_{\text{star}}}{b_{\text{Cephied}}} L_{\text{Cephied}}$. The apparent brightness of each star must be measured, and the luminosity of the Cepheid can be determined from the period of its brightness variation.

Stellar evolution

Following are the main features of the evolution of a star. Figure D.9 is a schematic summary of the main features of this evolution.

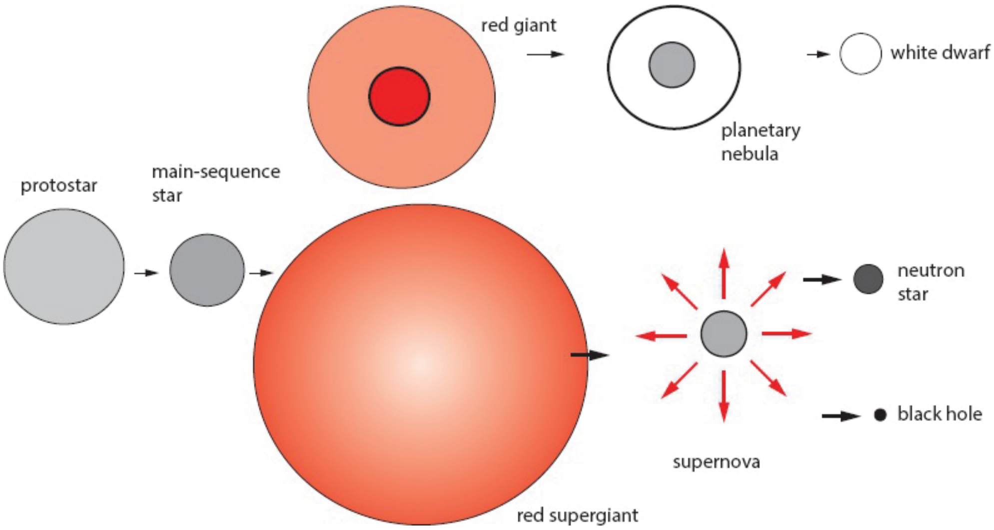


Figure D.9

- Stars spend most of their lifetime on the main sequence. There, they fuse hydrogen into helium.
- Once about 12% of its hydrogen is used up, a star will develop instabilities and will start to expand.
- As it expands, the surface temperature of the star decreases, but the increase in surface area more than compensates and so its luminosity actually increases. The star will become a **red giant** (if its mass is less than about 8 solar masses) or a **red supergiant** (if its mass is greater than about 8 solar masses).
- In higher-mass stars, nuclear reactions continue in the core, but now involve heavier and heavier elements. The more massive the star, the heavier the elements produced in the nuclear reactions. The process stops at the production of iron. No heavier elements can be produced by nuclear fusion, as this would require that additional energy be supplied from outside (see Chapter 7).
- The star then undergoes a catastrophic explosion, producing a **planetary nebula** if the mass of the star is low and a **supernova** if it is high.
- With most of its mass ejected into space, the star is left with just its **core**.
- If this core has a mass less than about $1.4M_{\text{Sun}}$, it will become a stable white dwarf star. It does not collapse further because of electron pressure. The limiting mass of $1.4M_{\text{Sun}}$ is known as the **Chandrasekhar limit**. Even a very massive star can end up as a white dwarf if the mass of its *core* is less than the Chandrasekhar limit.
- If the mass of the core is betweem $1.4M_{\text{Sun}}$ and $3M_{\text{Sun}}$, it will become a **neutron star**. It does not collapse further because of neutron pressure. The limiting mass of $3M_{\text{Sun}}$ is known as the **Oppenheimer–Volkoff limit**.
- If the mass of the core is larger than the Oppenheimer–Volkoff limit, then no known mechanism exists to stop its further collapse, and it becomes a **black hole** – a star from which nothing can escape.

Evolutionary paths and the mass-luminosity relation

Low-mass and high-mass stars follow quite different evolutionary paths. Figures D.10 and D.11 show the paths on the HR diagram which are followed by stars of $1M_{\text{Sun}}$ and $15M_{\text{Sun}}$, respectively, as they leave the main sequence.

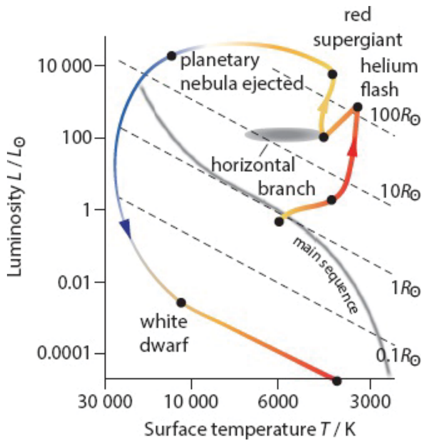


Figure D.10

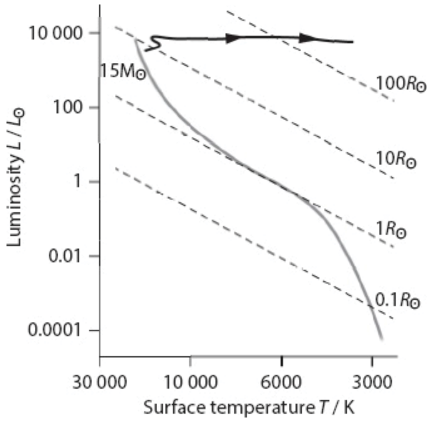


Figure D.11

Main-sequence stars show a relationship between their luminosity and their mass: $L = kM^{3.5}$, where k is a constant. A main-sequence star whose luminosity is 100 times that of our Sun, for example, has a mass given by $\frac{L}{L_{\odot}} = 100 = \frac{kM^{3.5}}{kM_{\odot}^{3.5}}$, or $\frac{M}{M_{\odot}} = \sqrt[3.5]{100} \approx 3.7$, so such a star has a mass about 3.7 times that of our Sun. (Only main-sequence stars obey this relationship.)

TEST YOURSELF D.5

A star has a mass that is 50 times as larger as the mass of the Sun, and a luminosity that is 80 000 times the luminosity of the Sun. Could this star be a main-sequence star?

D.3 Cosmology

The expanding universe and the Big Bang

Light from distant galaxies is received on Earth at a wavelength that is longer than that emitted – that is, it is red-shifted. Using the Doppler effect to explain this observation implies that the galaxies are moving away from us – the universe is expanding. This suggests that in the past the universe was smaller and had a beginning, which scientists refer to as the 'Big Bang'. This marks the beginning of time and space and the creation of all energy and matter. It happened everywhere in the universe (which was then a point). At the very early stages of the life of the universe, the temperature would have been very high; as the universe expanded, it cooled down.

A more accurate interpretation of the red-shift comes from Einstein's theory of general relativity, which says that space expands, giving the illusion of galaxies moving away from one another. As space expands, the wavelength of the photons emitted from distant galaxies also stretches by the time it gets to us (Figure D.12) R is the scale factor of the Universe – see later. Hence we measure a longer, red-shifted wavelength.

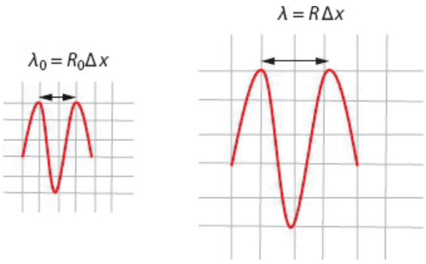


Figure D.12

If λ_0 is the emitted wavelength and λ the received wavelength, the quantity $z = \frac{\lambda}{\lambda_0}$ is the red-shift.

But the expansion of the universe does not imply that the Earth is at the centre of it all. An observer anywhere in the universe would reach the same – erroneous – conclusion.)

Hubble's law

Distant galaxies move away from each other with a speed that is proportional to their separation, as expressed by Hubble's law, $v = H_0 d$, where H_0, the slope of a graph of their speed versus their distance, is referred to as the **Hubble constant** (Figure D.13). The graph passes through the origin.

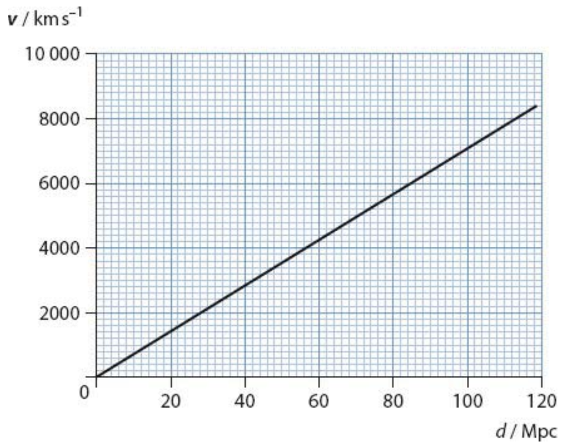


Figure D.13

The presently accepted value of the Hubble constant is about $H_0 = 68$ km s^{-1} Mpc^{-1}. (Until recently, the value has been uncertain because of the difficulty in measuring speeds and, especially, distances.)

The speed of a receding galaxy is obtained using the Doppler formula, $z = \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$. (This can only be used for small red-shifts, up to about 0.2.)

Hubble's law only applies to distant galaxies. The Andromeda galaxy, for example, is actually *approaching* our Milky Way because of the mutual gravitational attraction between the two galaxies.

Estimating the age of the universe: Hubble time

Imagine two points that were very close to each other at the time of the Big Bang but which are presently separated by a distance d and moving away from one another with a relative speed of $v = H_0 d$.

Assuming a constant speed of expansion, we also have $v = \frac{d}{T}$, where T is the present age of the universe.

Thus $\frac{d}{T} = H_0 d$, giving a value of $T = \frac{1}{H_0}$, about 14 billion years, for the age of the universe.

The fact that the expansion speed was greater at the beginning means that the age obtained in this way is an overestimate – that is, $\frac{1}{H_0}$, known as the **Hubble time**, is only an upper bound on the age of the universe.

The cosmic scale factor

As the universe expands, the physical distance between any two points increases. To quantify this, we introduce a quantity R, called the **scale factor**. This factor varies with time. The ratio of the physical distance d between two given points now and their separation d_0 in the past is $\frac{d}{d_0} = \frac{R}{R_0}$, where R is the scale factor now and R_0 the scale factor in the past.

The actual value of the scale factor is thus an indication of the 'size' of the universe. From $z = \frac{\Delta\lambda}{\lambda_0}$ we deduce that $z = \frac{\lambda-\lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 = \frac{R}{R_0} - 1$.

Worked Example D.2

Light of wavelength 434 nm is emitted from a distance galaxy. When this light arrives at Earth its wavelength is measured to be 486 nm. Estimate the ratio of the size of the universe now compared to when the light was emitted.

The red-shift is $z = \frac{\lambda-\lambda_0}{\lambda_0} = \frac{486-434}{434} = 0.1198$. From $z = \frac{R}{R_0} - 1$ we find $\frac{R}{R_0} = 1 + z = 0.1198 + 1 \approx 1.12$.

Cosmic microwave background radiation

The cosmic microwave background (CMB) radiation is the best evidence for the Big Bang theory. It is black-body radiation, coming from all directions, in the microwave region. It is the remnant of the radiation that filled the universe in its early stages when the temperature was very high. This means that in the past the peak wavelength was smaller and the temperature was very high; this is strong evidence for a hot big bang. As space stretched, the initial wavelength also stretched to its present large value.

The present peak wavelength of the CMB is about 1.07 mm, indicating a temperature of the universe of just below 3 K:

$$\lambda_0 T = 2.9 \times 10^{-3}\,\mathrm{m\,k}$$
$$T = \frac{2.9\times10^{-3}}{1.07\mathrm{mm}}\,\mathrm{m\,K}$$
$$T = 2.7\mathrm{K}$$

The CMB was discovered in 1964 by A. Penzias and R. Wilson, but had been predicted in 1948, on theoretical grounds, by R. Alpher and R. Herman, in work that went largely unappreciated and unacknowledged at the time.

TEST YOURSELF D.6

State and explain two pieces of experimental evidence in support of the Big Bang model of the universe.

TEST YOURSELF D.7

A line in the spectrum of hydrogen is measured to have a wavelength of 656.3 nm in the lab. The same line observed in the spectrum of a distant galaxy has a wavelength of 689.1 nm. Calculate the distance to the galaxy using H = 68 km s^{-1} Mpc^{-1}.

TEST YOURSELF D.8

a Estimate the age of the universe using a Hubble constant of H = 68 km s^{-1} Mpc^{-1}.

b Explain why this is actually an overestimate of the age of the universe.

The accelerating universe and type Ia supernovae

We know that the universe is expanding. Because of gravity we would except that the distant galaxies should be slowing down. So the discovery in 1999 that the universe is actually *accelerating* its expansion came as a big surprise.

A crucial role in this discovery was played by Type Ia supernovae. A Type Ia supernova occurs when a white dwarf that is part of a binary star system collects mass from its companion star, thus increasing its own mass. This is shown in an artist's impression in Figure D.14.

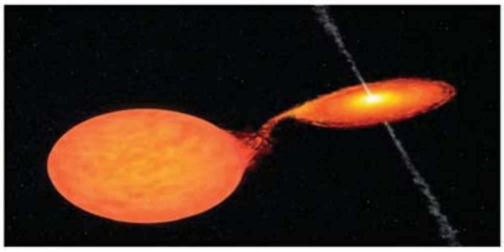


Figure D.14

Once the Chandrasekhar limit of 1.4 solar masses is exceeded, the white dwarf explodes because of runaway nuclear fusion reactions that release large amounts of energy, increasing its luminosity massively: it becomes a Type Ia supernova (see Figure D.15).

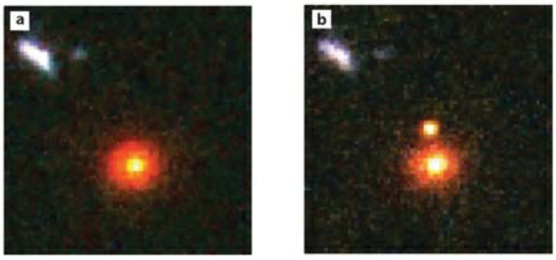


Figure D.15 Observation of a Type Ia supernova: a before and b after outburst.

Figure D.16 shows how luminosity varies with time for a large number of Type Ia supernovae. Note that they all have the same value of *peak* luminosity, *L*.

Therefore, if we can also observe the maximum apparent brightness *b* of such a supernova, from $b = \frac{L}{4\pi d^2}$ we can directly calculate its distance from Earth. From its Doppler shift we can also determine the speed at which it is moving away.

Taken together, the data for all the known Type Ia supernovae showed that they were dimmer than expected based on the Big Bang model. This could be explained by assuming that they were actually further away – the universe is accelerating.

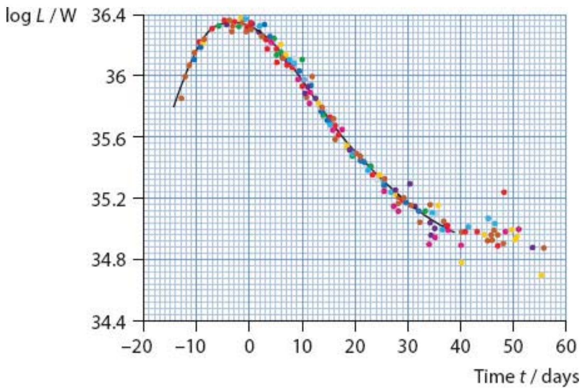


Figure D.16

D.4 Stellar evolution

Stars form when large clouds of dust collapse under their own weight. The temperature of the cloud must be low enough for that to happen – otherwise the random motion of molecules would prevent the collapse. Mathematically this condition is known as the **Jeans criterion**. It says that, for a cloud to collapse, its gravitational potential energy must be larger than the total random kinetic energy of its molecules:

$$\frac{GM^2}{R} > \frac{3}{2}NkT$$

where M is the mass of the cloud, R its radius, N the number of molecules in it and T the temperature. Using $N = \frac{M}{m}$, where m is the average mass of a molecule in the cloud, the Jeans condition becomes:

$$M > \frac{3}{2}\frac{kTR}{mG}$$

This so-called **protostar** then takes its place on the main sequence of the HR diagram.

Nuclear reactions on and off the main sequence

On the main sequence: We have seen that main-sequence stars fuse hydrogen into helium via the proton`-proton cycle. For stars more massive than our Sun there is also the so-called CNO cycle, in which helium is again produced from hydrogen through the intermediate production of carbon, oxygen and nitrogen. This requires higher temperatures and hence massive stars – massive stars have greater gravitational pressure that heats up the core.

Off the main sequence: The nuclear reactions taking place after a star leaves the main sequence depend on the mass of the star. For stars with mass between 0.25 and 8 solar masses, carbon is produced in the triple alpha process:

$$^4_2\text{He}+^4_2\text{He} \rightarrow^8_4 \text{Be}+\gamma$$
$$^4_2\text{He}+^8_4\text{Be} \rightarrow^{12}_6 \text{C}+\gamma$$

For even higher-mass stars, the temperature in the core is higher and this allows the fusing of heavier elements. Oxygen is the next element produced, followed by neon, sodium, magnesium and silicon:

$$^4_2\text{He} +^{12}{}_6\text{C} \rightarrow^{16}{}_6 \text{ O}+\gamma$$
$$^{12}{}_6\text{C}+^{12}{}_6\text{C} \rightarrow^{20}_{10} \text{Ne}+^4_2\text{He}$$
$$^{12}{}_6\text{C}+^{12}{}_6\text{C} \rightarrow^{23}_{11} \text{Na}+^1_1\text{H}$$
$$^{12}{}_6\text{C}+^{12}{}_6\text{C} \rightarrow^{23}_{12} \text{Mg}+^1_0\text{n}$$
$$^{16}{}_6\text{O}+^{16}{}_6\text{O} \rightarrow +^{28}_{14}\text{Si}+^4_2\text{He}$$

The process ends with the production of iron, which is near the peak of the binding energy curve.

This gives a star an 'onion-layered' structure with the heaviest elements closest to the core (Figure D.17).

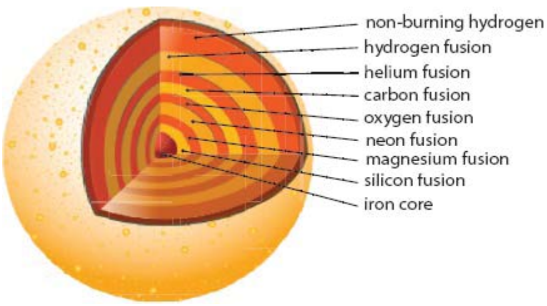


Figure D.17

Nucleosynthesis of elements heavier than iron

How are the rest of the elements produced? The answer lies in neutron absorption by nuclei. When a nucleus absorbs a neutron it will turn into an isotope of the original nucleus. This isotope is usually unstable and will decay. The issue is whether there is enough time for this decay to occur before the isotope absorbs yet another neutron.

The *s-process* (s for slow): In stars where the number of neutrons is small the isotope does have time to decay. This isotope will undergo a series of decays including beta decays. In beta decay the atomic number is increased by 1, thus producing a new element. This process accounts for the production of about half the nuclei above iron; it ends with the production of bismuth-209.

The *r-process* (r for rapid): In the presence of very large numbers of neutrons, nuclei that absorb neutrons do not have time to decay. They keep absorbing neutrons one by one, forming very heavy, neutron-rich isotopes. This happens during a supernova explosion. These neutron-rich isotopes are then hurled into space by the supernova, where they can now undergo beta decay, producing nuclei of even higher atomic number.

Beta decay is not the only way to turn a neutron into a proton and hence increase the atomic number. In supernova explosions, massive numbers of neutrinos are produced; a neutron may absorb a neutrino and turn into a proton according to the reaction $^1_0\text{n} + v \rightarrow^1_1 \text{p}+^0_{-1}\text{e}$.

Lifetime on the main sequence

Stars leave the main sequence after using about 12% of their hydrogen. We can use the mass–luminosity relation to estimate the time they spend on the main sequence:

$$L = kM^{3.5} = \frac{E}{T}$$

where E is the energy produced and T is the lifetime on the main sequence. But $E = 0.12Mc^2$ and so $kM^{3.5} = \frac{0.12Mc^2}{T} \Rightarrow T = \frac{0.12c^2}{k}M^{-2.5}$, thus $T \propto \frac{1}{M^{2.5}}$.

Thus, for example, a star with a mass 100 times that of the Sun will spend much less time on the main sequence:

$$\frac{T}{T_\odot} = \frac{M_\odot^{2.5}}{M^{2.5}} = \left(\frac{1}{100}\right)^{2.5} = \frac{1}{100\,000}$$

The total time to be spent by the Sun on the main sequence (its lifetime) is about 9×10^9 y, so

$$T = \frac{1}{100\,000} \times 9 \times 10^9 = 9 \times 10^4 \text{y.}$$

Supernovae

DEFINITIONS

TYPE II SUPERNOVA The explosion of a massive red supergiant star.

TYPE Ia SUPERNOVA The explosion of a white dwarf star as mass falls into it and pushes the mass past the Chandrasekhar limit.

The luminosity of type Ia supernova falls off more sharply than a type II supernova.

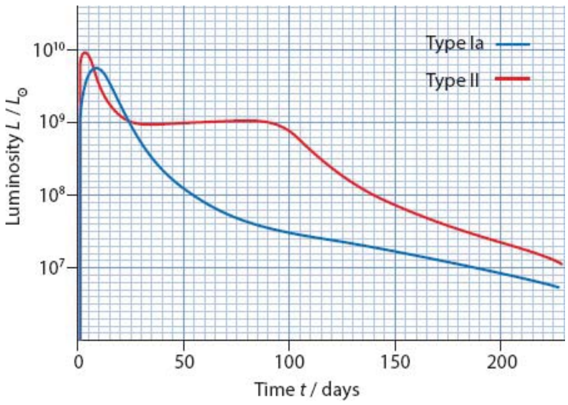


Figure D.18

Type Ia supernovae:

- do not have hydrogen lines in their spectra
- are produced when mass from a companion star accretes onto a white dwarf, forcing it to exceed the Chandrasekhar limit
- have a luminosity which falls off sharply after the explosion.

Type II supernovae:

- have hydrogen lines in their spectra
- are produced when a massive red supergiant star explodes
- have a luminosity which falls off gently after the explosion.

D.5 Further cosmology

The cosmological principle

On a small scale, the universe appears to be full of *structures*: planets, stars, galaxies, clusters and superclusters of galaxies. On a very large scale, however, we no longer see such structure. If we imagine cutting up the universe into cubes about 300 Mpc on a side, the interior of any one of these cubes would look the same as the interior of any other cube, anywhere else in the universe. In other words, on a *large scale* the universe looks *homogeneous* (uniform).

Similarly, if we look in different directions, we see essentially the same thing. No one direction is special in comparison with another. Thus on a *large scale* the universe is *isotropic.*

These two principles, homogeneity and isotropy, comprise the *cosmological principle.* It is important because most models of the universe assume this to be true.

Additional evidence for the cosmological principle comes from the great degree of isotropy of the CMB.

TEST YOURSELF D.9

Use the cosmological principle to deduce that the universe cannot have an edge or a centre.

Fluctuations in the CMB

The CMB is very uniform and isotropic, but not perfectly so. There are small variations ΔT in its temperature, of the order of $\frac{\Delta T}{T} \approx 10^{-5}$, where T = 2.723 K is the average temperature. Variations in temperature are related to variations in the density of the universe. In turn, variations in density are the key to how structures formed in the universe. With perfectly uniform temperature and density in the universe, stars and galaxies would not form.

The anisotropies in the CMB are crucial in understanding the formation of structures.

Figure D.19 shows results from the Planck satellite observatory. This spectacular map shows CMB fluctuations in temperature (different colours) as small as a few millionths of a degree. This is a map of the radiation filling the universe when it was only about 380 000 years old.

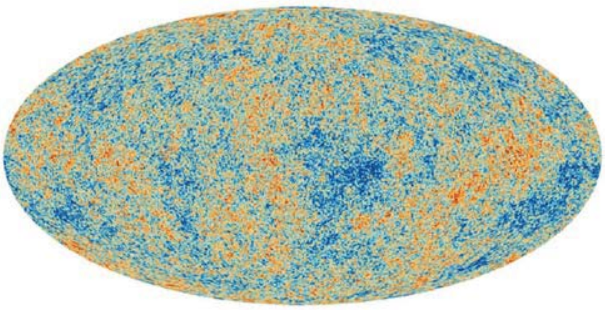


Figure D.19

Rotation curves

Consider a body of mass m moving in a circular path of radius r. We know from Chapters 6 and 10 that the orbital speed can be found from $\frac{GMm}{r^2} = \frac{mv^2}{r}$, where M is the total mass attracting m. This gives $v = \sqrt{\frac{GM}{r}}$.

The dependence of v on r is known as a **rotation curve** of the attracting mass. We may distinguish three separate cases of interest, each with a different rotation curve:

- m moves around a point mass M (for example a planet orbiting the Sun). In this case, $v \propto \frac{1}{\sqrt{r}}$.
- m moves within a cloud of dust of uniform density ρ (for example, a star at a radius r from the galactic centre). In this case the attracting mass M is the mass enclosed within a sphere of radius r only. Since $\rho = \frac{M}{V} = \frac{M}{\frac{4\pi r^3}{3}} = \frac{3M}{4\pi r^3}$, we find $M = \frac{4\pi r^3 \rho}{3}$ and so $v = \sqrt{\frac{G4\pi r^3 \rho}{3r}}$, thus $v \propto r$.
- m moves in a cloud of dust of non-uniform density, whose mass varies with distance from the axis according to $M = kr$, where k is a constant. In this case the density would be $\rho = \frac{M}{V} = \frac{kr}{\frac{4\pi r^3}{3}} = \frac{3k}{4\pi r^2}$, so $v = \sqrt{\frac{GM}{r}}$ gives $v = \sqrt{\frac{Gkr}{r}}$.

Figure D.20 shows the rotation curve for our galaxy. (Other galaxies show similar curves.)

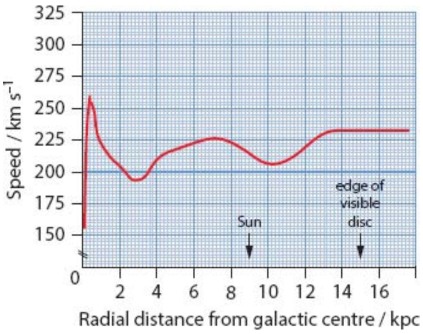


Figure D.20

The rotation curve shows an initial linear increase, suggesting a uniform density near the galactic centre. At larger distances the curve becomes flat, consistent with the third case above, in which there is substantial mass outside the galactic disc.

But we do not see any such matter. This is the main evidence for the existence of **dark matter** – matter that is too cold to radiate and so cannot be seen. It is estimated that, in our galaxy, dark matter forms a spherical halo around the galaxy and has a mass that is about 10 times larger than the mass of all the stars in the galaxy!

Dark matter

It is estimated that 85% of the matter in the universe is dark matter. It cannot be seen but we know of its existence mainly from its gravitational effects on nearby bodies.

Dark matter could be:

- **MACHOS (Massive Compact Halo Objects):** ordinary, cold matter that does not radiate – for example, brown dwarf or black dwarf stars or small planets
- **WIMPS (Weakly Interacting Massive Particles):** neutrinos fall into this class since they are known to have a small mass, although their tiny mass is not enough to account for all non-baryonic dark matter

Unconfirmed theories of elementary particle physics based mainly on the idea of *supersymmetry* predict the existence of various particles that would be WIMP candidates, but no such particles have been discovered.

So the answer to the question 'what is dark matter?' is mainly unknown at the moment.

Cosmological origin of the red-shift

Consider the emission of a photon of wavelength λ_0 at one time in the history of the universe, and its detection and measurement with a wavelength of λ at the present time. The formula $\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$, where R_0 is the value of the scale factor at the time of the photon's emission and R is its value now, offers a cosmological interpretation of red-shift that is not based on the Doppler effect: the space in between galaxies is expanding, so wavelengths will expand as well.

This has a direct bearing on the temperature of the CMB that fills the universe. The wavelength λ_0 corresponds to a CMB temperature of T_0, so, by Wien's law, $\lambda_0 T_0$ = constant. Likewise, λT = constant.

Therefore $\frac{\lambda}{\lambda_0} = \frac{T_0}{T}$, so $\frac{T_0}{T} = \frac{R}{R_0}$ or $T \propto \frac{1}{R}$.

This shows that, as the universe expands (R gets bigger), its temperature drops. This is why the universe is cooling down, and why the present temperature of the CMB (2.7 K) is so low now.

Worked Example D.3

The photons of the CMB observed today are thought to have been emitted at a time when the temperature of the universe was about 3.0×10^3 K. Estimate:

a the size of the universe then compared to the size now

b the red-shift of the received photons.

a From $T \propto \frac{1}{R}$ we find $\frac{T_0}{T} = \frac{R}{R_0}$, so $\frac{R}{R_0} = \frac{3.0\times10^3}{2.7} \approx 1100$, so the universe then was about 1100 times smaller.

b This corresponds to a red-shift of $z = \frac{R}{R_0} - 1 = 1100 - 1$, or about 1100.

TEST YOURSELF D.10

Photons are received on Earth with a red-shift of 3. Estimate the temperature of the universe at the time of emission of these photons, relative to its present temperature.

TEST YOURSELF D.11

It is possible to measure the temperature of the CMB by making observations in gas clouds in a distant galaxy. Estimate the temperature of the CMB made from such measurements in a galaxy whose red-shift is 2.

Critical density

Consider a spherical cloud of dust of radius *r*, and a mass *m* at the surface of this cloud which moves away from the centre with a speed *v* that satisfies Hubble's law, $v = H_0 r$.

The total energy of the mass is $E\frac{1}{2}mv^2 - \frac{GMm}{r}$, where *M* is the mass of the cloud. If we call the density of this cloud ρ, then $M = \rho\frac{4}{3}\pi r^3$.

Using this together with $v = H_0 r$, we find:

$$E = \frac{1}{2}mr^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$$

This energy is zero if the density is $\rho_c = \frac{3H_0^2}{8\pi G} \approx 10^{-26}$ kgm^{-3}. This is referred to as the **critical density** of the universe. (You should be able to derive this formula in an exam.)

The critical density plays a crucial role in models for the evolution of the universe. Comparison of the critical density with the sum of the matter and energy density of the universe (see next section) determines whether the universe has a flat or curved geometry.

The evolution of the universe

In 1915 Einstein published his general theory of relativity, replacing Newton's theory of gravitation by a revolutionary new theory in which the rules of geometry were dictated by the amount of mass and energy contained in the universe.

Applying the Einstein theory to the entire universe gives equations for the scale factor *R* which can be solved for the dependence of *R* on time – how the universe evolves.

The solutions depend on two parameters, called **density parameters** – Ω_m for matter and Ω_Λ for a possible **vacuum energy** filling all space:

$$\Omega_\mathrm{m} = \frac{\rho_\mathrm{m}}{\rho_\mathrm{crit}} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_\mathrm{crit}}$$

where ρ_m is the actual density of matter in the universe, ρ_{crit} is the critical density of the universe and ρ_Λ is the density of the vacuum energy.

In this course we are interested in just four cases, illustrated in Figure D.21.

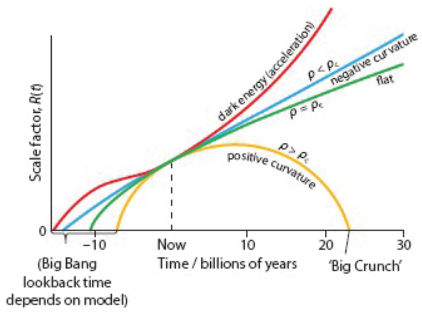


Figure D.21

In the first three, $\Omega_\Lambda = 0$; these are of historical interest only, because observations favour $\Omega_\Lambda \neq 0$.

In all three of these cases, the scale factor starts from zero, implying a Big Bang.

- In the first possibility (the orange line in the figure), $R(t)$ starts from zero, increases to a maximum value and then decreases to zero again – the universe collapses after an initial period of expansion. This is called the *closed* model. It corresponds to $\Omega_m > 1$, thus $\rho_m > \rho_{crit}$.
- The second possibility (the blue line) applies when $\Omega_m < 1$, thus $\rho_m < \rho_{crit}$. Here the scale factor $R(t)$ increases without limit – the universe continues to expand forever. This is called the *open* model.
- The third possibility (the green line) is that the universe expands forever, but the rate of expansion decreases, becoming zero at infinite time. This is called the *critical* model and corresponds to $\Omega_m = 1$. The density of the universe in this case is equal to the critical density, $\rho_m = \rho_{crit}$.

Remember, these three models have $\Omega_\Lambda = 0$ and so are *not* consistent with observations.

The fourth case (the red line) is the one that agrees with current observations. Data from the Planck satellite observatory (building on previous work by WMAP, Boomerang and COBE) indicate that $\Omega_m \approx 0.32$ and $\Omega_\Lambda \approx 0.68$. This implies $\Omega_m + \Omega_\Lambda \approx 1$ and implies a flat universe. This means that at present our universe has a flat geometry, 32% of its mass-energy content is matter and 68% is vacuum energy, and it will expand forever at an accelerating rate.

Dark energy

The discovery of the accelerating universe was contrary to expectations: gravity should be slowing the distant galaxies down. In the previous section we saw that an accelerating universe demands a non-zero value of the vacuum energy. This energy has been called **dark energy**. The presence of this energy creates a kind of repulsive force that not only counteracts the effect of gravity on a large scale, but actually dominates it, causing acceleration in distant objects rather than the expected deceleration. It is not yet known what type of energy this is.

TEST YOURSELF D.12

Discuss the significance of the critical density of the universe.

Cognitive bias

Nature of Science. When interpreting experimental results, it is tempting to dismiss or find ways to explain away results that do not fit with the hypotheses. In the late 20th century, most scientists believed that the expansion of the universe must be slowing down because of the pull of gravity.

Evidence from the analysis of Type Ia supernovae in 1998 showed that the expansion of the universe is accelerating - a very unexpected result. Corroboration from other sources has led to the acceptance of this result, with the proposed dark energy as the cause of the acceleration.

Checklist

After studying this Option you should be able to:

- ☐ **solve problems dealing with luminosity and apparent brightness**
- ☐ **discuss the nature and evolution of stars**
- ☐ **describe the Big Bang theory and the evidence for it**
- ☐ **describe the evolution of stars beyond the main sequence**
- ☐ **describe the evidence for dark matter**
- ☐ **describe the models for the evolution of the universe**.

GLOSSARY

A-scan (amplitude modulated) A map of the intensity of the reflected ultrasound signal from the skin and various organs inside the body.

Absolute (Kelvin) temperature scale A temperature scale on which the lowest possible temperature is zero degrees.

Absorption spectrum The set of wavelengths of light that are absorbed when white light is transmitted through a gas.

Accurate measurements Measurements that have a small systematic error.

Adiabatic compression or expansion A change in the state of a gas in which no energy enters or leaves.

Albedo The Earth as a whole has an average albedo of about 0.30, indicating that it reflects about 30% of the radiation incident on it. The albedo is different for different parts of the Earth, depending on soil type, water depth, forestation and cloud cover.

Amplitude, *A* The maximum displacement.

Angular acceleration The rate of change with time of the angular speed.

Angular magnification The ratio of the angle subtended at the eye by the image to the angle subtended by the object.

Angular position The angle swept by a rotating body relative to a reference line.

Anti-particles To every particle there corresponds an anti-particle of the same mass but with *opposite values of electric charge and all other quantum numbers.*

Archimedes' principle A body that is immersed in a fluid will experience an upward buoyant force given by $B = \rho_{fluid} g V_{imm}$, where V_{imm} is the immersed volume.

Asteroid One of very many small, rocky objects in orbit around the Sun in between the orbits of Mars and Jupiter.

Atomic mass unit, *u* One atomic mass unit is $\frac{1}{12}$ of the mass of the neutral atom of the carbon isotope $^{12}{}_{6}\mathrm{C}$. $1\,u = 1.66 \times 10^{-27}\,\mathrm{kg}$.

Atomic transitions A photon is emitted when an atom makes a transition from a higher energy level to a lower energy level. The photon carries away an energy equal to the *difference* ΔE in energy of the levels involved in the transition.

Attenuation coefficient, *μ* The probability per unit length that a particular X-ray photon will be absorbed.

Attenuation The loss of power when a signal travels through a fibre, due mainly to impurities in its glass core.

Attenuation The reduction in the intensity of the X-rays transmitted due to absorption and/or spreading of the X-ray beam.

Average acceleration The ratio $\frac{\Delta v}{\Delta t}$ of the change of the velocity vector to the time taken. This vector is in the same direction as $\Delta \boldsymbol{v}$.

Average speed The ratio $\frac{d}{t}$ of the **total** distance travelled d to the **total** time t taken.

Average velocity The ratio $\frac{\Delta r}{\Delta t}$ of the displacement to the time taken. This vector has the same direction as $\Delta \boldsymbol{r}$.

B-scan (brightness modulated) Conversion of an A-scan pattern into a pattern of dots whose brightness is proportional to the signal strength.

Baryon A hadron which is made of three quarks.

Best-fit line The curve or straight line that goes through all the error bars.

Binary star A pair of stars orbiting a common centre.

Black body A theoretical body that absorbs *all* the energy that is incident on it, reflecting none.

Black hole A 'singularity of spacetime', a point of infinite spacetime curvature.

Blue-shift A shift of received light to shorter wavelength (higher frequency) than that emitted.

Capacitance The charge per unit voltage that can be stored on a capacitor.

Centripetal acceleration The acceleration of circular motion, in which only the direction is changing.

Cepheid star or Cepheid variable A star whose brightness varies with a well-defined period. This period is related to the star's luminosity and so can be used to estimate the distance to the star.

Chandrasekhar limit The largest mass a white dwarf star can have. It is about 1.4 solar masses.

Closed system A system in which no mass can enter or leave.

Clusters and superclusters of galaxies Galaxies are often found in groups called clusters, and clusters are grouped together in superclusters.

Colour Quarks have a fundamental property which particle physicists call **colour** (though this has nothing to do with colour as we know it). Quarks have three colours: red, green and blue.

Comet A small object of ice, rock and frozen gases whose nucleus is about 10 km across.

Conductor A material, such as a metal, with lots of 'free' electrons.

Conservation of energy Total energy, mechanical and otherwise, cannot be created or destroyed. It only gets transformed from one form to another.

Conservation of momentum When the resultant (net) external force on a system is zero, the total momentum of the system stays constant.

Constellation A group of stars in a recognisable pattern as seen from Earth, though not necessarily close to each other in space.

Control rods These rods can be inserted to absorb excess neutrons in order to control the rate of the fission reaction.

Crest A point of maximum positive displacement.

Critical damping A degree of damping such that the system returns to its equilibrium position as fast as possible but without performing oscillations.

Critical mass The minimum mass of uranium-235 for a chain reaction to sustain itself; with a lower mass, many neutrons escape without causing further reactions.

Curved spacetime Means that the rules of classical geometry do not hold and have to be replaced by other rules.

Damped oscillations Oscillations in which mechanical energy is reduced because of the presence of fictional forces, so that amplitude decreases.

Degradation of energy Energy, while being always conserved, becomes less useful, meaning that it cannot be used to perform mechanical work.

Derived units All other quantities have **derived** units, that is, combinations of the fundamental units.

Direction of energy flow Heat, on its own, always gets transferred from a higher-temperature to a lower-temperature region.

Dispersion The variation of wave speed with wavelength.

Displacement A vector to the position of a point in a medium from the equilibrium position of the point.

Displacement The change Δr in the position vector.

Distance The length of the path travelled.

Doppler effect The change in the observed frequency of a wave whenever there is *relative* motion between the emitter and the receiver.

Driven oscillations Oscillations in which an external periodic force acts on a system.

Elastic collision A collisions in which the total kinetic energy before and after the collision are the same.

Electric current, *I* The charge that moves past a cross-section of a conductor per unit time.

Electric field, *E* The electric force per unit charge exerted on a point positive charge q:

Electrical potential energy at a point The work done (by you) to move a positive point charge q from infinity to that point in an electric field.

Electrical resistance The ratio of the potential difference *across* a conductor to the current *through* it.

Electron volt (eV) 1 eV is the energy required to move a charge of e through a potential difference of 1V.

EMF The total work done per unit charge in moving a charge from one terminal of a cell to the other:

Emission spectrum The set of wavelengths of light emitted by a gas.

Energy density The amount of energy that can be obtained from a unit volume of a fuel.

Energy level The energy of an atom is **discrete** - that is, it can only have certain specific values. The possible energies that an atom can have are called its energy levels.

Equilibrium The state when the net (resultant) force on a body or system is zero.

Equipotential surface A surface on which the potential is the same everywhere.

Escape speed The minimum launch speed of a projectile at the surface of a planet so that the projectile can move far away (to infinity).

Event horizon A surface enclosing a black hole such that nothing from within the surface can be communicated to the outside. Nothing can move from inside the event horizon to the outside. Its radius is the Schwarzschild radius.

Far point The furthest point from the eye where the eye can focus without strain.

Faraday's law The EMF induced in a loop is equal to the negative of the rate of change of magnetic flux.

Flowtube A set of neighbouring streamlines.

Focal length, *f* The distance from the focal point to the centre of the lens.

Focal point or focus (plural foci) The point on the principal axis through which all rays parallel to the principal axis pass, or appear to pass, after refraction through a lens.

Frequency The number of revolutions per second. It is the reciprocal of the period.

Fuel rods Rods containing the nuclear fuel in pellet or powder form.

Galaxy A collection of **very many** stars (of the order of 10^9) that attract one another gravitationally.

Geodesic The path followed by an object on which no net force acts. It is the path of least length in spacetime. Light travels along geodesics.

Gravitational field strength The gravitational force per unit mass exerted on a small, point mass m : $g = \frac{F}{m} \Leftrightarrow F = mg$.

Gravitational lensing Multiple images of an object caused by the bending of light by a very massive object in or near the path of the light.

Gravitational potential energy at a point The work done (by you) to move a point mass m from infinity to that point in a gravitational field.

Hadron A particle made of quarks. There are two kinds of hadrons: baryons and mesons.

Half-life, $T_{1/2}$ The time taken for the activity of a sample to halve.

Half-value thickness, $x_{1/2}$ The distance after which the intensity of the transmitted X-rays is reduced to half the original intensity.

Heat engine A device that uses heat to perform mechanical work.

Heat exchanger The kinetic energy of the reaction products is converted to thermal energy in the moderator.

Heat, *Q* 'Energy in transit' – energy that is transferred from one body to another due to a difference in temperature.

Hertzsprung-Russell (HR) diagram A plot of the luminosity of stars versus their surface temperature.

Ideal fluid A fluid whose flow is laminar, incompressible and non-viscous.

Impedance matching The reduction of reflected intensity when ultrasound travels between two regions, accomplished by making their impedances equal or similar.

Impedance The product of the density of a medium times the speed of sound in the medium

Incompressible flow Fluid flow such that fluid density is everywhere the same.

Inertial reference frame A reference frame that is not accelerating.

Instantaneous speed The rate of change with time of the distance travelled.

Instantaneous velocity The rate of change with time of the displacement. Instantaneous velocity is a vector that is tangent to the path.

Insulator A material with very few or no free electrons.

Intermolecular forces Forces, electromagnetic in origin, between any two molecules in any substance, whether solid, liquid or vapor.

Internal energy, *U* The total random kinetic energy of the molecules of a system plus its total intermolecular potential energy.

Invariant A quantity that has the same value in all reference frames.

Isobaric expansion or compression A change in the state of a gas in which its pressure stays constant.

Isolated system A system in which no energy can enter or leave.

Isothermal expansion or compression A change in the state of a gas in which its temperature stays constant.

Isotope Nuclei with the same proton number but different nucleon number are called *isotopes* of one another.

Isovolumetric change A change in the state of a gas in which its volume stays constant.

Laminar flow Fluid flow such that the fluid velocity at each point does not change with time.

Lenz's law The direction of the induced EMF is such as to oppose the change in flux that created it.

Lepton number, *L* A quantum number that applies to leptons only.

Light damping Oscillations in which a small frictional force leads to a gradual decrease in amplitude.

Light-dependent resistor (LDR) A resistor in which the resistance decreases as the incident intensity of light increases.

Magnetic flux linkage The product of the flux trough a loop times the number of turns of the wire around the loop.

Main-sequence star A star undergoing fusion of hydrogen into helium.

Meson A hadron made of one quark and one antiquark.

Moderator The neutrons produced in the fission reaction must be slowed down if they are to be used to cause further fissions. This is achieved through collisions of the neutrons with atoms of the moderator, the material surrounding the fuel rods.

Molar mass The mass in grams of one mole of a substance.

Mole The S.I. unit for quantity.

Momentum The product of the mass and the velocity of a body, $\boldsymbol{p} = m\boldsymbol{v}$. This is a vector with the same direction as that of velocity.

Near point The closest point to the eye where the eye can focus without strain.

Nebula A cloud of gases and dust extending for light years across space. It is formed by the mass ejected when a red giant or supergiant explodes.

Net (resultant) force The single force whose effect is the same as the combined effect of all the individual forces on the body.

Neutron star A very hot, very dense star which is the end stage in the evolution of massive stars.

Noise Power due to unwanted signals that travels through a medium along with the signal of interest.

Non-renewable energy source A finite energy source which is being depleted much faster than it can be produced, and so will run out.

Non-viscous flow Fluid flow without resistance forces.

Nuclear reactor A machine in which nuclear reactions take place, producing energy.

Nucleon number The number of protons and neutrons in a nucleus.

Nuclide A nucleus with a specific number of protons and neutrons.

Ohm's law At constant temperature, many metallic conductors have the property that the current through them is proportional to the potential difference across them.

Open system A system in which mass can enter or leave.

Oppenheimer-Volkoff limit The largest mass a neutron star can have. It is about 3 solar masses.

Overdamping A degree of damping such that the system returns to equilibrium without oscillations in a time longer than that for critical damping.

Pascal's principle The pressure applied to any part of an enclosed incompressible fluid will be transmitted to all other parts of the liquid.

Period, *T* The time taken for one full oscillation.

Photoelectric effect The emission of electrons from a metal surface when electromagnetic radiation is incident on the surface.

Photon The particle of light; a bundle of energy and momentum. The energy of a photon is given by $E = hf$, where f is the frequency of the light and h is the Planck constant.

Polarised wave A wave in which the displacement remains always in a single plane. This is a property of transverse waves only.

Position vector The vector $\boldsymbol{r}$ from some arbitrary fixed point (called the origin) to the position of a particle - that is, the distance in a given direction.

Power The rate at which energy is dissipated or produced.

Precise measurements measurements that have a small random error.

Pressure The normal force per unit area on a surface.

Primary cell A battery or cell that runs out of energy and must therefore be discarded.

Primary energy source A source of energy that has not been processed in any way.

Principal axis The line through the centre of a lens and at right angles to its face.

Principle of superposition When two waves (of the same kind) meet, the resultant displacement is the sum of the individual displacements.

Proper length The length of an object as measured in its rest frame.

Proper time interval The interval of time between two events that occur at the same point in space.

Proton or atomic number The number of protons in a nucleus.

Quantum number A number (or property) used to characterise a particle. Quantum numbers in the standard model include charge, baryon number, strangeness, lepton number and colour.

Quarks Elementary particles which combine to form a class of particles called **hadrons.**

Radioactive decay law The rate of decay of a nucleus is proportional to the number N of nuclei present that have not yet decayed.

Random uncertainties Uncertainties due to the inexperience of the experimenter and the difficulty of reading instruments.

Ray A line indicating the direction of energy transfer of a wave.

Rayleigh criterion Two sources are said to be *just resolved* if the central maximum of the diffraction pattern of one source falls on the first minimum of the other.

Real image An image formed by actual light rays.

Red giant A large, relatively cool, reddish star.

Red supergiant A very large, relatively cool, reddish star.

Red-shift A shift of received light to a longer wavelength (lower frequency) than that emitted.

Renewable energy source This includes solar energy (and the other forms indirectly dependent on solar energy, such as wind energy, wave energy and bio-fuels) and tidal energy. At present, about 7% of our energy comes from these sources.

Representation of forces Forces, being vectors, are represented by arrows whose length shows the magnitude of the force. The direction of the arrow gives the direction of the force.

Resolution A measure of the ability of a detection device such as a telescope, microscope or your eye to distinguish two nearby objects – that is, to *see* them as separate objects.

Resonance A condition in which the driving frequency is equal to the natural frequency of the system, leading to a maximum amplitude of oscillation.

Rest energy The energy needed to create a particle from the vacuum. In the frame of reference where the particle is at rest (its rest frame) there is still energy in the particle: $E_0 = mc^2$, since $\gamma = 1$.

Rest frame of an object The frame of reference in which the object is at rest.

Rest mass The mass of a particle as measured in its rest frame.

rms current The square root of the mean (average) value of the square of the current in an AC circuit. It is the equivalent DC current which would give the same power dissipation as the AC circuit.

Sankey diagram A pictorial energy-flow block diagram showing energy flows using arrows whose widths are proportional to the respective energy transfers.

Scalars A physical quantity with magnitude but not direction. A scalar can be positive or negative. Examples are distance, speed, mass, time, work/energy, electric/gravitational potential and temperature.

Secondary cell A battery or cell that can be recharged and used again.

Secondary energy source A source of energy that has been processed.

Signal-to-noise ratio (SNR) A logarithmic measure of the strength of a signal compared to the accompanying noise.

Solar constant The intensity of solar radiation at the Earth's distance from the Sun.

Spacetime diagram A graph with time t on the vertical axis (usually scaled to ct, giving it units of distance) and distance x on the horizontal axis, useful for representing motion and events.

Spacetime The four-dimensional continuum (three space dimensions and one time dimension) in which physical phenomena take place.

Specific energy The amount of energy that can be obtained from a unit mass of a fuel. In the S.I. system it is measured in J kg^{-1}.

Specific heat capacity, *c* The energy required to change the temperature of a unit mass by one degree.

Specific latent heat The energy required to change the phase of a unit mass at constant temperature. (It is important to mention that this definition is for a constant temperature).

Star A normal star is a large mass consisting mostly of hydrogen, with a temperature in its core that is high enough for nuclear fusion reactions to take place.

Stellar cluster A large number of stars relatively close to each other that affect each other through their gravitational forces. Stellar clusters are divided into **globular** and **open** clusters. Stars in globular clusters tend to be closer to one another and are mainly old, evolved stars; open clusters contain young stars further apart than those in globular clusters.

Stopping potential The potential at which the photocurrent in a photoelectric experiment becomes zero.

Strangeness, *S* A quantum number that applies only to hadrons. For every strange quark in a hadron we assign a strangeness quantum number $S = -1$. For every anti-strange quark we assign a strangeness $S = +1$.

Streamline The path followed by a small bit of a fluid as it moves.

Systematic uncertainties Uncertainties due mainly to incorrectly calibrated instruments. They cannot be reduced by repeated measurements.

Terminal voltage The voltage *V* at the terminals of a battery or cell.

The solar system Our solar system consists of eight **planets** that revolve around a **star** we call the Sun.

Thermal equilibrium When two bodies are at the same temperature they are said to be in thermal equilibrium, and no further net energy gets transferred between them.

Thermistor A resistor in which the resistance decreases as the temperature increases.

Time constant The time after which the stored charge has decreased to about 37% of its initial value.

Transformer A device that takes as input an alternating voltage and delivers as output an alternating voltage of the same frequency and different peak value.

Trough A point of maximum negative displacement.

Type Ia supernova The explosion of a white dwarf star as mass falls into it and pushes the mass past the Chandrasekhar limit.

Type II supernova The explosion of a massive red supergiant star.

Uranium enrichment The uranium commonly mined is ^{238}U, which contains small traces (0.7%) of ^{235}U. Enrichment is the process of increasing the concentration of the useful ^{235}U in uranium samples.

Vector A physical quantity that has both magnitude and direction. It is represented by arrows. The length of the arrow gives the magnitude of the vector. The direction of the arrow is the direction of the vector.

Virtual image An image formed by mathematical extensions of light rays. It cannot be displayed on a screen.

Wave speed, *v* The speed at which a crest moves past an observer at rest in the medium. It equals $\lambda = f$.

Wavefront A surface at right angles to rays is called as wavefront. All the points on a wavefront have the same phase.

Wavelength, λ The length of a full wave. It may be found from a displacement versus distance graph.

White dwarf A very small, hot, dense star occupying the lower left region of the HR diagram. This is the end stage in the evolution of less massive stars.

Wien's law The wavelength λ_0 at which the emitted energy of a black body is a maximum is related to its surface temperature T by $\lambda_0 T = 2.9 \times 10^{-3}$ m K.

Work done For a constant force, and motion along a straight line, the work done by this force is the product of its magnitude and the distance travelled in the direction of the force, $W = Fd \cos \theta$, where θ is the angle between the force and the direction of motion. Work is a scalar quantity. (Note that if the force is *not* constant, you cannot use this formula. In this case the work is the area under a graph of force versus distance.)

Work function, ϕ The minimum energy required to eject an electron from a metal.

Work–kinetic energy relation The work done by the **net** force on a body is equal to the change in the body's kinetic energy, $W_{net} = \Delta E_K$. (Notice that this refers to the **net** force. This is an important relation in mechanics, with many applications.)

Worldline A curve on a spacetime diagram, consisting of a sequence of events showing the position of a particle as time passes.

X-rays Short-wavelength (10^{-10} m) electromagnetic radiation.

ANSWERS TO TEST YOURSELF QUESTIONS

Chapter 1

1.1 $[\eta] = \frac{[F]}{[6\pi r v]} = \frac{\text{kg ms}^{-2}}{\text{m ms}^{-1}} = \text{kg m}^{-1}\text{s}^{-1}$.

1.2 The quantity $\frac{Et^2}{\rho}$ has units $\left[\frac{Et^2}{\rho}\right] = \frac{\text{Js}^2}{\text{kg m}^{-3}} = \frac{\text{N ms}^2}{\text{kg m}^{-3}} = \frac{\text{kg m s}^{-2}\text{ ms}^2}{\text{kg m}^{-3}} = \text{m}^5$, from which the result follows. Solving for the energy we get $E \approx \frac{R^5\rho}{t^2} = \frac{140^5 \times 1}{0.025^2} \approx 9 \times 10^{13}$ J.

1.3 The uncertainty should be given to 1 s.f. The percentage uncertainty in the resistance is $\frac{\Delta R}{R} = 40\% + 60\% = 10\%$ and so $\Delta R = \pm 10\% \times 24 = \pm 2.4\ \Omega \approx \pm 2\ \Omega$.

1.4 Since $V = a^3$ the fractional uncertainty is $3 \times 0.02 = 0.06$ so the percentage uncertainty is 6%.

1.5 The percentage uncertainty is $\frac{\Delta T}{T} = \frac{1}{2} \times (4\% + 6\%) = 5\%$.

1.6

Equation	Constants	Variables to be plotted to give straight line	Gradient	Vertical intercept
$P = kT$	k	P versus T	k	zero
$v = u + at$	u, a	v versus t	a	u
$v^2 = 2as$	a	v^2 versus s	$2a$	zero
$F = \frac{kq_1q_2}{r^2}$	k, q_1, q_2	F versus $\frac{1}{r^2}$	k	zero
$a = -\omega^2 x$	ω^2	a versus x	$-\omega^2$	zero
$V = \frac{kq}{r}$	k, q	V versus $\frac{1}{r}$	k	zero
$T^2 = \frac{4\pi^2}{GM}R^3$	G, M	T^2 versus R^3	$\frac{4\pi^2}{GM}$	zero
$I = I_0 e^{-aT}$	I_0, a	In I versus T	$-a$	In I_0
$\lambda = \frac{h}{\sqrt{2mqV}}$	h, m, q	λ versus $\frac{1}{\sqrt{V}}$ or λ^2 versus $\frac{1}{V}$	$\frac{h}{\sqrt{2mq}}$ $\frac{h^2}{2mq}$	zero zero
$F = av + bv^2$	a, b	$\frac{F}{V}$ versus b	b	a
$E = \frac{1}{2}m\omega^2\sqrt{A^2 - x^2}$	m, ω^2, A	E^2 versus x^2	$-\frac{m^2\omega^4}{4}$	$\frac{m^2\omega^4A^2}{4}$
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	f	$\frac{1}{u}$ versus $\frac{1}{v}$	-1	$\frac{1}{f}$

1.7 Plot In d versus In h to get a straight line with slope 0.8 and vertical intercept In c. Alternatively, d versus $h^{0.8}$ or $d^{1.25}$ versus h give straight lines through the origin.

1.8 **a** (53.44 ± 0.01) g.

b (67.5 ± 0.5) mL.

1.9 **a** Since r^2 is proportional to V the graph will be a straight line through the origin.

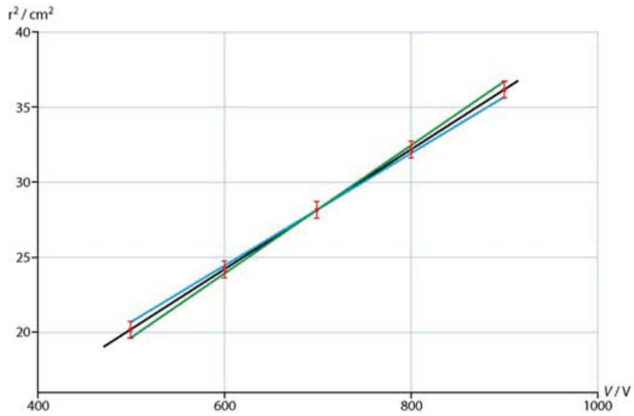
b The slope is $\frac{2m}{qB^2}$.

c We have that $\frac{\Delta r^2}{r^2} = 2\frac{\Delta r}{r} \Leftrightarrow \Delta r^2 = 2r\Delta r$. For the value $r = 4.5$ we thus have $r^2 = 4.5^2 = 20.2 \approx 20$ cm^2 to 2 s.f. $\Delta r^2 = 2 \times 4.5 \times 0.1 = 0.9$ cm^2 to 1 s.f. The least uncertain digit in the value of r^2 is in the units digit and so the uncertainty must express this fact. Hence the uncertainty must be rounded to the units digit, so $\Delta r^2 = 0.9 \approx 1$ cm^2. Hence $r^2 = 20 \pm 1$ cm^2. In the same way we can fill the rest of the table. (Note that the absolute uncertainties in all the values of r^2 are the same. This is not generally the case.)

Radius r/cm ±0.1 cm	Potential difference V / V	r^2 / cm^2
4.5	500	20 ± 1
4.9	600	24 ± 1
5.3	700	28 ± 1
5.7	800	32 ± 1

6.0	900	36 ± 1

d and **e** The error bars, best-fit line and minimum- and maximum-slope lines are shown here.



f The slope of the best-fit line is $\frac{(36-20)\times10^{-4}}{400} = 4.00\times10^{-6}\ \mathrm{m^2V^{-1}}$.

The slope of the maximum-slope line is $\frac{(37-19)\times10^{-4}}{400} = 4.50\times10^{-6}\ \mathrm{m^2V^{-1}}$.

The slope of the minimum-slope line is $\frac{(35-21)\times10^{-4}}{400} = 3.50\times10^{-6}\ \mathrm{m^2V^{-1}}$.

The uncertainty in the slope is therefore

$\frac{4.50\times10^{-6}-3.50\times10^{-6}}{2} = 0.50\times10^{-6} \approx 0.5\times10^{-6}\ \mathrm{m^2\ V^{-1}}$ to 1 s.f. We therefore quote the slope as $(4.0 \pm 0.5) \times 10^{-6}\ \mathrm{m^2\ V^{-1}}$.

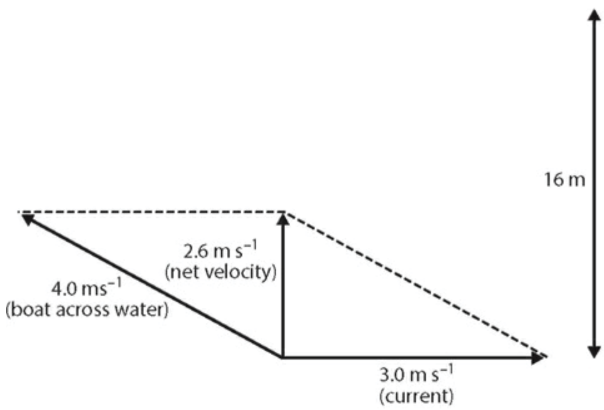
g The slope is equal to $\frac{2m}{qB^2}$ and so

$$\frac{q}{m} = \frac{2}{B^2\times\text{slope}} = \frac{2}{(1.80\times10^{-3})^2\times4.0\times10^{-6}} = 1.54\times10^{11}\ \mathrm{C\,kg^{-1}}.$$

Let U be the uncertainty in the measured value of $\frac{q}{m}$.

Then $\frac{U}{1.54\times10^{11}} = \frac{0.5\times10^{-6}}{4.0\times10^{-6}} \Rightarrow U = 0.193\times10^{11} \approx 0.2\times10^{11}\ \mathrm{C\,kg^{-1}}$. The uncertainty is in the first decimal place and so the measured value of $\frac{q}{m}$ should be rounded to the first decimal place, hence $\frac{q}{m} = (1.5 \pm 0.2)\times10^{11}\ \mathrm{C\,kg^{-1}}$.

1.10 The speed of the boat with respect to the shore is $\sqrt{4.0^2 - 3.0^2} = 2.646\ \mathrm{ms^{-1}}$ and the time taken to get across is $\frac{16}{2.646} = 6.0\ \mathrm{s}$.



1.11 The mass of an apple is about 200 g so its weight is about 0.2 × 10 ≈ 2 N.

1.12 The number of seconds in a year is of order $365\times24\times60\times60 \approx 400\times20\times50\times50 \approx 400\times20\times\frac{100}{2}\times\frac{100}{2} \approx 10^2\times20\times10^4 \approx 2\times10^7\ \mathrm{s}$.

1.13 The diameter of a nucleus is of order 10^{-15} m. Then the time taken by light to cross the diameter of a nucleus is of order $\frac{10^{-15}}{3\times10^8} = 0.33\times10^{-23} \approx 3.3\times10^{-24} \approx 10^{-24}\ \mathrm{s}$.

1.14 You should know that the heart beats at a rate of about 70 per minute. Then the interval between beats is $\frac{60}{70} \approx 1\ \mathrm{s}$.

1.15 The radius of the Earth is 6400 km and so the volume of the Earth is about

$$\frac{4\pi}{3}(6.4\times10^6)\ \mathrm{m^3} \approx \frac{4\times3\times(6\times10^6)^3}{3} \approx 4\times6\times40\times10^{18} \approx 10^{21}\ \mathrm{m^3}.$$

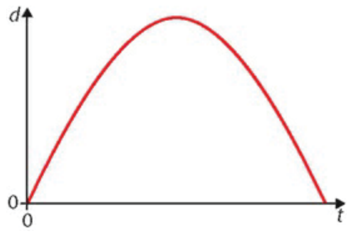
The volume of a grain of sand is about 1 mm^3 and so the number of grains of sand that can fit into a space the size of the Earth is $\frac{2\times10^{21}\ m^3}{1\ mm^3}=\frac{2\times10^{21}(10^3 mm)^3}{1\ mm^3}\approx 2\times10^{30}\approx 10^{30}$.

1.16 A glass contains about 200 g of water. The molar mass of water is 18 g per mole and so 200 g corresponds to $\frac{200}{18}\approx\frac{200}{20}=10$ moles. Each mole contains 6.02×10^{23} molecules, so the glass contains $6\times10^{23}\times10\approx10^{25}$ molecules of water.

1.17 We know that $I\propto\frac{T^4}{d^2}$ and so the percentage increase in the intensity received would be about 4 × 2% + 2 × 1% = 10%.

Chapter 2

2.1



2.2 The displacement is the vector shown in the figure.

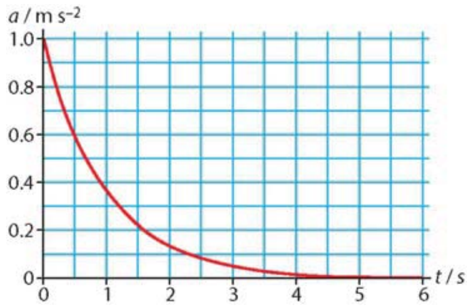
It has magnitude 10 m and so the average velocity is $\bar{v}=\frac{10}{5.0}=2.0\,\text{ms}^{-1}$ in the direction of the arrow.

The distance travelled is $\frac{1}{2}\times2\pi R\approx15.7\,\text{m}$ and so the average speed is $\frac{15.7}{5.0}=3.1\,\text{ms}^{-1}$.

2.3 Use $s=\left(\frac{u+v}{2}\right)t$, to get $20=\left(\frac{0+v}{2}\right)\times5.0\Rightarrow v=8.0\,\text{ms}^{-1}$.

2.4 **a** Drawing a tangent and finding its slope gives approximately 14 m s^{-2}.

b



c The area under the curve from 0 s to 6.0 s in Figure 2.4 can be obtained by counting squares, and is approximately 5.0 m. This is the displacement.

2.5 The area under the curve between 0s and 5.0s can be obtained by counting squares, and is 25 ms^{-1}. Since the initial speed is zero, the final speed is then 25 ms^{-1}.

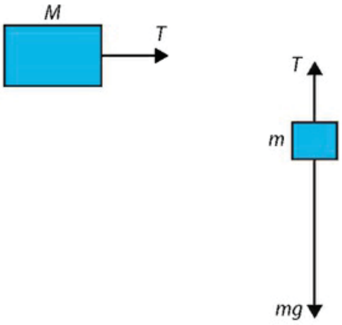
2.6 **a** The body will fall a vertical distance of 0.80m. So, from $\gamma=\frac{1}{2}gt^2$, $0.80=\frac{1}{2}\times9.8t^2$, giving t = 0.404s

b The horizontal distance travelled in this time is then $x=vt=15\times0.404=6.1$ m.

c The horizontal component of velocity stays constant at v_x = 15 ms^{-1}. The vertical component at impact with the floor is $v_y=u_y-gt=0-9.81\times0.404=-3.96$ ms^{-1}. The angle to the horizontal is then $\theta=\text{acrtan}\frac{v_y}{v_x}=\arctan\frac{-3.96}{15}=-14.8\approx-15^\circ$, or 15° *below* the horizontal.

2.7 The answer must be that either point of view gives the same time! Explicitly, let h be the height from the floor and let t be the time to hit the floor. Let u be the downward speed of the elevator. Then, for an observer outside, the ball will cover a distance $h+ut$ (the floor will move down a distance ut) and so $h+ut=ut+\frac{1}{2}gt^2$ (the ball has an initial speed equal to that of the elevator) and so $h=\frac{1}{2}gt^2$, the result we would get for an elevator at rest.

2.8 Look at the two masses separately, as in the diagram:



The net force on m is $mg - T$ and so $mg - T = ma$. The net force on M is T and so $T = Ma$. Thus:

$$mg - T = ma$$
$$T = Ma$$

Hence, by adding these equations, $mg = (M + m)a$ and so $a = \frac{mg}{M+m}$ and $T = \frac{Mmg}{M+m}$.

Alternatively you could consider the two masses as a single body. In this case the net force is mg and so the acceleration is $mg = (M+m)a \Rightarrow a = \frac{mg}{M+m}$. Then treat the bodies as separate again to find the tension T.

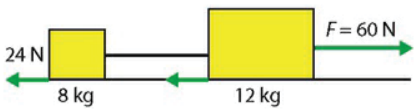
2.9 The tension in D pulls a bigger mass than the tensions in the other strings, and since all strings undergo the same motion, D has the greatest tension.

2.10 **a** Treating the two bodies as one, we see that the total mass is 20 kg and the net force 60 N so that the acceleration is 3.0 ms^{-2}.

b The net force on the 8.0 kg body is the tension in the string and so $T = ma = 8.0 \times 3.0 = 24$ N.

c If the bodies are reversed, the acceleration will stay the same but the tension will now be $T = ma = 12 \times 3.0 = 36$ N.

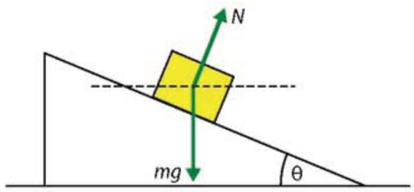
d The forces are now as shown below, and we have equilibrium.



The maximum frictional force on the larger block is $0.30 \times 120 = 36$ N. Hence the tension is $60 - 36 = 24$ N. The frictional force on the 8.0 kg block is 24 N.

2.11 The scale exerts a force R on the girl. Hence by Newton's third law, the girl exerts a force R on the scale, which is what the scale reads. The answer is **c.**

2.12 The forces on the block are as shown below.



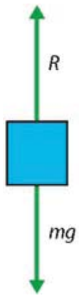
We must take components along the horizontal and vertical axes (because motion takes place horizontally). The vertical component of $\boldsymbol{N}$ is $N \cos\theta$, and this must equal the weight of the block, mg. The horizontal component is $N \sin\theta$, and this must equal ma. So,

$$N \sin\theta = ma$$
$$N \cos\theta = mg$$

This gives $\tan\theta = \frac{a}{g}$.

Treating the two bodies as one, we see that the net force is just F and so the acceleration is $a = \frac{F}{M}$, where m is the combined mass. Hence $\tan\theta = \frac{F}{Mg} = \frac{F}{W}$, as required.

2.13 The body is accelerating upwards, so R is bigger than mg.



The net force is thus $R - mg$ and so $R - mg = ma$, or $R - ma + mg$.

2.14 **a** $\Delta p = \left(0.20 \times 2.5 - 0.20 \times (\underbrace{-4.0}_{\text{notice the sign}})\right) = 1.3\,\text{Ns}$ to the right.

b $F = \frac{\Delta p}{\Delta t} = \frac{1.3}{0.14} = 9.3\,\text{N}$.

c If the system is taken to be just the ball, conservation of momentum does not apply, since there is a force on the block from the wall. If the system is taken to be the ball and the wall, then it does apply. The wall must have acquired a momentum of 13 Ns to the left.

d The net force on the ball is now $F = R - mg$ and so $F = \frac{\Delta p}{\Delta t} = R - mg$, giving $R = 9.3 + 2.0 \approx 11$N.

2.15 **a** In time Δt a mass $\mu \Delta t$ of gas will be ejected from the rocket. The momentum of this mass is $(\mu \Delta t)u$. Since everything was assumed to be initially at rest, $(\mu \Delta t)u$ is also the *change* in momentum Δp, and so the force on the gases is $F = \frac{\Delta p}{\Delta t} = \frac{(\mu \Delta t)}{\Delta t} - \mu u$.

b The force exerted on the gases is $2.2 \times 3.0 \times 10^2 = 660$N.

c By Newton's third law, this force is also the force acting on the rocket in the opposite direction, and so it accelerates the rocket.

d The initial mass of the rocket is 200 kg so the initial acceleration is $a = \frac{660}{200} = 3.3\,\text{ms}^{-2}$.

2.16 **a** About 2.0 ms.

b The area is (by counting squares) about 95×10^{-3} Ns.

c The average force is given by $F\Delta t$ = impulse = 95×10^{-3} Ns, or $F = 48$ N.

d The impulse is also $\Delta p = mv - m(-v) = 2mv$ and so $m = 2.6 \times 10^{-3}$ kg.

2.17 The change in the kinetic energy of the body is $\Delta E_K = 0 - 48 = -48$J. The only force doing work on the body is the frictional force, and $W_{net} = f \times d \times \cos 180° = -6.0 \times d$. Hence $-6.0d = -48 \Rightarrow d = 8.0$m.

2.18 The work is $W = Fd \cos \theta$ with $\theta = 0°$ and so equals $W = 20 \times 2\pi \times 5.0 \approx 630$J.

2.19 Since elastic potential energy $E_E = \frac{1}{2}kx^2$, extending the spring from its natural length to an extension $2e$ would require work equal to $4W$. So to extend it from e to $2e$ requires $4W - W = 3W$, hence the answer is **c.**

2.20 **a** The total momentum before the collision is $5.0 \times 3.0 + 7.0 \times 0 = 15$ Ns. After the collision it is $(5.0 + 7.0) \times v$. Thus $12v = 15$ Ns $\Rightarrow v = 1.25$ ms^{-1}.

b The total kinetic energy before the collision is $E_E = \frac{1}{2}5.0 \times 3.0^2 + 0 = 22.5\,\text{J}$. After the collision it is $E_K = \frac{1}{2}12 \times 1.25^2 = 9.4\,\text{J}$. The collision is inelastic since kinetic energy has not been conserved.

2.21 **a** The frictional force is 520 N and the power it dissipates is $P = 520 \times 8.0 = 4160$W.

b The rate of increase of the gravitational potential energy is $P_G = mgv \sin \theta = 14000 \times 8.0 \times \sin 6.0 = 11{,}707$ W. The total power that must be provided is therefore 4160 + 11,707 = 15.9 kW.

c The efficiency is $\frac{15.9\,\text{kW}}{45\,\text{kW}} = 0.35$.

Chapter 3

3.1 **a** 238g

b 4g

c The molar mass of hydrogen $\left({}^1_1\text{H}\right)$ is 1 g and that of oxygen $\left({}^{16}_8\text{O}\right)$ is 16g. Hence, the molar mass of water is $2 \times 1 + 16 = 18$g.

3.2 Consider a mole of lead. Its mass is 207 g or 0.207 kg. The volume of one mole is therefore $\frac{0.207}{1.13 \times 10^4} = 1.83 \times 10^{-5}\,\text{m}^3$ (density = mass/volume). Since we have one mole, there are 6.02×10^{23} molecules of lead. Thus to each molecule there corresponds a volume

$\frac{1.83\times10^{-5}}{6.02\times10^{23}} = 3.04\times10^{-29}\ \mathrm{m}^3$. Assuming this volume to be a cube, we find that the cube side is $a = \sqrt[3]{3.04\times10^{-29}} \approx 3\times10^{-10}\ \mathrm{m}$, and, as Figure 3.1 shows, this is also the average separation of the molecules.

3.3 The change in temperature is 10 K and the final temperature is 32 + 273 = 305 K.

3.4 The thermal energy lost by the aluminium is 0.080 × 900 × (250 − *T*). The thermal energy gained by the water and calorimeter is 0.75 × 4200 × (*T* − 15) + 0.200 × 900 × (*T* − 15). From conservation of energy we have 0.080 × 900 × (250 − *T*) = 0.75 × 4200 × (*T* − 15) + 0.200 × 900 × (*T* − 15). Solving for *T*, we find that *T* = 20°C.

3.5 **a** It takes 140s to warm the paraffin from 20°C to 48 °C, so $P = mc\frac{\Delta\theta}{\Delta t} = 0.120\times2500\times\frac{28}{140}$ = 60W.

b The constant temperature at which paraffin melts is 48°C.

c It takes the paraffin 560 − 140 = 420 s to melt. The energy provided is *Q* = *Pt* = 60 × 420 = 25200J and so the specific latent heat of fusion is $L = \frac{Q}{m} = \frac{25200}{0.120} = 2.1\times10^5\ \mathrm{J\,kg^{-1}}$.

d It takes an additional 600 − 560 = 40s to warm liquid paraffin at 48°C to a temperature of 58°C, so $P = mc\frac{\Delta\theta}{\Delta t} = 0.120\times c\times\frac{58-48}{40} \Rightarrow c = \frac{60\times40}{10\times0.120} = 2.0\times10^3\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

e The temperature stays constant even though energy is provided because the energy is used to separate the molecules, thus increasing their intermolecular potential energy. No energy goes into kinetic energy and so the temperature does not change.

3.6 Since the temperature increases, the average molecular speed increases. Hence the pressure will go up. In addition, the higher speed means the frequency of collisions will increase as well, so the pressure increases even further.

3.7 **a** Boiling takes place at a particular temperature, evaporation at any temperature. Boiling takes place throughout the liquid volume, evaporation only from the surface of a liquid.

b The molecules that leave the surface are the faster ones and so the kinetic energy of the molecules left behind is reduced. Since temperature is a measure of the kinetic energy of the molecules, the temperature of the liquid is reduced.

3.8 $pV = nRT$ so $n = \frac{pV}{RT} = \frac{2.0\times10^5\times4.0\times10^{-3}}{8.31\times300} = 0.32$.

3.9 From $\frac{p_1V_1}{n_1T_1} = \frac{p_2V_2}{n_2T_2}$, because *N* and *V* stay the same we have $\frac{p_1}{T_1} = \frac{p_2}{T_2}$. So $\frac{p_1}{300_1} = \frac{p_2}{400} \Rightarrow p_2 = \frac{400}{300}\times3.0\times10^5 = 4.0\times10^5\ \mathrm{Pa}$.

3.10 Common sense says that the pressure has to be between *P* and 2*P*, so the answer must be **c.** Explicitly, the pressure is $\frac{3RT}{2V} = \frac{3}{2}\frac{nRT}{V} = \frac{3}{2}P$, where *N* = 3 is the total number of moles and *V* is the volume of one-half of the container.

3.11 From $\frac{p_1V_1}{n_1T_1} = \frac{p_2V_2}{n_2T_2}$, because *p* and *T* stay the same we have $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. So $T_2 = \frac{V_2}{V_1}T_1 = \frac{5.0\times10^{-3}}{2.0\times10^{-3}}\times300 = 750\,\mathrm{k}$.

3.12 **a** The internal energy of an ideal gas is the total random kinetic energy E_k of the molecules of the gas.

b The absolute temperature is proportional to the *average* kinetic energy of the molecules, $T \propto \frac{F_K}{N}$. Therefore the internal energy is proportional to the absolute temperature (and the number of molecules), $U = E_K \propto = NT$.

3.13 The piston, moving rapidly inwards, collides with molecules, so the molecules rebound with speeds greater than before the collision. The average kinetic energy of the molecules – and hence the temperature – increases.

3.14 Use $\frac{1}{2}mv^2 = \frac{3}{2}kT$. Since *k* is a constant and *T* is held constant, $\frac{1}{2}m_{\mathrm{Ne}}v_{\mathrm{NC}}^2 = \frac{1}{2}m_{\mathrm{Ar}}v_{\mathrm{Ar}}^2$. So $\frac{v_{\mathrm{Ne}}^2}{v_{\mathrm{Ar}}^2} = \frac{m_{\mathrm{Ar}}}{m_{\mathrm{Ne}}}$ and $\frac{v_{\mathrm{NC}}}{v_{\mathrm{Ar}}} = \sqrt{\frac{m_{\mathrm{Ar}}}{m_{\mathrm{Ne}}}}$. Hence $\frac{v_{\mathrm{Ne}}}{v_{\mathrm{Ar}}} = \sqrt{\frac{40}{20}} = \sqrt{2}$, so the answer is **b.**

Chapter 4

4.1 **a** Yes, since acceleration is opposite to, and proportional to, displacement.

b No, since acceleration is not opposite to displacement.

c No, since acceleration is not proportional to displacement.

4.2 **a** The graph is a straight line through the origin with negative slope and so the motion satisfies the defining equation for SHM.

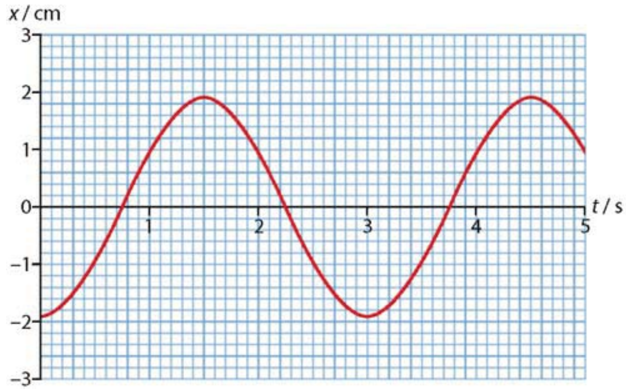
b 2.0 cm

4.3 **a** 3.0s.

b At any maximum or minimum, for example 0.75s.

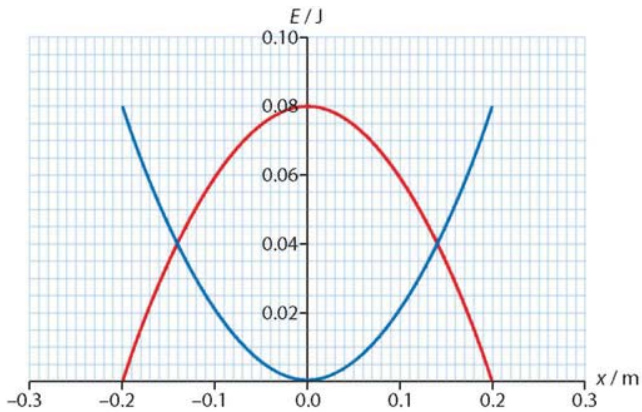
c At any time when the speed is zero, for example 1.5s.

d See the graph below:



4.4 **a** 0.80 ms^{-1}

b See the blue curve below:



c 0.14m

4.5 **a** $1.33 = \frac{3.0\times10^8}{v} \Rightarrow v = 2.3 \times 10^8\ \text{ms}^{-1}$.

b The frequency of light must stay the same and so $\lambda_{\text{water}}\frac{\lambda_{\text{air}}}{1.33} = 406\ \text{nm}$.

c The angles in the refraction formula are measured relative to the normal to the surface. From $n_1 \sin \theta_1 = n_2 \sin \theta_2$, we have $1 \times \sin 40° = 1.33 \sin \theta_2 \Rightarrow \sin \theta_2 = 0.483 \Rightarrow \theta_2 = 29$.

4.6 Total internal reflection can occur when light passes from glass into water. $1.45 \times \sin \theta = 1.33 \times \sin 90° \Rightarrow \sin \theta_c = 0.9172 \Rightarrow \theta_c = 66.5°$.

4.7 The path difference at P is $S_2P - S_1P = 0.60$ m, and since $\frac{S_2P-S_1P}{\lambda} = \frac{0.60}{0.30} = 2$, we will have constructive interference at P. (At any point that is equidistant from the sources, the path difference is zero and so we have constructive interference.) At Q, $S_2Q - S_1Q = 0.45$ m, and $\frac{S_2Q-S_1Q}{\lambda} = \frac{0.45}{0.30} = 1.5$, we will have destructive interference at Q.

4.8 The energy gets redistributed in the interference pattern. The energy that would appear at the minima appears at the maxima.

4.9 The intensity is reduced by a factor of 2 after going through the first polariser. There is a further reduction by $\cos^2 45°$ after transmission through the second polariser and then another $\cos^2 45°$ after transmission through the third. Overall the intensity is reduced by $\frac{1}{2} \times \cos^2 45^\circ \times \cos^2 \times 45^\circ = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

4.10 The wavelength of the first harmonic is $\lambda_1 = 2L$ and that of the second is $\lambda_2 = L$. If c is the speed of a wave on the string, then $f = \frac{c}{L}$. Hence $\frac{f_2}{f_1} = \frac{\frac{c}{L}}{\frac{c}{2L}} = 2$.

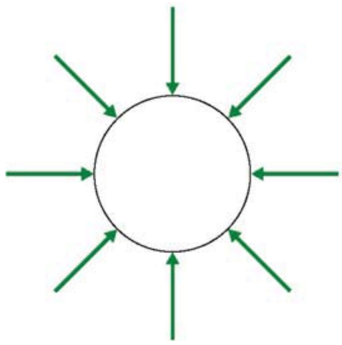
4.11 The wavelength in tube × is $\lambda_X = 2L$ and that in tube Y is $\lambda_Y = 4L$. Hence $\frac{f_x}{f_Y} = \frac{\frac{c}{\lambda_x}}{\frac{c}{\lambda_Y}} = \frac{\lambda_Y}{\lambda_x} = \frac{4L}{2L} = 2$.

4.12 In the first case we have the first harmonic, so $\lambda = 4L_1$. In the second case $\lambda = \frac{4L_2}{3}$, so $3\lambda = 4L_2$. Subtracting, $2\lambda = 4(L_2 - L_1) = 4x = 1.20 \Rightarrow \lambda = 0.60$m.Thus $v = \lambda f = 0.60 \times 560 = 336 \approx 340\ \text{ms}^{-1}$.

Chapter 5

5.1 According to Coulomb's law, if the charge doubles so does the force. But the magnitude of the force is the same on both charges, hence **b** is correct.

5.2



5.3 If Q is positive, a negative charge is induced on the near side of the sphere, leaving a positive charge on the far side. from Coulomb's law, the point charge will experience a stronger force from the nearer negative charge than from the positive charge, leading to a net attraction. If Q is negative, negative charge is pushed to the far side of the sphere, again leading to a net attraction. So **d** The force will always be attractive.

5.4 **b** To the left.

5.5 When the voltage is 0.30 V the current is 1.1 A and so $R = \frac{V}{I} = \frac{0.30}{1.1} = 0.27\Omega$.

5.6 The current is unchanged (conservation of electric charge), so the possible answers are **a** and **b**. From the current formula, where cross-sectional area is four times smaller the drift speed must be four times larger, so the answer is **b**.

5.7 Using $P = \frac{V^2}{R}$ we find $R = \frac{V^2}{P} = \frac{240^2}{6} 960\,\Omega$. The new voltage is *half* the original and so $P = \frac{60}{4} = 15\,\text{W}$.

5.8 **a** If the radius doubles, the cross-sectional area increases by a factor of $2^2 = 4$ and so the resistance becomes $\frac{1}{4} \times 2 = \frac{1}{2}$ as large.

b Each piece will have resistance $R/2$. Hence the combination has resistance $R/4$ by applying the law of parallel resistors.

5.9 The work required to push one electron through the resistor is $W = qV$ and so $7.2 \times 10^{-19} = 1.6 \times 10^{-19} \times V$, giving $V = 4.5\text{V}$ The potential difference across the internal resistor is $0.80 \times 1.4 = 1.12$ V and so the EMF is $4.5 + 1.12 = 5.6$ V

5.10 **a** We have the top two resistors in series, for a total of 120ω, and this is in parallel with the bottom 60 Ω resistor, for a grand total resistance between points A and B of $\frac{1}{120} + \frac{1}{60} = \frac{1}{40} \Rightarrow R_T = 40\Omega$.

b The bottom resistor must have burnt out – that is, the circuit there is broken.

5.11 **a** The potential difference across the 25 Ω resistor is $25 \times 0.20 = 5.0$ V and this is also the potential difference across R_2. Hence $R_2 = \frac{5.0}{0.60} = 8.3\Omega$.

b The current through R_1 is 0.80 A and the potential difference across it is $12 - 5.0 = 7.0\text{V}$. Hence $R_2 = \frac{7.0}{0.80} = 8.8\Omega$.

5.12 **a** We have 2 resistors in series for a total of $6.0 + 6.0 = 12\Omega$ and this is in parallel with the bottom resistor for a total of $\frac{1}{12} + \frac{1}{6.0} = \frac{3}{12} \Rightarrow R_T = 4.0\,\Omega$. In turn this is in series with the internal resistor for a grand total of $4.0 + 2.0 = 6$ Ω. The current is therefore is $I = \frac{12}{6.0} = 2.0\,\text{A}$.

b If the bottom resistor burns out the total becomes $6.0 + 6.0 + 2.0 = 14$ Ω.

5.13 **a** The lamps are in series so they take the same current. Since $P = RI^2$ it follows that lamp A has the greater resistance.

b The lamps are in parallel so they have the same potential difference across them. Hence the convenient power formula is $P = \frac{V^2}{R}$. Lamp B has the smaller resistance and so the greater power.

5.14 The total resistance of $R_1 + R_2 = 100$ Ω and $R = 5.0\Omega$ is $\frac{1}{100} + \frac{1}{5.0} = \frac{21}{100} \Rightarrow R_T = 4.76\,\Omega$. This is in series with the internal resistance and so the overall total resistance of the circuit is $R_T = 2.0 + 4.8 = 6.8$ Ω. The current is then $I = \frac{12}{6.8} = 1.8\,\text{A}$ and so the reading of the voltmeter is $V = 12 - 1.8 \times 2.0 = 8.4\text{V}$.

5.15 **a** From $P = \frac{V^2}{R}$ we find $R = \frac{V^2}{P} = \frac{36}{12} 3.0\,\Omega$.

b From $P = VI$ we get $I = \frac{P}{V} \frac{12}{6.0} = 2.0\,\text{A}$.

c The potential difference across wire AB is 6.0V and so the potential difference across BC is also 6.0V. Since the resistance of AB is double that of BC, the current in BC is double that in AB. Let x stand for the current in AB. Then, Hence $x + 2.0 = 2x$ and so $x = 2.0\text{A}$. The resistance of AB is then $R_{AB} = \frac{6.0}{2.0} = 3.0\,\Omega$ and so $R_{AC} = 4.5\Omega$.

5.16 **a** When the temperature is increased the resistance of T will decrease. Therefore the potential difference across T will decrease as well. We can see this as follows: think of the circuit as a potential divider. Since the resistance of T is less than R and the currents in T and R are the same, the potential difference drops.

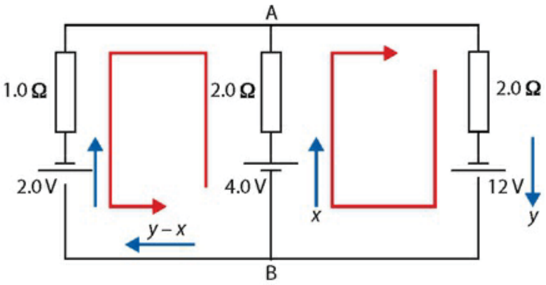
b The LDR will decrease in resistance as the light gets brighter and so the answer is the same as in **a.**

5.17 We define currents x and y through the two cells as shown. The current in the 2.0V cell is $y - x$ by applying the current law to junction B.

The left-hand loop gives: $4.0 + 2.0 = 2x - (y - x)$, so $3x - y = 6.0$.

The right-hand loop gives: $12 + 4 = 2x + 2y$, so $x + y = 8.0$.

Solving gives $x = 3.5$A and $y = 4.5$A.



a The current through the 4.0V cell is 3.5A in the direction of the arrow. In the 12V cell it is 4.5A in the direction of the arrow. In the 2.0V cell it is 1.0A in the direction of the arrow.

b The current in the 2.0V cell goes 'backwards' – that is, from the positive to the negative terminal – which means that the cell is being charged.

c We may take the potential at B to be anything we like, say 12V. The potential at A is $V_A = 16 - 2.0 \times 3.5 = 9.0$V. The potential difference is then 3.0V with B at the higher potential.

d Power is generated by the 4.0V and 12V cells. This power is
$P = \sum \varepsilon \times I = 12 \times 4.5 + 4.0 \times 3.5 = 68$W. The power dissipated in the resistors is
$P = \sum RI^2 = 1.0 \times 1.0^2 + 2.0 \times 3.5^2 + 2.0 \times 4.5^2 = 66$W and in the 2.0V cell it is
$P = \varepsilon \times I = 2.0 \times 1.0 = 2.0$W. The total dissipated power is 68 W and energy is conserved.

5.18 Applying the right-hand rule to find the magnetic fields at Z from each current and adding as vectors gives **d**, a leftward net field.

Chapter 6

6.1 Both points complete one revolution in the same time and so have equal angular speeds. The linear speed is given by $v = \omega r$ and so Y has double the linear speed, so the answer is **b.**

6.2 **a** For maximum speed we will need the maximum frictional force $f_{max} = \mu_S N$, where $N = mg$. This provides the centripetal force, so:

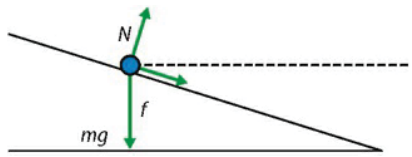
$$f_{\text{max}} = ma$$

$$\mu_S N = m\frac{v^2}{R}$$

$$\mu_S mg = m\frac{v^2}{R}$$

$$v = \sqrt{\mu_S gR} = \sqrt{0.60 \times 9.81 \times 45} = 16\,\text{ms}^{-1}$$

b If the road were banked the normal force would be at an agle to the vertical, as shown:



We see that there is now an additional force towards the centre of the circle, namely the horizontal component of the normal reaction force N. This provides additional centripetal force; with greater centripetal force the centripetal acceleration is greater, and from $a = \frac{v^2}{r}$ the speed can be greater as well. (The careful reader will have noticed that, at the same time, the frictional force towards the centre is reduced, since only its component $f \cos\theta$ points towards the centre of the curve. Therefore, strictly speaking a more detailed analysis is needed. The correct analysis does in fact support this conclusion.)

6.3 Referring to Figure 6.6, the net centrifugal force on the car is $mg - N$, and this equals $ma = m\frac{v^2}{R}$.

The car would lose contact with the road if $N \to 0$, which corresponds to $mg = m\frac{v^2}{R} \Rightarrow v = \sqrt{gR}$.

6.4 The block is moving in a circle (of radius L). This requires a net force, so it is not in equilibrium. As the string moves past the vertical position, the net force on the block points towards the other end of the string – that is, upwards. A net upwards force means that the tension in the string is larger than the weight of the block. Applying Newton's second law, $F = ma$ with $F = T - mg$ and $a = \frac{v^2}{L}$, we have $T - mg = m\frac{v^2}{L}$. To find the tension we therefore need to know the speed of the block. For this we can apply conservation of energy, $\frac{1}{2}mv^2 = mgL$, so $m\frac{v^2}{L}$, leading, finally, to $T = 3mg$. (Note that the result doesn't depend on the length of the string.)

6.5 The body'a weight is the gravitational force acting on it. At the planet's surface, Newton's law of gravitation gives $20\text{N} = \frac{GMm}{(2R)^2} = \frac{1}{4}\frac{GMm}{R^2} = 5.0\,\text{N}$. At a height R above the surface, the body is $2R$ from the centre of the planet, and Newton's law gives $W = \frac{GMm}{(2R)^2} = \frac{1}{4}\frac{GMm}{R^2} = 5.0\,\text{N}$.

6.6 Write the equation for the gravitational field strength of the Earth: $g = \frac{GM}{R^2}$, where M is the Earth's mass and R is its radius. Do the same for the other planet: $g_\text{P} = \frac{GM_\text{P}}{R_\text{P}^2}$. Divide side by side:

$$\frac{g_\text{P}}{g} = \frac{\frac{GM_\text{P}}{R_\text{P}^2}}{\frac{GM}{R^2}} = \frac{M_\text{P}}{M}\frac{R^2}{R_\text{P}^2} = 2 \times \frac{1}{2^2} = \frac{1}{2}\text{. Then } g_\text{P} = \frac{g}{2},$$

Alternatively, write $g_\text{P} = \frac{GM_\text{P}}{R_\text{P}^2} = \frac{G(2M)}{(CR)^2} = \frac{2}{4}\frac{GM}{R^2} = \frac{1}{2}g$.

6.7

$$g_\text{total} = \underbrace{\frac{GM_\text{E}}{x^2}}_{\text{due to Earth}} - \underbrace{\frac{GM_\text{M}}{(3.8\times10^8 - x)^2}}_{\text{due to Moon}} = 0.$$

Because $M_\text{E} = 81\,M_\text{M}$,

$$\frac{G81M_\text{M}}{x^2} = \frac{GM_\text{M}}{(3.8\times10^8 - x)^2}$$

$$\frac{81}{x^2} = \frac{1}{(3.8\times10^8 - x)^2}$$

$$81 \times (3.8 \times 10^8 - x)^2 = x^2$$

$$9 \times (3.8 \times 10^8 - x) = x$$

$$10x = 9 \times 3.8 \times 10^8$$

$$x = 3.4 \times 10^8\ \text{m}$$

6.8 For small values of r (i.e. close to the larger mass), the field must be directed to the left, thus it must be negative. This eliminates B and C. The field must be zero closer to the smaller mass and so the answer is A.

Chapter 7

7.1 The longest wavelength means the smallest energy (since $\Delta E = hf = \frac{hc}{\lambda}$). Hence it is the transition from $N = 3$ to $N = 2$.

7.2 Using $\Delta E = hf$ we find $1.9 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times \frac{3\times10^8}{\lambda}$ and so $\lambda = 6.5 \times 10^{-7}$m.

7.3 $\frac{K_\text{P}}{K_\text{Pb}} = \frac{\frac{p^2}{2m_\text{p}}}{\frac{p^2}{2m_\text{Pb}}} = \frac{m_\text{Pb}}{m_\text{p}} = \frac{214}{4} = 53.5$ since the momenta are equal and opposite.

7.4 12 days corresponds to 3 half-lives and so the activity is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ of the original.

7.5 The table shows the fraction of each isotope present initially and after 2.0. 4.0 and 6.0 minutes.

Time / min	X	Y	$\frac{Y}{X}$
0	1	0	0
2.0	$\frac{1}{2}$	$\frac{1}{2}$	1
4.0	$\frac{1}{4}$	$\frac{3}{4}$	3
6.0	$\frac{1}{8}$	$\frac{7}{8}$	7

So 6.0 minutes must go by.

7.6 **c** There is a 50% chance of decaying in any half-life interval.

7.7 **a** The energy released is given by $Q = (M_\text{Po} - (M_\text{Pb} + M_\text{He}))c^2$. Now, $\text{BE}_\text{Po} = (84m_\text{p} + 134m_\text{n} - M_\text{Po})c^2 \Rightarrow M_\text{Po}c^2 = (84m_\text{p} + 134m_\text{n})c^2 - \text{BE}_\text{Po}$

$\text{BE}_\text{Pb} = (82m_\text{p} + 132m_\text{n} - M_\text{Pb})c^2 \Rightarrow M_\text{Pb}c^2 = (82m_\text{p} + 132m_\text{n})c^2 - \text{BE}_\text{Pb}$

$\text{BE}_\text{He} = (2m_\text{p} + 2m_\text{n} - M_\text{He})c^2 \Rightarrow M_\text{He}\,c^2 = (2m_\text{p} + 2m_\text{n})c^2 - \text{BE}_\text{He}$.

Hence, $Q = \text{BE}_\text{Pb} + \text{BE}_\text{He} - \text{BE}_\text{Po}$.

This shows that the energy released is the difference between the total binding energy of the products minus the binding energy of the decaying nucleus.

b The binding energy of polonium (Po) is 218 × 7.8 ≈ 1700 MeV, that of helium (He) 4 × 7.2 ≈ 29 MeV and that of lead (Pb) 214 × 7.9 ≈ 1690 MeV. The energy released is then about 29 × 1690 − 1700 = 19 MeV.

7.8 There are 11 nucleons in $^{11}_{5}\mathrm{B}$ and so the energy is 11 × 7.0 = 77 MeV.

7.9 The total mass on the left is 18.005 67 *u* and is **smaller** than the total on the right of 18.00696 *u*, by 0.00129 *u*. Roughly speaking, the missing energy of 0.00129 × 931.5 = 1.20MeV must be provided by the alpha particle's kinetic energy. (In fact, because momentum must be conserved, the alpha's kinetic energy must be somewhat larger than this.)

7.10 **a** The beam must be very narrow in order that the scattering angle can be measured acccurately.

b The foil must be very thin in order to avoid absorption of the alpha particles by the foil and also in order to avoid multiple scatterings.

7.11 No, because the K meson has non-zero strangeness quantum number.

7.12 **a** $p \to e^- + \gamma$, baryon number, electric charge, lepton number

b $p - \pi^- + \pi^+$, electric charge, baryon number

c $n \to p + e^-$, lepton number

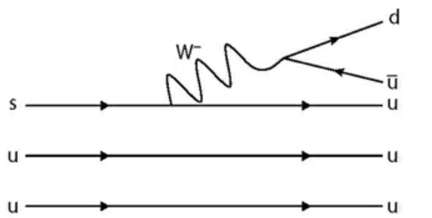
d $e^- + e^+ \to \gamma$, momentum

e $e^+ \to \mu^+ + \nu_\mu$, energy, lepton number

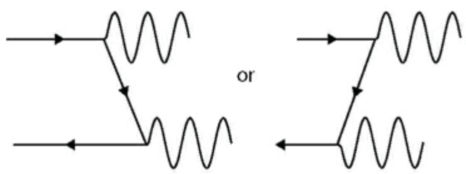
f $p \to n + e^+ + \nu_e$, energy (but this can occur within a nucleus where the required energy is provided by the binding energy of the nucleus).

7.13 **a** Strangeness is violated, since $S = -1$ on the left-hand side of the reaction equation but $S = 0$ on the right-hand side. Hence the decay must be a weak interaction process.

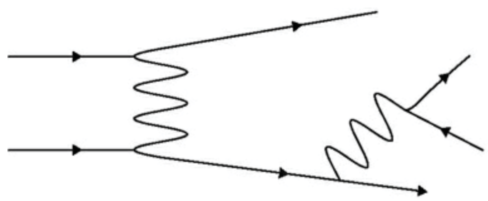
b A possible diagram is the following:



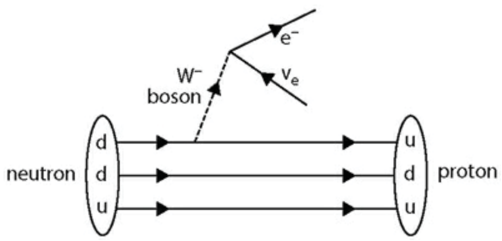
7.14



7.15



7.16



Chapter 8

8.1 **a** The energy per second – that is, the power – extracted from burning *M*kg per second is *ME*. The efficiency is *e* and so the power produced is *eME*.

8.2 The useful energy is 30% and the efficiency is 0.30.

8.3 The power to be provided by nuclear fission is $\frac{400}{0.40} = 1000\,\mathrm{MW}$. If the number of reactions per second is *N*, then *N* × 200 × 10^6 × 1.6 ×

10^{-19} = 1000 × 10^6 and so $N = \frac{1000\times10^6}{200\times10^6\times1.6\times10^{-19}} = 3.1\times10^{19}\ \mathrm{s}^{-1}$. In a single reaction, a mass of 235 *u* or 235 × 1.66 × 10^{-27} = 3.9 × 10^{-25} kg is used. Hence the mass used at a rate of 3.1 × 10^{19} × 3.9 × 10^{-25} = 1.2 × 10^{-5} kgs^{-1}. In one year we need 1.2 × 10^{-5} × 365 × 24 × 3600 ≈ 380 kg.

8.4 Without a moderator the neutrons would be too fast to cause any reactions, so very little energy would be produced.

8.5 The mass in the basin is 2.0 × 20 × 1000 = 4.0 × 10^4 kg. The *average* potential energy lost as the water descends is 4.0 × 10 × 9.8 × 10 = 3.92 × 10^6J (using the average depth, 10m).The basin empties in 40 minutes and so the power is $P = \frac{3.96\times10^6}{40\times60} = 1.6\,\mathrm{kW}$.

8.6 **a** From the formula, we have simply $P = \frac{1}{2}\rho A v^3 = \frac{1}{2}\times1.2\times\pi\times\left(\frac{12}{2}\right)^2\times8.0^3 = 35\,\mathrm{kW}$. The number of wind turbines needed is therefore $\frac{5.0\times10^3}{35} = 143$.

b In practice, each wind turbine will generate less than 35kW because of friction and other losses such as turbulence, and because the wind turbine does not extract all of the wind's kinetic energy. The calculation also assumes that the wind blows at right angles to the blades, which is not always the case unless the wind turbine can be turned into the wind.

8.7 We want the largest ratio of temperature difference to width and that is **c.**

8.8 The power is proportional to the 4th power of the Kelvin temperature, so doubling *T* makes the power 2^4 = 16 times larger.

8.9 **a** The power received by 1m^2 of the Earth's surface $(A = 1\,\mathrm{m}^2)$ is $P_{\text{in}} = (1-\alpha)\frac{S}{4}A = (1-\alpha)\frac{S}{4}$. The second term arises because a power $\alpha\frac{S}{4}A$ is reflected back into space. The Earth radiates power from the entire surface area of its spherical shape and so the power radiated from 1m^2 is (by the Stefan–Boltzmann law) $P_{out} = \sigma T^4$. (We are assuming that the emissivity of the Earth's surface is 1.

Of the 30% of reflected radiation only about 4% to 5% is reflected from the surface itself and so the emissivity is actually about 0.95–0.96.) Equating the two: $(1-\alpha)\frac{S}{4} = \sigma T^4 \Rightarrow (1-\alpha)S = 4\sigma T^4$.

b We find $T = \frac{\sqrt[4]{(1-\alpha)S}}{4\sigma}$, is $T = \frac{\sqrt[4]{(1-0.30)\times1400}}{4(5.67\times10^{-8})} \approx 256\,\mathrm{K}$. This temperature is −17°C.

c A temperature of 256K is 32K lower than the Earth's actual average temperature of 288K, so obviously the model is wrong. One reason this model is too simple is precisely because we have not taken account of the fact that not all the power radiated by the Earth actually escapes. Some is absorbed by gases in the atmosphere and re-radiated down to the Earth's surface, causing further warming. In other words, this model neglects the *greenhouse effect.* Another drawback of this simple model is that it is essentially a zero-dimensional model – the Earth is treated as a point without interactions between the surface and the atmosphere (latent heat flows, thermal energy flow in oceans through currents, thermal energy transfer between the surface and the atmosphere due to temperature differences between the two are all ignored). Realistic models must take all these factors (and many others) into account and so are very complex.

Chapter 9

9.1 **a** The graph is a straight line through the origin with negative slope so it satisfies the defining equation for SHM, $a = -\omega^2 x$.

b From the graph, the slope is $\frac{-6.0-6.0}{15\times10^{-2}-(-15\times10^{-2})} = -40\,\mathrm{s}^{-2}$.This equals $-\omega^2$ and so $\omega \approx 6.3246\mathrm{s}^{-1}$. Hence $T = \frac{2\pi}{\omega} = \frac{2\pi}{6.3246} = 0.99\,\mathrm{s}$.

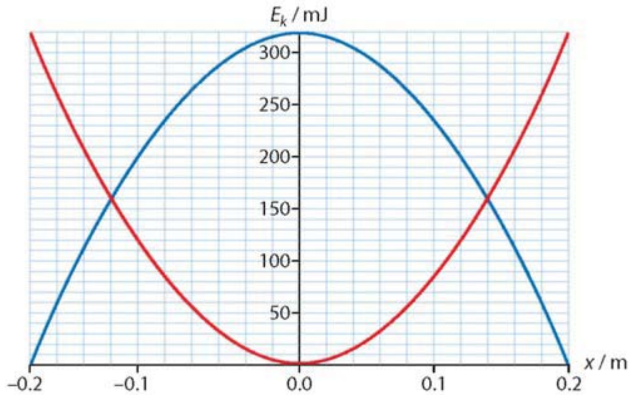
c The maximum speed is $v = \omega x_0 \approx 6.3246 \times 0.15 = 0.95\ \mathrm{ms}^{-1}$.

d From $v = \omega\sqrt{x_0^2 - x^2}$ we find $v = 6.3246\times\sqrt{0.15^2-0.05^2} \approx 0.89\,\mathrm{ms}^{-1}$.

9.2 **a** The maximum kinetic energy is 0.32 J and so $E_{\max} = \frac{1}{2}m\omega^2 x_0^2 = 0.32 \Rightarrow \omega = \sqrt{\frac{2\times0.32}{0.25\times(0.20)^2}} = 8.0\,\mathrm{s}^{-1}$. Hence $T = \frac{2\pi}{\omega} = \frac{2\pi}{8.0} = 0.79\,\mathrm{s}$.

b The maximum speed is $v_{max} = \omega x_0 = 8.0 \times 0.20 = 1.6\ \mathrm{ms}^{-1}$. The maximum acceleration is $a_{max} = \omega^2 x_0 = 8.0^2 \times 0.20 \approx 13\mathrm{ms}^{-2}$.

c See the red line in the graph.



d The two curves intersect at $x \approx \pm 14$ cm.

9.3 Use $\theta_D \approx \frac{\lambda}{b}$. From Figure 9.3, θ_D = 0.028 rad and so $b \approx \frac{\lambda}{\theta_D} = \frac{4.8\times10^{-7}}{0.028} = 1.7\times10^{-5}\,\mathrm{m}$.

9.4 The angular separation of two points at the ends of the diameter of Mars is $\theta_\mathrm{A} = \frac{7\times10^6}{7\times10^{10}} = 1\times10^{-4}$. The angle of the first diffraction minimum at the eye is $\theta_\mathrm{D} = 1.22\frac{650\times10^{-9}}{3\times10^{-3}} = 3\times10^{-4}$. The angular separation is less than this ($\theta_\mathrm{A} < \theta_\mathrm{D}$) and so the points are not resolved. Mars appears as a point source.

9.5 The number n of lines per mm gives d as $d = \frac{1}{N}$ mm. Hence $d = \frac{1}{700}$ =1.4286 × 10^{-3} mm=1.4286 × 10^{-6}. The n = 1 primary maximum is observed at θ = 20° and so from $d \sin\theta = n\lambda$ we find $1.4286\times10^{-6}\times\sin 20° = 1\times\lambda$, giving $\lambda = 4.9\times10^{-7}$m.

9.6 The smallest difference in wavelength that can be resolved is $\Delta\lambda = \frac{\bar{\lambda}}{mN} = \frac{589.2937}{2\times1000} = 0.2946\,\mathrm{nm}$. This is smaller than the difference in wavelength of the two lines (0.5974 nm) and so the lines can be resolved.

9.7 We have a phase change at only one boundary. The condition of constructive interference is therefore $2d\left(m+\frac{1}{2}\right)\frac{\lambda}{n}$. For the minimum thickness we choose m = 0, so $d = \frac{\lambda}{4n} = \frac{520}{4\times1.33} \approx 98\,\mathrm{nm}$.

9.8 The car is approaching the emitter so the frequency it receives is $f_1 = 300\times\frac{340+u}{340}\,\mathrm{Hz}$, where u is the unknown car speed. The car now acts as an emitter of a wave of this same frequency (f_1), and the original emitter now acts as the new receiver. The frequency received at this new receiver is $315 = f_1\times\frac{340}{340-u} = \left(300\times\frac{340+u}{340}\right)\times\frac{340}{340-u}$, from which we find u = 8.29ms^{-1}.

9.9 The highest frequency measured will be when the child and whistle are approaching the observer, so $510 = f_\mathrm{s}\frac{340}{340-v_\mathrm{s}}$, and the lowest when they are moving away, so $510 = f_\mathrm{s}\frac{340}{340+v_\mathrm{s}}$. Hence $\frac{510}{504} = 1.0119 = \frac{f_\mathrm{s}\frac{340}{340-v_\mathrm{s}}}{f_\mathrm{s}\frac{340}{340+v_\mathrm{s}}} = \frac{340=v_s}{340-v_s}$. Solving for the speed v_s of the carousel: $0.0119\times340 = 2.0119v_\mathrm{s} \Rightarrow v_\mathrm{s} = \frac{0.0119\times340}{2.0119} = 2.0\,\mathrm{ms}^{-1}$.

9.10 The received wavelength is longer than that emitted and so the galaxy is moving away.

Using $v = \frac{c\Delta\lambda}{\lambda}$, the speed

is $v = \frac{3.00\times10^8\times(720-658)}{658} = 2.8\times10^7\,\mathrm{ms}^{-1}$.

Chapter 10

10.1 Since the gravitational potential is the sum of the potentials from each mass separately, and since it is negative, we must choose the point that involves the largest distances from the masses, thus **d**.

10.2 All points on the x-axis are equidistant from ± pairs of charges and so the potential due to those charges is zero. This is not true for points along the y-axis, so the answer is B.

10.3 **a** The gravitational potential at the midpoint is $V = -\frac{GM}{d} - \frac{GM}{d} = -\frac{2GM}{d}$ and so the work done to bring the small mass there from infinity is $W = mV = -\frac{2GMm}{d}$.

b The potential at the new position is $V = -\frac{GM}{3d/2} - \frac{GM}{d/2} = -\frac{8GM}{3d}$. Hence the work required is

$$W = m\Delta V = m\left(-\frac{8GM}{3d} - \left(-\frac{2GM}{d}\right)\right) = -\frac{2GMm}{3d}.$$

10.4 The field is directed from high to low potential and it gets stronger as we move to the right. This means that the equipotential lines must get closer as we move to the right. Hence the answer is **d.**

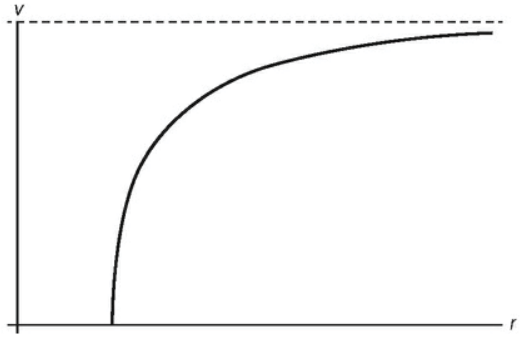
10.5 $E_\mathrm{P} = \frac{9\times10^9\times(1.6\times10^{-19})(-1.6\times10^{-19})}{2\times10^{-10}} = -1.2\times10^{-18}$ J.

10.6 **a** The change in potential is 90 V, so from $\frac{1}{2}mv^2 = q\Delta V$ we get

$$v = \left(\frac{2q\Delta V}{m}\right) = \sqrt{\frac{2\times1.6\times10^{-19}\times90}{1.67\times10^{-27}}} = 1.3\times10^5\ \text{ms}^{-1}.$$

b At infinity the potential is zero and so the change in potential is 180 V, or double the change in **a.** Hence $v_\infty = \sqrt{2}\times1.3\times10^5 = 1.8\times10^5\ \text{ms}^{-1}$.

c



10.7 $E_P = \frac{GMm}{r} = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times3500}{(6400+520)\times10^3} = -2.0\times10^{11}\ \text{J}.$

10.8 The period is one year and so

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} \Rightarrow M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.5\times10^{11})^3}{(6.67\times10^{-11}\times(365\times24\times3600)^2)} = 2.0\times10^{30}\ \text{kg}.$$

10.9 The launch speed is $v = \frac{v_{esc}}{2} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$ and so by energy conservation, with r being the distance the probe gets to, $\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{r}$ (no kinetic energy when the mass stops), or

$$\frac{1}{2}\times\frac{1}{4}\times\frac{2GMm}{R} - \frac{GMm}{R} = -\frac{GMm}{r} \Rightarrow -\frac{GMm}{r} = -\frac{3GM}{4R} \Rightarrow r = \frac{4R}{3}.$$

Chapter 11

11.1 **a** The flux is decreasing and so there is an induced current. As explained for Example 2, the current is counterclockwise. Then there is a magnetic force on the rod directed to the left and so the speed of the rod decreases.

b The loss in kinetic energy of the rod gets transformed into the electrical energy that lights up the lamp.

11.2 The device is probably a coil. When the switch is closed, the current in the coil induces an EMF. By Lenz's law this induced EMF has opposite sign to the battery's EMF, so the current rises only gradually to a final constant value. Since the constant final current is 1.0 A, the resistance of D must be

$$R = \frac{12}{1.0} = 12\,\Omega.$$

11.3 **a** **i** $V_{ms} = \frac{16}{\sqrt{2}} = 11.3 \approx 11\,\text{V}.$

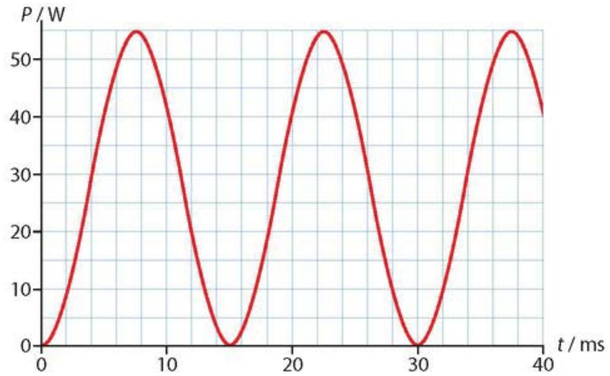
ii $\frac{200}{900} = \frac{16}{N_s} \Rightarrow N_s = 72.$

b **i** The resistance is $R = \frac{V_{ms}}{I_{ms}} = \frac{11.3}{2.4} \approx 4.7\,\Omega.$

ii The average power is $\overline{P} = V_{ms}I_{ms} = \frac{16}{\sqrt{2}}\times2.4 \approx 27\,\text{W}.$

iii The peak power is $2\times\overline{P} = 54\,\text{W}.$

c



11.4 $E=\frac{q_0^2 e^{-\frac{2t}{\tau}}}{2C}$ and so $\frac{1}{2}\frac{q_0^2}{2C}=\frac{q_0^2 e^{-\frac{2t}{\tau}}}{2C}$, giving $\frac{1}{2}=e^{-\frac{2t}{\tau}}$ or $2=\frac{2t}{\tau}$ and hence $t=\frac{\tau\ \ln\ 2}{2}$, answer **c.**

Chapter 12

12.1 **a** At the threshold frequency, $E_k = 0$ and so $hf=\phi \Rightarrow f=\frac{1.8\times1.6\times10^{-19}}{6.63\times10^{-34}}=4.3\times10^{14}\ \text{Hz}$.

b The kinetic energy of the emitted electrons is

$E_k=hf-\phi=6.63\times10^{-34}\times\frac{3.0\times10^{8}}{4.8\times10^{-7}}-1.8\times1.6\times10^{-19}=1.26\times10^{-19}\ \text{J}$. Hence, from $E_k=\frac{1}{2}mv^2$ we find $v=\sqrt{\frac{2\times1.26\times10^{-19}}{9.1\times10^{-31}}}=5.3\times10^{5}\ \text{ms}^{-1}$.

12.2 From $E_k=hf-\phi$, and $E_k = qV$ we find $qV=hf-\phi \Rightarrow V=\frac{h}{q}f-\frac{\phi}{q}$. Hence the vertical intercept is $-\frac{\phi}{q}$ and so $-\frac{\phi}{q}$, giving ϕ = 4.0 eV = 4.0 × 1.6 × 10^{-19} J = 6.4 × 10^{-19} J.

The slope of the graph equals $\frac{h}{q}$ and is measured to be 4.25 × 10^{-15} JsC^{-1}. Hence h = 4.25 × 10^{-15} × 1.6 × 10^{-19} = 6.8 × 10^{-34} Js.

12.3 The kinetic energy of the accelerated electron is $E_k=\frac{p^2}{2m}$ and equals the work done in moving through the potential difference, namely qV. Hence $p=\sqrt{2mqV}$ and so

$$\lambda=\frac{h}{\sqrt{2mqV}}=\frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times250}}=7.8\times10^{-11}\ \text{m}.$$

12.4 To resolve two objects, the wavelength of the light used must be of the same order of magnitude as the separation of the objects. The de Broglie wavelengths of the electrons used in an electron microscopes are much smaller than visible light and so can resolve very small objects.

12.5 **a** A wavefunction is a mathematical function which, when squared and multiplied by a small volume element, gives the probability of finding a particle within that volume element.

b **i** The wavelength is $\lambda=\frac{1.0\times10^{-10}}{2}=0.5\times10^{-10}\ \text{m}$. The momentum is therefore

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}}{0.5\times10^{-10}}=1.3\times10^{-23}\ \text{Ns}.$$

ii The kinetic energy is $E_k=\frac{p^2}{2m}=\frac{p^2}{2m}=\frac{(1.3\times10^{-23})^2}{2\times9.1\times10^{-31}}=9.3\times10^{-17}\ \text{J}$.

c Any point where the wavefunction is zero.

12.6 $\frac{E_{\text{atom}}}{E_{\text{nucleus}}}=10^{-6}=\frac{m_n {L_n}^2}{m_e {L_a}^2}$, so $\frac{L_a}{L_a}=\sqrt{10^6\frac{m_n}{m_e}}\approx\sqrt{10^6\frac{1}{10^{-4}}}\approx10^5$.

12.7 **a** The measurement must be completed in less than 0.1 ns – that is, $\Delta t \approx 10^{-10}$ s. Then

$$\Delta E\approx\frac{h}{4\pi\Delta t}=\frac{6.6\times10^{-34}}{4\pi\times10^{-10}}\approx5\times10^{-25}\ \text{J}.$$

b The answer to **a** means that the energy difference between levels is not precisely known. Hence there will be a range of wavelengths for a given transition rather than a strictly monochromatic photon.

12.8 After acceleration the kinetic energy of the proton is eV, and this is all converted to electrical potential energy when it just stops at the nuclear surface. Hence $eV=\frac{ke(42e)}{R}$, where R is the radius of the nucleus. The radius is $R \approx 1.2\times10^{-15}\times96^{1/3}=5.5\times10^{-15}$ m. Hence,

$$V=\frac{ke(42e)}{eR}=\frac{8.99\times10^{9}\times42\times1.6\times10^{-19}}{5.5\times10^{-15}}\approx11\ \text{MV}.$$

12.9 The decay constant is $\lambda=\frac{\ln\ 2}{2.45}=0.283\ \text{min}^{-1}$.Thus $A=A_0e^{-\lambda t}=A_0e^{-0.283\times1.0}=A_0 0.753$, or about 75% of the original activity.

12.10
$$N_X=N_0e^{-\lambda t}$$
$$N_Y=N_0-N_0e^{-\lambda t}$$
$$\frac{N_X}{N_Y}=\frac{N_0e^{-\lambda t}}{N_0-N_0e^{-\lambda t}}=\frac{e^{-\lambda t}}{1-e^{-\lambda t}}=\frac{1}{4}$$

Hence

$$4e^{-\lambda t}=1-e^{-\lambda t}\Rightarrow5e^{-\lambda t}=1\Rightarrow\frac{1}{5}\Rightarrow\lambda t=\ln\ 5$$
$$t=\frac{\ln\ 5}{\lambda}=\frac{\ln\ 5}{\ln\ 2}\times2.0=4.6\ \text{min}$$

12.11 For X we go to half the initial activity and find a time of 0.35 min. For Y we must wait until most of X has decayed away so that no new Y nuclei are produced. Looking at how long it takes for the activity to halve (after 2.5 min) we find a Y half-life of about 0.65 min.

Option A

A.1 Light will cover the distance $\Delta x = 6 \times 10^8 - 3 \times 10^8 = 3 \times 10$ m in a time of $\frac{\Delta x}{c} = \frac{3\times10^8}{3\times10^8} = 1\,\text{s}$.

The event 'lightning strikes' thus occurred at $t = 3 - 1 = 2$s, at position $x = 3 \times 10^8$m. **All** observers in frame S agree that these are the coordinates of the event 'lightning strikes'.

A.2 Take the ground as frame S, and A as frame S′. Then $v = 0.80c$ and $u = -0.40c$, and we need to find u':

$u' = \frac{u-v}{1-\frac{uv}{c^2}} = \frac{-0.40c-0.80c}{1-\frac{(-0.40c)(0.80c)}{c^2}} = \frac{1.20c}{1+0.32} = -0.91c$. The speed of B relative to A is $0.91c$.

A.3 The two strikes are simultaneous in frame S (i.e. $\Delta t = 0$), and $\Delta x = 900$ m. The time separating these events in frame S′ is $\Delta t' = t_2{}' - t_1{}' = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = \gamma(0 - \frac{v}{c^2}\Delta x) = -\frac{5}{3} \times \frac{0.80c}{c^2} \times 900 = -4.0\,\mu\text{s}$.

A.4 The total energy is $E = \sqrt{938^2 = 2800^2} = 2953\,\text{MeV}$. Hence the gamma factor is

$\gamma = \frac{2953}{938} = 3.148$. So $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3.148$, and $1 - \frac{v^2}{c^2} = \frac{1}{3.148^2} \Rightarrow \frac{v}{c} = \sqrt{1 - \frac{1}{3.148^2}} = 0.948$.

A.5 The gamma factor is $\frac{1}{\sqrt{1-0.98^2}} = 5.025$ and so the total energy after acceleration is

$E = \gamma mc^2 = 5.025 \times 0.511 = 2.568$ MeV. The kinetic energy therefore is

$E_k = (2.568 - 0.511) = 2.057$ MeV and thus the voltage is $V = 2.057 \approx 2.06$ MV.

A.6 The gamma factor is $\frac{1}{\sqrt{1-0.95^2}} = 2.203$. The total energy is thus

$E = \gamma mc^2 = 3.203 \times 938 = 3004$ MeV and so the kinetic energy is

$E_k = (3004 - 938) = 2066$ MeV.

A.7 Use $E^2 = (mc^2)^2 + p^2c^2$ to get $540^2 = 106^2 + p^2c^2 \Rightarrow p = 529 \approx 530\ \text{MeV}c^{-1}$.

A.8 Since $E = \gamma mc^2$ the gamma factor is $\gamma = \frac{6500}{1800} = 3.61$. Hence $1/\sqrt{1 - \frac{v^2}{c^2}} = 3.61$, so $1 - \frac{v^2}{c^2} = \frac{1}{3.61^2} \Rightarrow \frac{v^2}{c^2} = 0.923 \Rightarrow \frac{v}{c} = 0.961$.

A.9 **a** Combining $E = \gamma mc^2$ and $p = \gamma mv$ we find $p = \frac{\gamma mc^2 v}{c^2} = \frac{Ev}{c^2} \Rightarrow v = \frac{pc^2}{E}$. For a massless particle, $E = pc$ and so $v = \frac{pc^2}{pc} = c$.

b $v = \frac{(640\ \text{MeV c}^{-1})c^2}{654\ \text{MeV}} = 0.979c$.

c $mc^2 = \sqrt{E^2 - p^2c^2} = \sqrt{654^2 - 640^2 = 135\,\text{MeV}}$, so $m = 135\ \text{MeV c}^{-2}$.

A.10 Before acceleration, $E_0 = mc^2$. After acceleration, $E = \gamma mc^2$. Hence the change is $\Delta E = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$ and this equals $qV = 3.2 \times 10^9$ eV = 3200 MeV. So, $(\gamma - 1) = \frac{3200}{938} \Rightarrow \gamma = 4.41$. Hence $4.41 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1 - \frac{1}{4.41^2}} = 0.97$, and $p = \gamma mv = 4.41 \times 938 \times 0.97 = 4.0 \times 10^3\ \text{MeV c}^{-1}$. This can also be expressed as $p = 4.0\ \text{GeV c}^{-1}$.

A.11 The best way to proceed is to find the total energy of the proton through $E^2 = (mc^2) + p^2c^2$; thus, $E = \sqrt{0.938^2 + 2.40} = 2.58\,\text{GeV}$. Then use $E = \gamma mc^2$ to find $\gamma = \frac{2.58}{0.938} = 2.75$ and finally $2.75 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1 - \frac{1}{2.75^2}} = 0.93$.

A.12 By the equivalence principle, the situation is equivalent to a rocket at rest on the surface of a massive body. In this case we know that the ray of light will bend towards the massive body and so the ray will follow a curved, downward path. It will therefore hit the opposite side of the rocket at a height less than h.

A.13 Working backwards from the previous example, the situation is equivalent to a rocket accelerating in outer space with an acceleration 9.8 m s^{-2}. The time for light to cross the distance of 300 m is $t = \frac{s}{c} = \frac{300}{3\times10^8} = 10^{-6}\,\text{s}$. In this time the rocket has moved up a distance $x = \frac{1}{2}at^2 = \frac{1}{2} \times 9.8 \times (10^{-6})^2 \approx 5 \times 10^{-12}\,\text{m}$ (!).

A.14 Newton would claim that the path of the planet is the result of the gravitational force between the Sun and the planet. Einstein would claim that the path is due to the curvature of spacetime: the planet follows a geodesic – that is, a path of least length – in the curved spacetime around the massive Sun.

A.15 We have that $5 = \frac{1}{\sqrt{1-\frac{R_S}{r}}}$ and so $r = \frac{25}{24} R_S = 5.2 \times 10^8$ m. Hence the distance h from the event horizon is $h = \frac{25}{24} R_S - R_S = \frac{R_S}{24} = 2.1 \times 10^7$ m.

Option B

B.1 Using $\omega^2 = \omega^2 + 2\alpha\theta$ we find $0 = 36^2 - 2 \times 0.54 \times \theta$ and so $\theta = 1200$ rad or $\frac{1200}{2\pi} = 190.986 \approx 191$ full revolutions.

B.2 The net force is zero and the net torque is $FR + FR = 2FR$.

B.3 The net force on the cylinder in the direction of the plane is $Mg \sin\theta - f$ and so

$$F_{net} = Ma$$

$$Mg \sin\theta - f = Ma$$

The net torque about a horizontal axis through the centre of mass is fR (N and Mg have zero torques about this point) and so, because the moment of inertia of a cylinder about its axis is $I = \frac{1}{2} MR^2$:

$$\Gamma = I\alpha$$
$$fR = \frac{1}{2} MR^2 \alpha.$$

We assume rolling without slipping and so $\alpha = \frac{a}{R}$. So our two equations become

$$\begin{aligned} Mg \sin\theta - f &= Ma \\ fR &= \frac{1}{2} MR^2 \frac{a}{R} \end{aligned} \quad \text{or} \quad \begin{aligned} Mg \sin\theta - f &= Ma \\ f &= \frac{1}{2} Ma \end{aligned}.$$

and so $Mg \sin\theta - \frac{1}{2} Ma = Ma$, giving $a = \frac{2g \sin\theta}{3}$.

B.4 Applying angular momentum conservation, $\frac{2}{5} MR^2 \omega = \frac{2}{5} \frac{M}{50} \left(\frac{R}{200}\right)^2 \omega'$, where ω' is the angular velocity after the explosion. Hence $\omega' = 50 \times 200^2 = 2.0 \times 10^6 \omega'$. The number of revolutions per second is then $\frac{2.0 \times 10^6}{24 \times 60 \times 60} \approx 23\ \text{s}^{-1}$.

B.5 Use $\Delta U = \frac{3}{2} V \Delta p$ to find $\Delta U = \frac{3}{2} \times 3.1 \times 10^{-3} \times (4.6 - 2.2) \times 10^5 = 1.1 \times 10^3$ J.

B.6 **a** From $\frac{p_1 V_1}{n_1 T_1} = \frac{p_2 V_2}{n_2 T_2}$ we see that $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ and so the new temperature is $T_2 = \frac{V_2}{V_1} T_1 = \frac{5.0 \times 10^{-3}}{2.0 \times 10^{-3}} \times 300 = 750\ \text{K}$. The pressure is constant so we may calculate the work done from $W = p\Delta V = 4.0 \times 10^5 \times 3.0 \times 10^{-3} = 1.2 \times 10^3$ J.

b Since the temperature has increased, $\Delta U > 0$, and the gas has expanded. So from $Q = \Delta U + W$ we see that $W > 0$, so $Q > 0$. Thermal energy has been supplied to the gas.

B.7 **a** The internal energy of an ideal gas is the total random kinetic energy E_k of the molecules of the gas.

b The absolute temperature is proportional to the *average* kinetic energy of the molecules, $T \propto \frac{E_k}{N}$.

Therefore the internal energy is proportional to the absolute temperature (and the number of molecules): $U = E_k \propto = NT$.

B.8 The piston, moving rapidly inwards, collides with molecules and the molecules rebound with a speed greater than before. The average kinetic energy of the molecules increases, and hence so does the temperature.

B.9 At constant volume, we have $Q = \Delta U$ since no work is being done. At constant pressure, $Q = \Delta U' + W$, with $W > 0$. Hence the change in internal energy is larger in the constant-volume case, and there we have the larger temperature increase.

B.10 **a** AB is the isothermal since it is less steep than the adiabatic CA.

b BC, because the work done is negative (on the gas) and the temperature drops so that $\Delta U < 0$. Since $Q = \Delta U + W$ we have $Q < 0$.

c Work is the area under the curve in a pressure–volume diagram. This is largest for the isothermal expansion.

B.11 Applying $Q = \Delta U + W$ to the *entire cycle*, we see that $\Delta U = 0$. The total work done is the area within the loop, and this is positive. The total thermal energy transferred is $Q_1 - Q_2$. Hence $Q_1 - Q_2 = W$ and so Q_1 is larger.

B.12 The temperature at B is higher than that at A so $\Delta U > 0$. Work is done by the gas since it is expanding and so $W > 0$. Hence $Q > 0$.

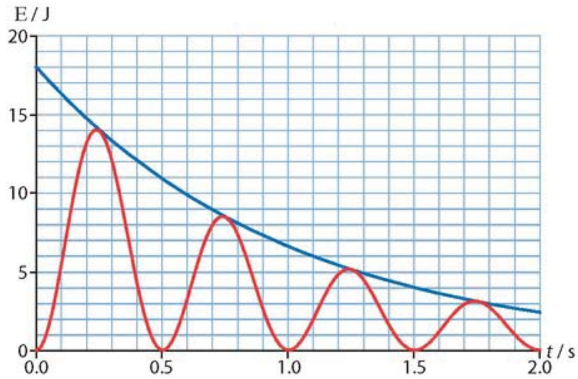
The temperature at D is lower than that at C and so $\Delta U < 0$. The volume stays constant and so $W = 0$. Hence $Q < 0$.

B.13 The efficiency is $1 - \frac{400}{600} = \frac{1}{3}$. Hence the work done is $1200 \times \frac{1}{3} = 400$ J.

B.14 We have equilibrium and so $mg = \rho_{\text{fluid}} g V_{\text{imm}}$ or $\rho_{\text{body}}\, gV = \rho_{\text{fluid}} g V_{\text{imm}}$, so $\frac{V_{\text{imm}}}{V} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} = \frac{850}{950} = 0.89$.

B.15 Applying Bernoulli's equation, we find $p_{\text{atm}} + 0 + \rho gh = p_{\text{atm}} + \frac{1}{2}\rho v^2 + 0$. Hence $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.5} = 5.42\,\text{ms}^{-1}$. Hence $Q = Av = \pi r^2 v = \pi \times (2.0 \times 10^{-2})^2 \times 5.42 = 6.8 \times 10^{-3}\,\text{m}^3\,\text{s}^{-1}$.

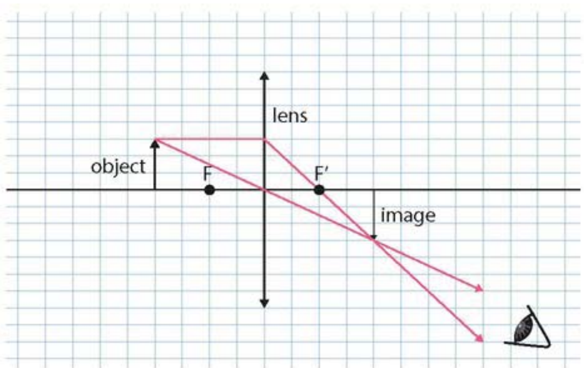
B.16



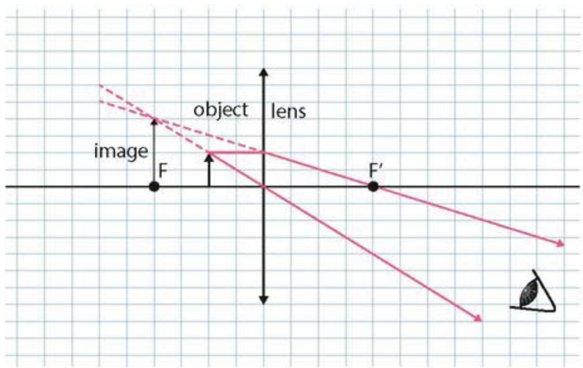
Option C

C.1 $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{6.0} - \frac{1}{8.0} = \frac{1}{24} \Rightarrow v = 24\,\text{cm}.\ M = -\frac{24}{8.0} = -3.0$. The image is real, inverted and 9.0 cm high.

C.2



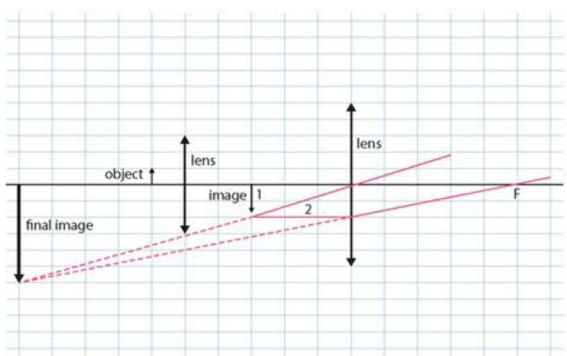
C.3



C.4 Since half the amount of light now goes through the lens the image formed will be half as bright, but there will be no other changes to the image.

C.5 From the top of the first image, draw line 1 to the right through the centre of the second lens (it continues without change of direction), and also extend it to the left. Draw line 2 parallel to the principal axis to the second lens (from where it continues through the focus of the second lens), and also extend this to the left. Where these two lines intersect at the left is the final (virtual) image.

For a look at the mathematics of the compound microscope, see Section C.2.



C.6 $-16 = 10\log\frac{P_{\text{final}}}{P_{\text{initial}}} \Rightarrow -1.6 = \log\frac{P_{\text{final}}}{8.0} \Rightarrow \frac{P_{\text{final}}}{8.0} = 10^{-1.6} \approx \frac{1}{40}$. So

$P_{\text{final}} \approx \frac{1}{40}P_{\text{initial}} \Rightarrow P_{\text{final}} = \frac{8.0}{40}\,\text{mW}{=}0.20\,\text{mW}$.

C.7 The *loss* is 3.0 × 12 = 36 dB. Then

$-36 = 10\log\frac{P_{\text{out}}}{P_{\text{in}}}, \frac{P_{\text{out}}}{P_{\text{in}}} = 10^{-3.6}, P_{\text{out}} \approx 180 \times 2.51 \times 10^{-4} = 0.045\,\text{mW}$.

C.8 $G = 10\log\frac{P_{\text{out}}}{P_{\text{in}}} = 10^{G/10}$, $P_{\text{out}} = P_{in}\,10^{G/10} = 0.25 \times 10^{6.8/10} = 1.2\text{mW}$.

C.9 The total power loss without any amplifiers would be −4.0 × 400 = −1600 dB. The net gain of the amplifiers must then be +1600 dB, and so we need $\frac{1600}{20} = 80$ amplifiers.

C.10 **a** The attenuation after a distance of 3.0 km is 3.0 X 14 = 42 dB. The power is then found from $-42 = 10\log\left(\frac{P}{200}\right), \log\left(\frac{P}{200}\right) = -4.2, \frac{P}{200} = 10^{-4.2} \approx 6.3 \times 10^{-5}, P = 0.013\,\text{mW}$.

b Attenuation in coaxial cables is very frequency-dependent. There is much more attenuation at higher frequencies.

C.11 $I = I_0 e^{-\mu x} = I_0 e^{-0.52\times 1.2} = 0.54 I_0$.

C.12 The photoelectric effect is more useful, since we need to have contrast between, for example, tissue and bone that have different atomic numbers.

C.13 The transducer acts as both emitter and receiver. The emissions must stop so that the reflected ultrasound may be detected.

Option D

D. 1 This is a ratio problem. Write $L_A = \sigma A_A(5000)^4$ and $L_B = \sigma A_B(25\ 000)^4$. Take ratios side by side to get $\frac{L_A}{L_A} = \frac{\sigma A_A(5000)^4}{\sigma A_B(25000)^4} \Rightarrow 36 = \frac{A_A}{A_B}\left(\frac{1}{5}\right)^4 \Rightarrow \frac{A_A}{A_B} = 22500$. Since area is related to radius by $A = 4\pi R^2$, it follows that $\frac{R_A}{R_B} = \sqrt{22500} = 150$.

D.2 The wavelength is found from $\lambda_0 = \frac{2.9\times 10^{-3}}{3000} = 9.7 \times 10^{-7}\,\text{m}$ and so the star will appear reddish. (The peak wavelength is in the infrared but the left end of the black-body curve is mainly over red wavelengths.)

D.3 Achernar and EG129 have the same temperature but Achernar has a much higher luminosity so it must have a very large area: approximately,

$\frac{L_A}{L_E} \approx \frac{10^4}{10^{-3}} = \frac{\sigma A_A T^4}{\sigma A_E T^4} = \frac{A_A}{A_E} \Rightarrow \frac{A_A}{A_E} \approx 10^7 \Rightarrow \frac{R_A}{R_E} \approx 3 \times 10^3$.

Achernar is a blue giant and EG129 a white dwarf; so Achernar is much larger.

D.4 From the HR diagram, its luminosity is about 10^{-4} X L_Q = 3.8 X 10^{22} W. Hence

$b = \frac{L}{4\pi d^2} = \frac{3.8\times 10^{22}}{4\pi\times(6.0\times 9.46\times 10^{15})^2} = 9.4 \times 10^{-13}\,\text{W m}^{-2}$.

D.5 If it were a main-sequence star, its luminosity would have to be $50^{3.5} \approx 9 \times 10^5$ times larger than the luminosity of the Sun. So it not a main-sequence star.

D.6 One piece of evidence is the existence of the cosmic microwave background (CMB) radiation. This is radiation that existed when the universe was very young. As the universe has expanded, its original high temperature has fallen to about 2.7 K, shifitng the peak wavelength of this radiation to its present value. A second piece of evidence is the expansion of the universe according to Hubble's law. If the distance between galaxies is getting larger, then in the past the distance must have been smaller, indicating a violent start of the universe from a small region.

D.7 From $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$, the speed of the receding galaxy is $\frac{689.1-656.3}{656.3} = \frac{v}{c} \Rightarrow v = 1.5 \times 10^4\,\text{kms}^{-1}$.

From $v = Hd$, $1.5 \times 10^4 = 68d \Rightarrow d \approx 220$ Mpc.

D.8 **a** $T = \frac{1}{H} = \frac{1}{68\,\text{kms}^{-1}\text{Mpc}^{-1}} = \frac{\text{Mpc}}{68\times 10^3\,\text{m}}\text{s} = \frac{10^6\times 3.09\times 10^{16}\text{m}}{38\times 10^3\,\text{m}}\text{s}$.

$= 4.54 \times 10^{17}\text{s} = \frac{4.54\times 10^{17}}{365\times 24\times 60\times 60} = 14.4$ billion years

b This estimate is based on a *constant* rate of expansion equal to its *present* value. In the past the universe expanded faster and so the universe is younger than the estimate in **a.**

D.9 It cannot have an edge, for if it did, the part of the universe near the edge would look different from a part far from the edge, violating the homogeneity principle. It cannot have a centre, for if it did, observing from the centre would show a different picture from observing from any other point, violating the principle of isotropy.

D.10 $z = 3 = \frac{R}{R_0} - 1 = \frac{T_0}{T} - 1 \Rightarrow \frac{T_0}{T} = 4.$

D.11 $z = 2 = \frac{T_0}{T} - 1 \Rightarrow \frac{T_0}{T} = 3 \Rightarrow T_0 = 3T = 3 \times 2.7 = 8.1\,\mathrm{K}.$

D.12 The critical density is the density to which the present matter and energy densities of the universe must be compared, in order to determine its future evolution. If the *combined matter and energy density* is p then the geometry of the universe is flat if $\rho = \rho_c$, curved like a sphere if $\rho > \rho_c$ and curved like a saddle if $\rho < \rho_c$).

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ISBN 978-1-107-49575-3